Optimization of Vehicle Structure Considering Torsion Stiffness Using Simple Structural Beam Frame-Approach

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ABSTRACT

Vehicle structural design is an important component of automotive design since the structure of a vehicle plays complex interactions with the other vehicle components and has significant impact on the performance of the vehicle. Structural design is usually completed after many iterations and the design changes in the late design stages effect many other parameters in the design of vehicle. Therefore, it is highly valuable for designers to employ simple but effective analyses at the early design stages. One method of analysis is using Simple Structural Surfaces. This method utilizes planar sheets to model the vehicle structure and allows the determination of the forces in each sheet. The major drawback of this method is its inability to determine deflections in a structure. To overcome this drawback a method that uses beam elements to represent the vehicle structure has been developed. This method uses a numerical finite element method and is able to determine unknown deflections and reaction forces as well as the internal loading on each member. This method can also be readily adapted to allow for parametric optimization for torsion stiffness. The parameters associated with each beam element are the length, orientation and the beam characteristics of beams' cross-sectional area and moment of inertia. An automated process is developed that manipulates some of these parameters to develop a structure that will have the greatest torsional stiffness.

Keywords: stiffness matrix method, automotive structure, FEA, beam-frame model.
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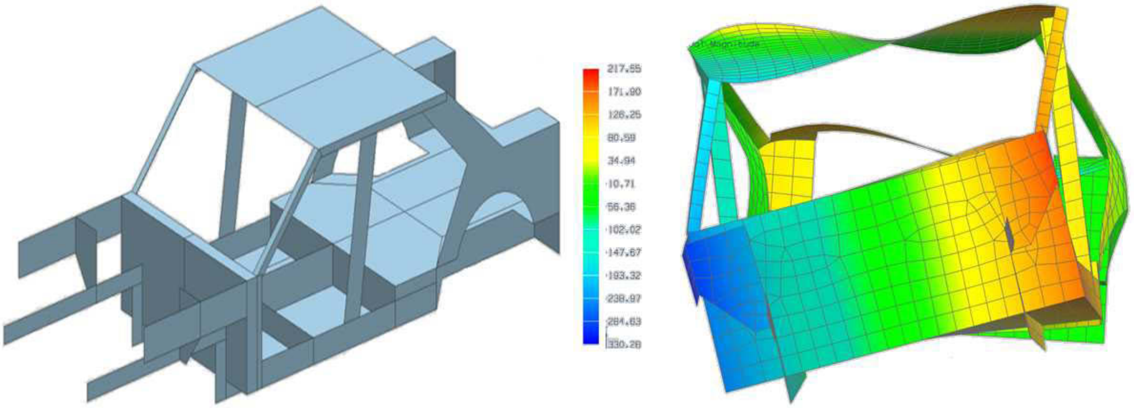
1 INTRODUCTION

The design of an automotive structure is critical to the overall performance of a vehicle. The structure of the vehicle is important to ensure it can satisfactorily carry the applied loads that occur [1]. The structure of a vehicle interacts with all other vehicle sub-components and it has complex influence one their functionality. Due to its design complexity, the structural design process is traditionally conducted by trial and errors and is subject to too many changes even till the latest stages of vehicle design. However, some of the changes in the design of structure may cause significant re-design of the other vehicle components and this may become very costly. Typically, it is much more desired to maximize design changes during the early design stages and particularly before the detail design activities [2]. However, employing a very comprehensive and detailed process of analyses at the conceptual design stage, when there are long ranges of design choices still available, may become very time consuming and computationally expensive. Therefore, it is very valuable for designers to employ simple but effective analyses at the early design stages. The objective of this paper is to present a method of analyzing a vehicle structural model and implement an optimization process to improve the structural design. A simplified model is used to test the analysis and optimization processes which reduces the accuracy but can be used to test the implemented methods.

One of the most important criteria in automotive structural design is structural stiffness. The chassis stiffness, both in bending and torsion, has significant impacts on the ride and comfort characteristics as well as the overall dynamic vehicle performance [3], [4], [5]. For this reason the stiffness values are used as design parameters to be optimized. Increasing the structural stiffness is highly demanding to enhance the vehicle performance. However, due to economical constrains increasing vehicle stiffness by increasing the structural weight is not recommended. An optimised solution is desired that maximizes structural stiffness while it keep the structural weight as low as possible.

