Mesh Simplification Based on Feature Preservation and Distortion Avoidance for High-Quality Subdivision Surfaces

Tetsuo Oya[1], Toshiyuki Tanamura[1], Hideki Aoyama[1] and Masatake Higashi[2]

[1]Keio University, oya@sd.keio.ac.jp, tanamura420@gmail.com, haoyama@sd.keio.ac.jp
[2]Toyota Technological Institute, higashi@toyota-ti.ac.jp

ABSTRACT

We propose a mesh simplification scheme that can reduce the number of meshes to the requested number while uniform and high-quality base meshes for subdivision surfaces. In this paper, a modified QEM (Quadric Error Metrics) scheme is proposed to obtain high-quality subdivision surfaces on which sharp features exist. To meet the requirements for surface quality and precise feature expressions, we combined mesh distortion check and feature preservation with the QEM scheme. We demonstrated our algorithm using some models including complex shapes, and the demonstrations showed the effectiveness of the proposed method.

Keywords: RE, subdivision surface, mesh simplification, and feature preservation.
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1 INTRODUCTION

To design industrial products such as an automotive body, tensor product parametric surfaces have been used in conventional CAD (Computer-Aided Design) systems. These parametric surfaces are suitable for presenting free-form surfaces with appropriate smoothness; however, it is cumbersome to create a complex model from scratch. Moreover, satisfying geometrical continuity between patches is usually difficult. Therefore, reverse engineering has emerged as a powerful candidate that meets the requirements of both rapid modeling and high-quality surfaces.

In the context of reverse engineering, there are many methods of recovering the original shape from scanned data that is usually a cloud of sampled points. Spline surfaces, polygonal meshes, and implicit surfaces have been the major frameworks for reconstructing the original shapes from sampled points. Spline surfaces have an advantage in presenting smooth surfaces; however, as stated above, they require many patches to cover complex shapes, and connecting each patch with high continuity is rather difficult. Mesh modeling is very popular in complex organic expressions such as character models in movies or games. However, it is no more than an approximation with finite number polygons; thus, it is not suitable for expressing product design that requires smoothness. Implicit surface is a powerful method of representing complex surfaces with sufficient smoothness; but its data structure is not directly convertible to that of the conventional CAD. Therefore, this technique may not be suitable for industrial design. On the other hand, the subdivision surface is an effective surface representation scheme because this technique can deal with shapes of arbitrary topology and obtain a sufficiently smooth curvature almost everywhere. Catmull and Clark [3] developed the subdivision technique for quadrilateral meshes; then, that for triangular meshes was introduced by Loop [7]. Hoppe et al. [6] proposed a method of reconstructing accurate piecewise smooth surface models from scattered range data. They also presented a scheme to express sharp features on the reconstructed subdivision surfaces.

Although there are many methods of reconstructing an original shape from the sampled points, we focused on the subdivision surface [8] scheme because of its flexibility to topology and its ability to produce smooth surfaces from the control mesh that is directly obtained from the scanned data. In the fitting process with subdivision surfaces, a dense triangle mesh is obtained from scanning; then, the control mesh is computed to generate subdivision surfaces, which would approximate the shape of the target. Generally, the obtained points are too many to handle efficiently; therefore, the number of meshes is reduced by the mesh simplification.

Here, we have two problems with the conventional mesh simplification. The first is the low-quality surface due to the distorted mesh. The well-known mesh simplification method called QEM (Quadric Error Metrics) [4] reduces the number of meshes using the edge collapse procedure. While simplifying the mesh, distorted triangles are generated because this method only handles a quadric error of the mesh geometry without considering the quality of the subdivision surface. The second problem is the disappearance of sharp features by mesh simplification. The shape of a subdivision surface is strongly affected by the initial mesh. Thus, if the initial mesh is simplified without preserving the sharp features, the obtained subdivision surface cannot represent the sharp features.

We modified the QEM scheme so as to address these two problems. To deal with the first problem, we introduced a mesh distortion criterion for a vertex. This criterion can be used to detect the mesh that is excessively distorted by the simplification. Therefore, this criterion does not allow the production of overly distorted meshes during the simplification. This is expected to improve the quality of the subdivision surface. Next, we combined the feature detection with the QEM. As a preprocess, a discrete Gaussian curvature was used to carry out feature detection. By omitting the feature edge from the simplification process, sharp features are maintained, which leads to the preservation of the original shape. Then, we demonstrated our algorithm using some models including complex shapes. The proposed method worked well compared with the case wherein only QEM is used. The quality of the subdivision surfaces is improved, and the features are preserved.