Being able to efficiently analyze the body structure during the conceptual design stages is important to determining the performance characteristics. A primary method used to analyze the structure is the Simple Structural Surfaces (SSS) [1], [6]. This method utilizes planar sheets to model the body structure. SSS method can be used to determine the load-paths present in a body structure, but it is not able to analyze indeterminate static conditions. Alternatively, the method utilized in this work, uses beam-frame elements to represent the structure as an equivalent space frame. The approach of using beam elements has the advantage of being able to determine displacements due to these forces by using the Finite Element Method (FEM). The use of the beam-frame finite element model can be used for basic analysis of a vehicle structure and as an initial estimate of some important vehicle parameters such as bending and torsion stiffness as well as some vibration characteristics. Using analogy of names, this method is referred in this paper as Simple Structural Beam-Frames (SSB) method. This paper presents an approach to optimize design parameters of a SSB model to optimize the torsion stiffness of a conceptual model. The optimization of the model will improve the stiffness to weight ratio compared with an initial model that has been used in previous analysis.

An SSS model has been previously analyzed using commercial finite element software [6]. A diagram of this geometry is shown below in Fig. 1(a). The deflection results that were produced using this model are shown in Fig. 1(b).



(a) (b)

Fig. 1: (a) SSS Model (b) Deflection Result of SSS Model [6].

2 BACKGROUND

A simpler model is often utilized since the finite element method can be computationally intense with a trade-off between accuracy and computation time [7]. Both SSS and SSB methods can be used to determine forces that are present throughout the structure and assist with preliminary design decisions. These methods are still used, despite their limitations, as they can provide insight into how the initial geometry of the structure interacts for different loading conditions. A brief explanation of both of these analysis methods is presented here as background information. Another important aspect of this structural analysis is the utilized finite element method that is also briefly presented here.

The Simple Structural Surface method uses planar surfaces to model a structure. It was developed initially to analyze the load path of a vehicle [1]. The surfaces are able to react in plane loads only and transfer the forces from one surface to another via edge shear loads. The original intent of this method is to analyze the structure and determine a suitable load path. This method of analysis has a few limitations which restrict the benefits however as an initial estimation before the development of improved techniques it is sufficient. One of the major limitations of this method is that it cannot analyze structures with redundancy which occurs regularly in automotive vehicles. This requires that the structure to be statically determinant throughout. This may not be able to fully represent the structure and simplifications may be required. The second major limitation is this method does not have the capacity to determine deflections that will occur due to different loading conditions. This disadvantage prevents the method from significantly contributing to the design process since it doesn't allow an initial analytical estimation of some important design parameters such as stiffness. Overall the SSS method is only of interest as part of early automotive structural design and has been replaced by improved models that allow for a greater range of analysis such as the SSB method presented here.

In order to overcome some of the limitations of the SSS method the SSB method is employed in this work. A beam-frame uses beam elements to model the structure of the vehicle [8]. An example of simple beam frame model is shown in Fig. 2. The beam frame model was developed primarily because it can be easily implemented in the Finite Element Method. This method allows the determination of the deflection of the vehicle based on applied loading conditions. Once the deflections have been found, it is possible to determine torsion stiffness of the chassis. This method neglects the sheet components that occur in a structure however where necessary an extra beam element is implemented in the model to account for missing sheets [9]. The beam frame model also has the added flexibility of allowing for optimization of the design by improving the cross-section type and dimensions [10]. Finally the beam element model allows for the determination of the vibration characteristics [11].

More complete models have been developed that utilize plate and shell elements to more accurately model the vehicle structure [9]. However, their application may become computationally too expensive for an optimization process when there are too many design variables. And typically this is the case during the early stages of the design process. It is more appropriate to use a simplified conceptual model during the early optimization process to roughly select values for majority of the structural design parameters and then use the more accurate models for a few more important parameters and the final tuning during detail design. The SSB method presented here is a trade-off between accuracy and time, and is sufficient for the purposes of preliminary design estimation of majority of the design parameters.

The finite element solver developed for the SSB method uses typical beam elements with linear shape functions and Galerkin's Method is used for deriving the beam element equations [12]. The method divides the structure into nodes and beams (elements). Nodes occur wherever elements intersect and are associated with the degrees of freedom. The nodes for the beam element each have six degrees of freedom, three in translation along each axis and three for rotation about each axis. Each individual beam element will have a corresponding stiffness matrix that relates the element forces with the nodal displacements. The beam element's stiffness matrix employed in this work is as follows:

$$K^e=\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\
0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 & 0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & 0 \\
0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\
\frac{-EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{-6EI_z}{L^2} \\
0 & 0 & \frac{-12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{12EI_z}{L^3} & 0 \\
0 & 0 & 0 & \frac{-GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & 0 & \frac{-6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{12EI_z}{L^3} & 0 & \frac{4EI_y}{L} & 0 \\
0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & \frac{-6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L}
\end{bmatrix} \tag{1}$$

In the above equation A is the cross-sectional area, I_z and I_y are the moments of inertia about the local z and y axes respectively, J is the polar moment of inertia, E is Young's modulus and is equal to 206×10^9Pa G is the shear modulus and is equal to 79.8×10^9Pa. As Eqn. 1 is standard for the beam element analysis the equations to calculate the element stress and strain are also standard for this element type and thus not shown here [12].

The overall structure will have a global stiffness matrix that is a combination of individual stiffness matrices. The solution procedure is called the stiffness method where the displacements are unknown and related to the global forces by the stiffness matrix. The stiffness method is the most common solution method and is used in commercial finite element solvers. The FEM used here is a system of linear equations that can be solved using the developed computer program and implemented iteratively for the optimization process.

Optimization is one of the oldest fields in mathematics and has found modern application in a variety of scientific and engineering disciplines [13]. Most optimization methods are based on principles from calculus and have a strong connection to inequalities. A number of algorithms can be applied depending on the objective of optimization and what constraints exist. Generally an optimization program requires the definition of an objective function to be optimized by varying the parameters associated with the objective [14]. The constraints are applied to the optimization parameters since the parameters can be interrelated by physical laws or must be constrained to ensure physical compatibility, or to simplify the model. A problem that has no inequality constraints is said to be unconstrained, however there will still be bounds on the parameter values. Some examples of available optimization algorithms are simplex method, sequential quadratic programming and interior point methods. Optimization is applied to the design of the SSB model in order to improve the torsion stiffness to weight ratio. The stiffness can be calculated based on the following equation.

$$K_T = \frac{T}{\varphi} = \frac{FB}{(\varphi_d + \varphi_p)} \tag{2}$$

Where T is the torque being applied on the chassis, the force applied at the front nodes is represented by F, and B represents the track width of the structure, the angle of rotation of the passenger side is given by φ_p while the driver side angle of rotation is given by φ_d. These values are calculated using the following equations where δ_d will be the vertical deflection of the driver side and similarly δ_p is the vertical deflection of the passenger side:

$$\varphi_d = \tan^{-1}\left(\frac{\delta_d}{B/2}\right) \tag{3}$$

$$\varphi_p = \tan^{-1}\left(\frac{\delta_p}{B/2}\right) \tag{4}$$

3 METHODOLOGY

The analysis and optimization of the beam-frame structure is a multi-step process. The first step is to determine appropriate loads to be applied to the structure. This is done by utilizing the existing analysis of a vehicle model based on the SSS method [6]. The SSB model is shown below, in Fig. 2, as it would appear in a commercial solver.

The loads that are applied are based on assumed weights for the vehicle components such as the drivetrain and passengers as well as the weight of the structure itself. The initial weight of the beam frame model is not known so the weight of the structure is based on the SSS model. Based on the initial applied loads and target torsion stiffness the initial radiuses can be determined using the following equation.

$$K_{T_{Target}} = K_{T_{SSB}} \tag{5}$$

In the above equation K_T is the torsion stiffness of the target value and the torsion stiffness of the initial SSB model respectively. Using Equation 1, a set of radius values, $R_{Original}$ which is of the form of a vector shown below, will be found. Only solid circular cross-sections are considered throughout the process.

$$R_{Original}=[r_1 \quad r_2 \dots \quad r_n] \tag{6}$$

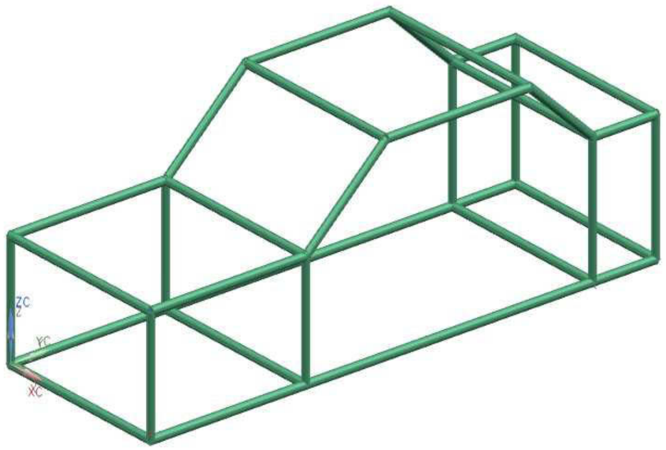


Fig. 2: Beam element model.