2 RELATED TECHNIQUES

2.1 Subdivision Surface

Subdivision surfaces [11] can be considered as a generalization of spline surfaces because a coarse control mesh can be used to control them. In contrast to spline surfaces, subdivision surfaces can represent arbitrary topology surfaces. Subdivision surfaces are generated by the repeated refinement of control meshes: after each topological refinement, the positions of the old and new vertices are adjusted on the basis of a set of local averaging rules [1]. There are several rules of subdivision; however, we adopted the Loop scheme since this scheme is applicable to a triangle mesh that is preferred in terms of topological flexibility.

The Loop subdivision scheme subdivides a triangle into four smaller triangles: one new vertex is inserted at each edge, and four triangles are obtained by connecting these new vertices. These newly introduced vertices are called odd vertices, and the original vertices are called even vertices. In the Loop subdivision scheme, let an arbitrary vertex be v and a newly computed vertex be v'; then, we have the position of an odd vertex as the weighted average of its neighboring vertex positions:

$$v' = \frac{1}{8}(3v_1 + 3v_2 + v_3 + v_4) \tag{1}$$

Here, v_1 and v_2 are end points of an edge, and v_3 and v_4 are the opposite-side points of triangles that share the concerned edge. Similarly, the positions of even vertices are obtained by calculating the weighted average of its adjacent vertices as

$$v' = (1 - k\beta)v + \beta\sum_{j}^{k} v_j \tag{2}$$

Here, v_j represents 1-neighbor vertices that are directly connected with the vertex v. k denotes a valence of the vertex v: if $k = 6$, the vertex is called regular; otherwise, it is a singular vertex. β is a value dependent on the valence of the vertex v, and it is computed as

$$\beta = \begin{cases} \dfrac{3}{16} & (k = 3) \\ \dfrac{1}{k}\left\{\dfrac{5}{8} - \left(\dfrac{3}{8} + \dfrac{1}{4}\cos\dfrac{2\pi}{k}\right)^2\right\} & (k > 3) \end{cases} \tag{3}$$

By conducting the subdivision scheme repeatedly on the input control mesh, a smooth surface can be obtained as a result of convergence. This limit surface is called the subdivision surface, and the limit position of the surface point v^∞ is calculated using the weighted average of its adjacent vertices as

$$v^\infty = \alpha v + \chi\sum_{j=1}^{k} v_j, \quad \alpha = 1 - k\chi, \quad \chi = \frac{1}{\dfrac{3}{8\beta} + k} \tag{4}$$

In subdivision surface fitting, a set of control points whose subdivision limit surface coincides with the input points (surface) is required. In other words, the calculated control mesh converges, by subsequent subdivision steps, to the scanned point data in the limit. To obtain the control mesh vertices v_i, a simultaneous equation should be solved with respect to the variable v_i so that the subdivision limit point of each control point is equal to the input points s_i. Namely, by using a matrix M, we have the simultaneous equation as

$$v_i^\infty = Mv_i. \tag{5}$$

In this equation, the matrix M provides a limit position v_i^∞ by inputting the initial control mesh v_i. By replacing the limit position v_i^∞ with the target surface points, this equation is changed to

$$s_i = Mv_i. \tag{6}$$

Therefore, by computing the inverse of the matrix M, the control mesh whose limit surface is fitted to the target shape is obtained by solving

$$v_i = M^{-1} s_i. \quad (7)$$

When the number of input points is large, the cost of solving the simultaneous equation becomes crucial in practice; therefore, some efficient methods have been proposed to obtain the control mesh by an appropriate convergence check. Suzuki et al. [10] presented a fast algorithm to obtain a refined control mesh. They introduced an energy metric to move the control vertex to achieve good convergence. Maekawa et al. [9] proposed a new method of obtaining a better fitting result. They attempted to find the nearest position on the limit surface for each input point. Then, the initial control points were offset in the normal direction by the error vector. Higashi et al. [5] presented a novel fitting scheme in which they conducted a convergence check with respect to the weight of 1-neighbor vertices because the magnitude of the inverse matrix M^{-1} of the 2-neighbor of the objective vertex is negligibly small. By this method, they succeeded in obtaining a good fitting result in a relatively reliable and fast way.

2.2 QEM and Edge Collapse

Garland and Heckbert [4] presented a mesh simplification algorithm, known as Quadric Error Metrics (QEM), which is able to generate a high-quality approximated model of an input mesh. This method is based on the edge contraction scheme. As shown in Fig.1, the highlighted edge whose both ends are vertices v_1 and v_2 is contracted into a single vertex $\bar{v}$. The old vertices v_1 and v_2 are moved to the position of the new vertex, connecting all the injected edges with the vertex v_1. This edge contraction scheme, keeping the original shapes, can reduce the number of initial meshes. A more detailed procedure is as follows. First, a 4x4 symmetric matrix Q is computed for each vertex, and the error at vertex v is defined to be the quadratic form $\Delta(v) = v^T Q v$. Then, all the valid pairs are selected. Next, the optimal contraction target $\bar{v}$ is computed for each valid pair (v_1, v_2). The error $\bar{v}^T (Q_1 + Q_2) \bar{v}$ of this target vertex becomes the cost of contracting that pair. After this, all the pairs are placed in a heap that is keyed on cost with the minimum cost pair at the top. Finally, the pair (v_1, v_2) of least cost is iteratively removed from the heap and contracted. Then, the costs of all the valid pairs involving v_1 are updated. The quality of the obtained subdivision surface depends on the initial mesh or base mesh of control vertices. When the lengths of adjacent edges of a vertex in the simplified mesh are very different, the interpolated subdivision surface has undesirable undulations or large distortions, as shown in Fig.2. Therefore, we introduced a criterion to avoid generating distorted meshes around the target vertex after the mesh collapse. The details are described in section 3.

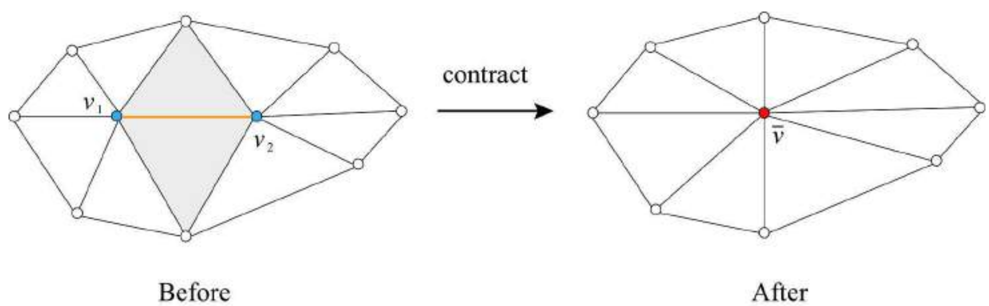


Fig. 1: Edge collapse.

Fig. 2: Example of a failed mesh simplification. Leftmost is the input mesh. Center is the simplified mesh wherein remarkable undulations are seen. Rightmost is the resultant subdivision surface.

3 MESH DISTORTION CHECK

We introduce a criterion of mesh distortion for internal vertices. Distortion of an interpolated subdivision surface is caused because some parts of the base control mesh determined by uniform subdivision interpolation protrude or cross each other. For an inner vertex v_0, we check the level of distortion using the centroid of one-neighbor vertices of v_0. Let the centroid be v_c and the minimum height of triangles around v_0 be $h_{\min}$, as shown in Fig.3; then, we define a criterion of distortion for an inner vertex as

$$\kappa = \frac{|v_c - v_0|}{h_{\min}} . \tag{8}$$

When an edge is collapsed into a new vertex v_0, the value κ is calculated on the meshes around this new vertex. If the value κ is greater than a designated threshold value κ_0, it indicates that the edge collapse this time failed because the resultant mesh is distorted. Basically, it is not necessarily effective to attempt this check on all the edges to find the vertex that generates a poor-quality surface; because there are situations wherein distorted meshes do not generate a poor-quality surface. For example, when an edge is located at a relatively flat area, a smooth surface could be generated in spite of the existence of distorted 1-neighbor meshes. This indicates the requirement of extraction of edges located at a region with a large shape variation; in other words, sharp feature detection is needed. In our mesh simplification algorithm, the mesh distortion check is conducted appropriately, combined with feature detection on edges. The feature detection is presented in the next section, and the whole algorithm is described in section 5.