In the above equation n is number of unique elements and r refers to radius of each of these elements. A unique element is any element that can potentially have a different radius from all other elements. For the initial analysis and estimation these values are assumed to be uniform for all elements. Before the optimization process the initial results are validated by comparing them with the results developed by commercial finite element software. After developing the initial estimates and validation of results the developed numerical method is used to generate data for different combinations of radius values. The generated data is used to estimate an empirical model for stiffness and to estimate a suitable initial condition for the optimization process. The data generation is conducted based on a full factorial design of experiments in terms of radius and elements. There are m levels of radius that are determined based on the initial estimates and n unique elements. The total number of trials is therefore $N= m^n$ trials.

The next step in the process is filtering the data to reduce the number of data points to be utilized in the model generation process. Before conducting the filtering process, the data is sorted in ascending order based on the weight. The filtering process is a multistep process. The first filter excludes data based on the equation below.

$$[T'] = [T_i \quad i \in [1 \dots N] \ \wedge \ W^i_{SSB} < \alpha_1 W_{Original}] \tag{7}$$

In the above equation T_i represents the data associated with the i^{th} trial. If the weight of that trial is less than the acceptable weight the data is stored in the matrix T'. The acceptable weight can be considered as the original weight multiplied by a factor, α_1, which represents an increase or decrease over the original weight. This filter will reduce the amount of trials stored from N to N'. The next step in the filtering process is described by the following equation.

$$[T''] = [T_i' \quad i \in [1 \dots N'] \ \wedge \ K^i_{T_{SSB}} > \alpha_2 K_{T_{Original}}] \tag{8}$$

In the above equation T_i' represents the data associated with the i^{th} trial of the previously filtered date. If the torsion stiffness of that trial is larger than the acceptable stiffness the data is stored in the matrix T”. The acceptable target stiffness can be considered as the original stiffness multiplied by a factor, α_2, which represents an increase or decrease over the original stiffness. This filter will reduce the amount of trials stored from N’ to N’’. The next step in the filtering process is described by the following equation.

$$[T'''] = [T_i''\ \ i \in [1 \ldots N'']\ \ \wedge\ \ \frac{K^i_{TSSB}}{W^i_{SSB}} > \frac{\alpha_2 K^i_{TOriginal}}{\alpha_1 W^i_{Original}}] \tag{9}$$

The final filter also sorts the data according to the ratio between the stiffness and weight of the i^{th} iteration to find the best trial as the initial condition for the optimization process. The filtering process can be illustrated graphically with the following flow chart.