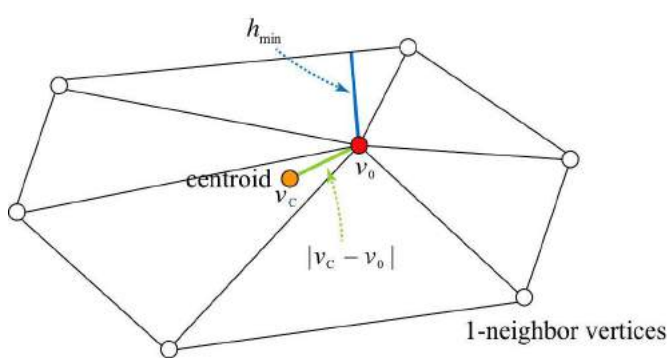


Fig. 3: Definition of the criterion for the mesh distortion check.

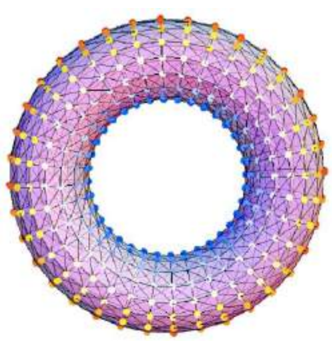
4 FEATURE DETECTION

To make the mesh simplification efficient, as explained, mesh distortion check is carried out before the simplification. By doing this, we can improve the surface quality; however, conducting this check on whole edges would not be efficient. Thus, this check should be carried out only on the edge that is assumed to affect the surface quality. In this paper, we conduct feature edge detection for all the edges on the input mesh. A discrete Gaussian curvature is utilized as a geometrical measure. Although there are many other sharp feature detection algorithms such as using normals or discrete Laplacians, we began with the discrete Gaussian curvature owing to its simplicity and geometrically rich information. On a vertex v_i, the discrete Gaussian curvature K_i [2] is computed using the following expression:

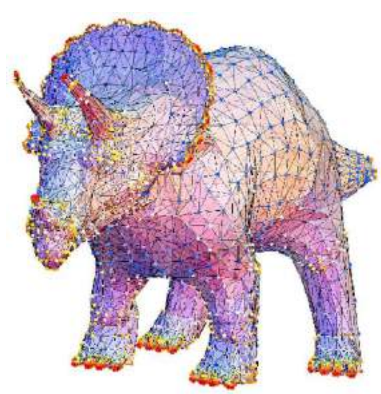
$$K_i = \frac{1}{S_i}\left(2\pi - \sum_{v_j \in N_1} \theta_j\right), \tag{9}$$

where S_i is a third of the total area of 1-neighbor around vertex v_i and θ_j denotes the angle of the incident triangles at vertex v_i. The absolute value $|K|$ becomes large on uneven regions and becomes small on relatively flat regions. Therefore, by designating a threshold, sharp edges can be detected as a large $|K|$ point. Figures 4(a) and 4(b) show the calculation results of a discrete Gaussian curvature on a torus model and a triceratops model, respectively. In these figures, $|K|$ is expressed as a colored point on each vertex; its magnitude is represented by red, white, and blue in that order. Thus, vertices in red represent points on sharp edges, which should be treated specially.

Here, we show another result of feature extraction. The fandisk model was selected. Figure 5(a) show the result of the Gaussian curvature calculation on whole vertices. In figure 5(b), edges having at least one vertex that is recognized as feature are shown. From these results, it is clear that the feature extraction has successfully been conducted.

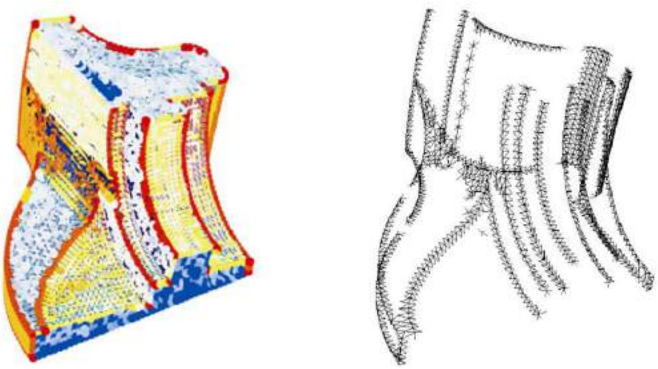


(a) Torus



(b) Triceratops

Fig. 4: Results of the discrete Gaussian curvature calculation on whole vertices.



(a) Result of the Gaussian curvature calculation. (b) Results of the edge extraction.

Fig. 5: Example of feature extraction on the fandisk model.