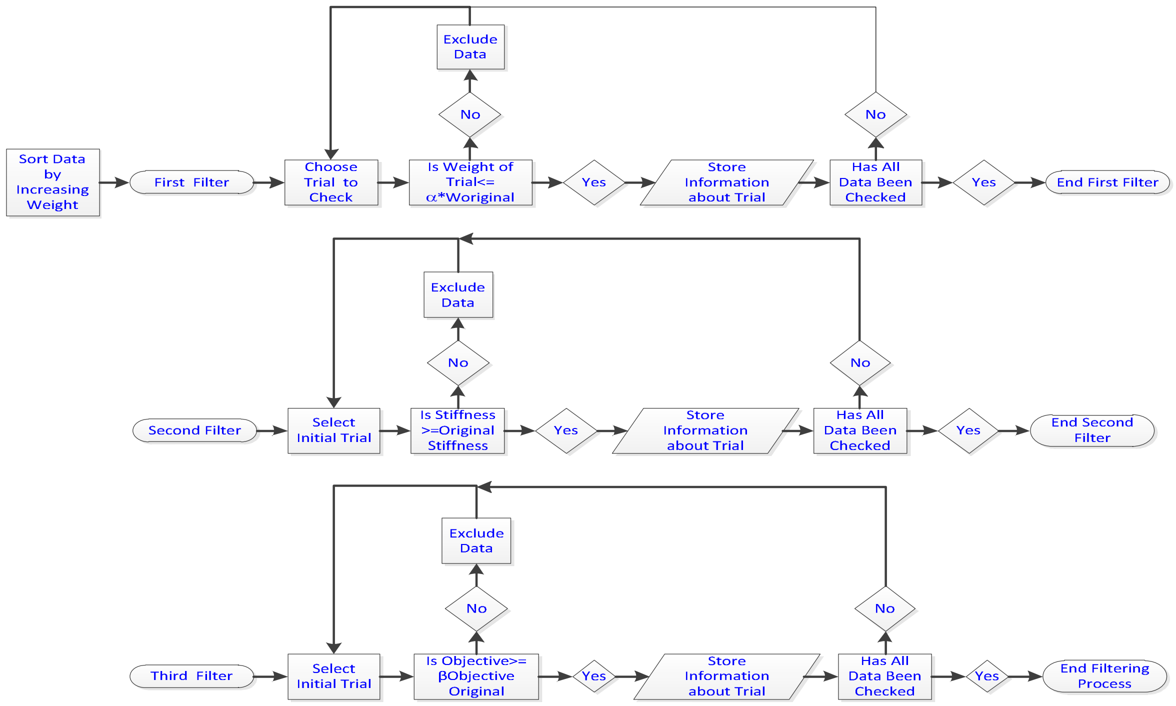


Fig. 3: Flow chart of filtering process.

The filtered data is used to form a non-linear model in terms of stiffness and weight. The model selected for the stiffness is a quadratic form of the square of radiuses (fourth order). The model contains quadratic components, their interactions, and forth order components since the elements of stiffness matrix in Equation 1 only include second and forth orders of r based on the following equations:

$$A = \pi r^2 \tag{10}$$

$$I_y = I_z = \frac{\pi r^4}{4} \tag{11}$$

$$J = I_y + I_z = \frac{\pi r^4}{2} \tag{12}$$

In the above equation r is the element radius, A is the area, I_z and I_y are the moments of inertia and J is the polar moment of inertia. The desired format for the non-linear model of the torsion stiffness is shown below.

$$K_T = \beta_1 + \sum_{i=1}^{n} \beta_{i+1} r_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{\frac{(2n-i+1)i}{2}+j+1} r_i^2 r_j^2 + \sum_{i=1}^{n} \beta_{1+n+\frac{n(n-i)}{2}+i} r_i^4 \tag{13}$$

The coefficients, β, are the coefficients that fit the non-linear model for stiffness and n is the number of unique elements as described above. The weight is a function of square of the radius and an incomplete second order model is used for the weight. The format of the equation is shown below.

$$W=\gamma_1+\sum_{i=1}^{n}\gamma_{i+1}r_i^2 \tag{14}$$

The coefficient, γ, is the coefficient that fits the non-linear model for weight and n is again the number of unique elements.

The optimization process is used to determine the radius values that will give the largest ratio between stiffness and weight. The optimization process uses a constrained multi-function process that utilizes four different algorithms. The algorithms used for the process are interior point, SQP, active set and trust region reflective [15-23]. The process is a constrained optimization that attempts to minimize a non-linear multivariable function. The variables that can be adjusted are the 'n' unique solid radius values. The initial step in the optimization uses the set of radius values that were found to give the largest ratio between torsion stiffness and structure weight based on the data that was generated and filtered. The radius values are restricted based on the results of the data generation and filtering process. The restrictions provide an upper bound and lower bound for the radius of each unique element. These restrictions ensure that the radius values being tested are feasible and that the design will have good compatibility between different element members. The optimization process can be summarized as follows.

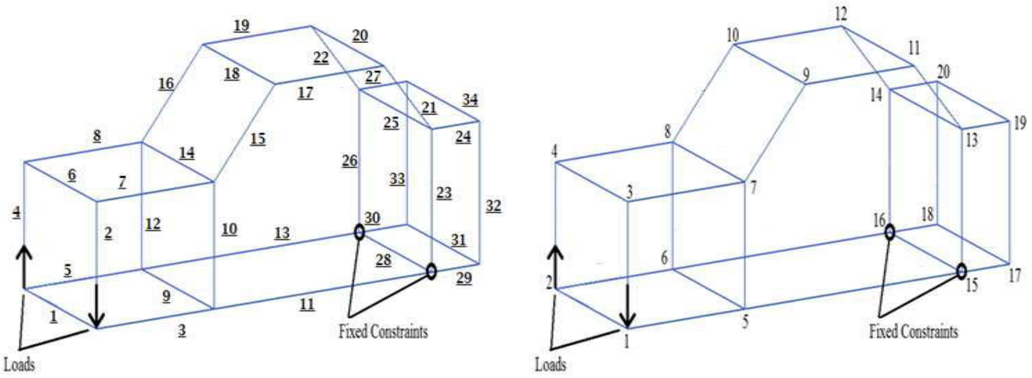
$$\text{Objective}= Min_{[r_1,r_2\ldots r_n]}\left\{\frac{W_{SSB}^i}{K_{T_{SSB}}^i}\right\} \quad i\in[1\ldots N''] \tag{15}$$

The output of the optimization process is the element radius values that will give the smallest ratio between the structure weight and torsion stiffness. This is analogous to having the structure with the largest torsion stiffness for a fixed limit of weight. Bounds are set for the radius values based on the initial analysis so that the values being chosen are reasonable. The process uses a Hessian to drive the direction of each step and the process ends when a set number of consecutive trials show no improvement to within a specified tolerance.

4 IMPLEMENTATION

The presented method is implemented for validation purpose. As stated in the Methodology Section, the first step in the process is preliminary analysis to determine initial radius values as well as the original stiffness and weight. A torsion load is created by applying vertical forces on the front two points. One force will be in the positive vertical direction and the other load will be in the negative vertical direction. A fixed boundary condition is applied at the rear of the structure. The load and boundary conditions can be seen in Fig. 3 which is shown below. The figure also shows the labelled nodes and elements of the structure. As can be seen, a total of twenty nodes and thirty four elements are present in the model. Each element utilizes a solid circular cross-section with the radius as a design parameter.

The first step of the process was to determine initial loads. These loads are based on assumed loads that are commonly found in a vehicle such as passengers, the power train and the other components. The structure loads are found based on existing analysis of the SSS method and can be found in Table 1.



(a) (b)

Fig. 3: Beam Element Geometry with Constraints and Loads: (a) elements' numbers (b) nodes' numbers.

Component	Weight (N)	Centre of Gravity Position (m)
Front Bumper	200	0
Powertrain	3000	0.65
Front Passengers & Seats	2000	2.2
Rear Passengers & Seats	2500	3
Fuel Tank	500	2.95
Luggage	950	4
Rear Bumper	300	4.4
Exhaust	350	2.5
Front Structure	2227.5	0.675
Passenger Compartment	3870	2.425
Rear Structure	1170	3.95

Tab. 1: Component weights for initial analysis.

The initial analysis generates the loads that are applied. The next step is to determine uniform radius values for each of the elements that will yield sufficient torsion stiffness (12000Nm/ radian) for the initial applied loads [24]. This uniform radius will be used to determine the radius values, m, that are used as part of the process. The resulting uniform radius was found to be 15mm. This uniform radius is used to calculate the original torsion stiffness and weight that are used for the filtering process later. The initial torsion stiffness was found to be 12475Nm/radian and the initial weight was found to be 2225N, these results are based on the uniform radius of 15millimetres. As the weight of the structure changes the load applied to the structure will change and for this reason the load needs to be re-calculated before performing the analysis at each iteration during the data generation step. This gives a ratio between the torsion stiffness and structure weight of 5.77. A three level design was utilized, m=3, and the radius values used are 7.5mm which is half the initial value, 30mm which is twice the initial value, and finally 18.75millimetres which is halfway between the two extremes. The performance of the numeric method was evaluated by comparing the results with an analytical approach where possible as well as commercial FEA software. A sample of the displacement result from the commercial solver is shown below.

As shown in Fig. 1 there are a large number of elements and if each one was to be considered unique, where n is equal to 34, a total of $m^n=3^{34}$ trials would be required. This number of trials is computationally expensive and unnecessary. In order to reduce the number of trials to be completed symmetry was introduced. Any element on the driver side of the structure will have the same radius value as the corresponding element on the passenger side. Also all lateral elements that connect the two sides to each other will have a common radius. Using these simplifications the number of unique elements was reduced from 34 to 13 which results in 3^{13}= 1549323 total trials. A table showing the unique elements is shown below based on the image shown in Fig. 1.

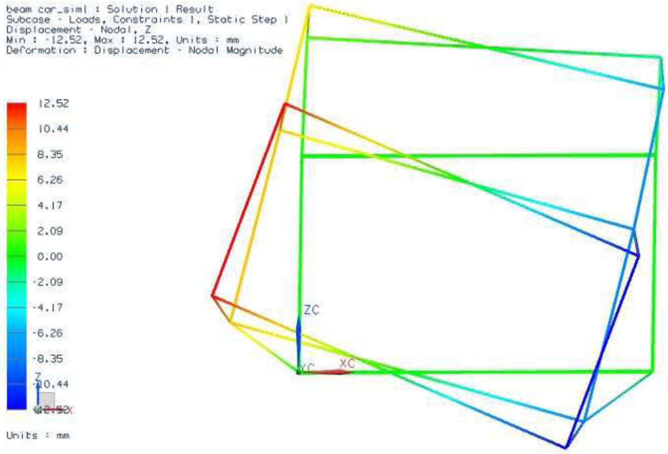


Fig. 4: Torsion Deflection of NX model.

	Nodes	Element Number		Nodes	Element Number
Unique Element 1	1,2	1	Unique Element 6	5,15	11
	3,4	6		6,16	13
	5,6	9	Unique Element 7	7,9	15
	7,8	14		8,10	16
	9,10	18	Unique Element 8	9,11	17
	11,12	20		10,12	19
	13,14	25	Unique Element 9	11,13	21
	15,16	28		12,14	22
	17,18	31	Unique Element 10	13,15	23
	19,20	34		14,16	26
Unique Element 2	1,3	2	Unique Element 11	13,19	24
	2,4	4		14,20	27
Unique Element 3	1,5	3	Unique Element 12	15,17	29
	2,6	5		16,18	30
Unique Element 4	3,7	7	Unique Element 13	17,19	32
	4,8	8		18,20	33
Unique Element 5	5,7	10			
	6,8	12			

Tab. 2: Number of unique elements.

After the data was generated it was sorted according to weight. The results for all trials are shown below in order of increasing weight. As can be seen there are a large number of data points that are in the high weight range. These points represent the structure where most elements have larger radiuses. The larger radiuses will drastically increase weight, but will also improve the torsion stiffness. The next step in the process is filtering the data in order to reduce the amount that is to be analyzed as part of the modelling process. The first filter is governed by Eqn. 6 where α_1 was selected as equal to 1.1. This alpha value ensures the weight can increase by only ten percent over the original weight. Filtering the initial data reduces the number of points to 83882. The result of this filter is shown in Figure 6.

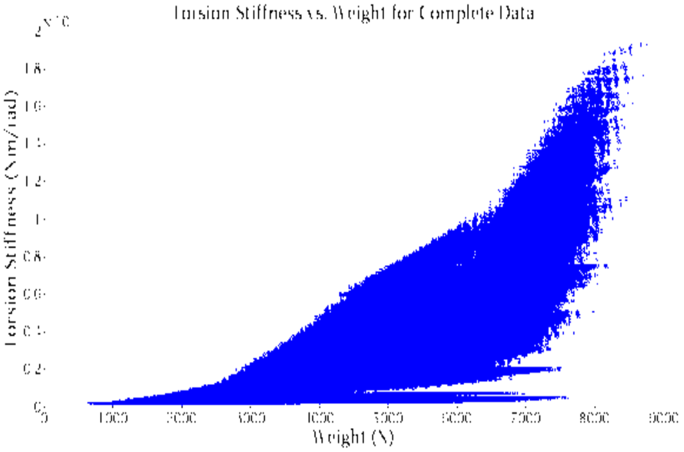


Fig. 5: Unfiltered data.

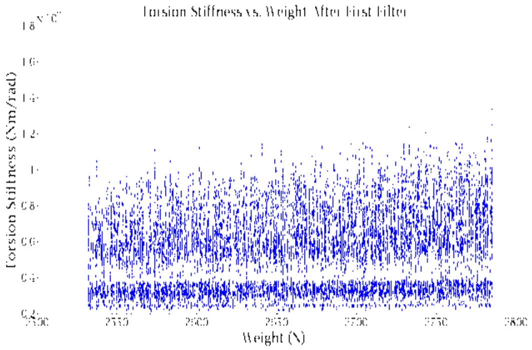


Fig. 6: Data filtered by weight.

The first filtering process significantly reduced the amount of data however a further reduction is required. This filtering process was based on Eqn. 7 where α_2 was chosen to be 1. This α_2 value ensures that only data points that have torsion stiffness greater than the initial stiffness are stored. This filtering process reduced that amount of data to 176 points. The results are shown in Figure 7.

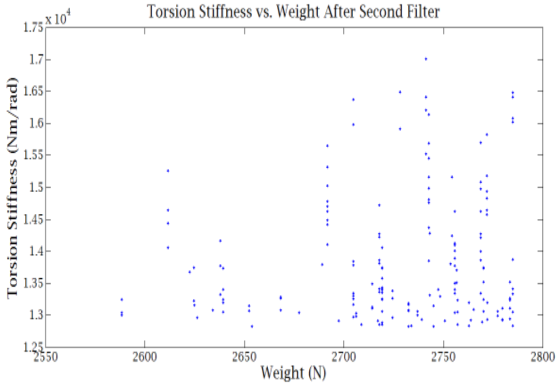


Fig. 7: Data Filtered by weight and torsion stiffness.

The next process was modelling the data. As mentioned a non-linear model was used for both the stiffness and weight. A fourth order model in terms of radius was used for the stiffness while a second order model in terms of radius was used for the weight. A sample of the modelling is shown below.

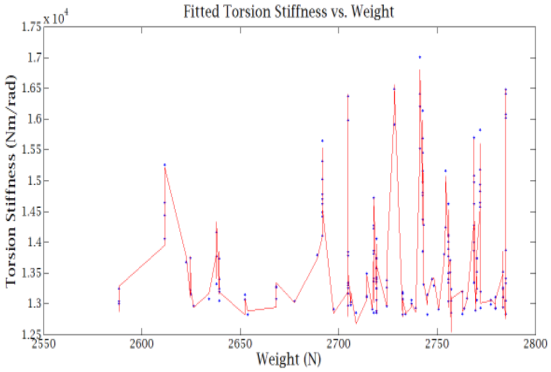


Fig. 8: Filtered data with fitted model.

The equation generated to model the stiffness also gave some indication of the sensitivity of the stiffness to each element based on the derivative.

The final step in the procedure is to perform the optimization. As stated the optimization is a non-linear constrained optimization that seeks to minimize the ratio between the weight and the stiffness. The initial condition for the optimization process was the radius values corresponding to the best observed data run, which is to say that the radius values chosen gave the largest ratio between stiffness and weight. After running the optimization process the following results were found.

Property	Optimization Initial Condition	Optimization's Initial Condition	Optimized Model
Stiffness K_T (Nm/rad)	12475	17008	27163
Weight W (N)	2225	2741	3035
Objective Ratio, K_T/W (m/rad)	5.607	6.205	8.947

Tab. 4: Results of the optimization process.

As can be seen the objective ratio for the optimum trial is almost twice that of the initial data. This represents a significant improvement over the initial values. The radius values, in millimetres, that produced this result are shown in Table 5.

Radius	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}
Original Condition	15	15	15	15	15	15	15	15	15	15	15	15	15
Optimization's Initial Condition	18.8	7.5	7.5	18.8	18.8	18.8	18.8	18.8	18.8	7.5	7.5	7.5	7.5
Optimum Result	15.7	9.2	24.78	12.8	14.1	30.7	12.8	12.9	12.8	1.9	12.8	10.9	5.9

Tab. 5: Optimal radius values in millimeter (mm).

5 CONCLUSION

A new approach to model and FEA-based selection of the structural design parameters of a vehicle called Simple Structural Beam-frame (SSB) is introduced and is used to optimize the design based on Torsion stiffness of the structure. The optimization objective is to determine the radius of each beam frame element that would give the largest ratio between stiffness and structure weight. The optimization process sought to determine the dimensions that minimize the introduced objective function by maximizing the overall torsion stiffness of the vehicle when its weight is remained in a constant range.

Implementation of the methodology and the conducted case-study successfully demonstrates more than 60% increase of the structural torsion stiffness to weight ratio when comparing an initial design to a design with optimum selection of the design parameter. Implementing only solid circular cross-sections are considered in this work, however a number of other cross-sectional geometries can be introduced to the algorithm. The use of different cross-sectional shapes, specifically hollow shapes, would closer match the design of actual vehicles. The use of hollow shapes would also serve to reduce the weight. This method can be efficiently employed for initial design of vehicle structure when weight reduction and enhance of structural stiffness are the major objectives.

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