5 PROPOSED METHOD AND EXAMPLES

5.1 Algorithm

The proposed algorithm for efficient mesh simplification is explained as follows.

1. The sharp feature extraction is conducted by using a discrete Gaussian curvature.
2. The QEM cost is computed for whole initial vertices.
3. Edge collapse is carried out on the basis of the QEM cost. On a vertex obtained by the edge collapse, the 1-neighbor mesh distortion criterion κ is calculated. Then, the calculated κ is compared with the initial threshold κ_0. In this paper, as a default value, κ_0 is set to 1.5 in the following demonstrations.
4. If $\kappa > \kappa_0$, edge collapse on that edge is skipped; and the process goes to the next edge as the QEM cost order.
5. If $\kappa < \kappa_0$, edge collapse on that edge is conducted.
6. For all the edges that do not exist in the feature list, edge collapse is conducted on the basis of the QEM cost.
7. When the number of meshes reaches the designated number, the simplification process stops.

5.2 Application Results

In this section, we present two application results of the proposed method: a torus model and a triceratops model. The input meshes are the same as those in Fig.4. The results of feature extraction for both models are shown Fig.6. We compared two cases, namely, the one using only QEM and that using the proposed method.

The first case is a torus. The number of input meshes of a torus model, 1152, was reduced to 570. In Figs.7 and 8, the simplified mesh and its approximated surface by subdivision surface fitting are presented. In the case of the torus model, the feature edges are chosen as the region where the Gaussian curvature K equals 0 as shown in Fig.6. Although the ridges where $K = 0$ do not show sharp features, these are geometrically special shapes; thus, these ridges should be preserved. As seen in Fig.7, when only the QEM is applied, distortions are generated at the ridge of the torus model. In contrast, as shown in Fig.8, when the proposed method is applied, no distortions are seen and a smooth surface is produced. This is because the proposed method preserves the designated shapes during mesh simplification, yielding no vertices that cause a distorted mesh around it.

Next, as a larger and more complex example, a triceratops model was used. In this case, the number of input meshes, 5600, was reduced to 1700; and the edges whose absolute value of the Gaussian curvature was greater than the designated threshold value were recognized as having sharp features to be preserved. As shown in Figs.9 and 10, both schemes seem to generate good results; however, there are apparent differences in the regions such as the tips of the frill behind the head or the fingers and toes of the arms and legs, as shown in Fig.11. Namely, the scheme wherein only QEM was used failed to preserve these complicated shapes.

These two examples and discussions on them suggest that a mesh simplification scheme in which the geometrical information extracted from the objective model is reflected is very effective in generating high-quality approximated surfaces.

The presented method is disadvantageous in computational cost because of the necessity of extra calculations for the distortion check. First, feature edge detection should be carried out. Since this process deals with all of the edges, the feature preservation is conducted as a pre-process in advance. While the simplification process is running, two judgments have to be done; namely, the feature edge classification and the 1-neighbor distortion check. The latter is conducted only on the edges that are judged as feature lines. Therefore, when there are many feature edges in a model, computation time will be greater than a simpler model case. Because the torus model is a relatively simple model, thus, the difference of the computing time between the QEM-only case and the proposed-method case was not great. On the other hand, the computing cost gap of the triceratops model was great because of its complicated shapes. The authors deem that this is an inevitable increase of computational cost to obtain a better fitting result.

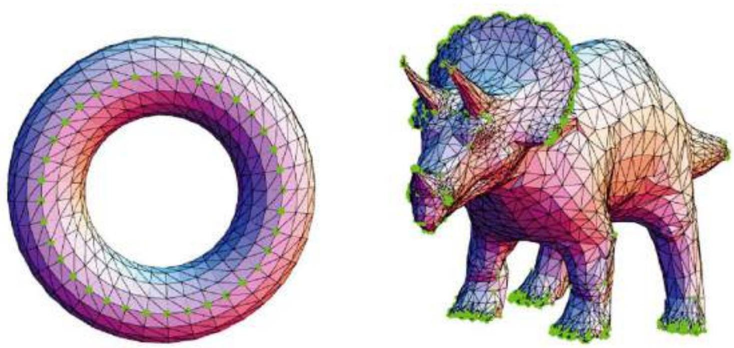


Fig. 6: Feature vertices; green vertices were recognized as feature points in these demonstrations.

Fig. 7: Input mesh, simplified result by only QEM, and the resultant subdivision surface.

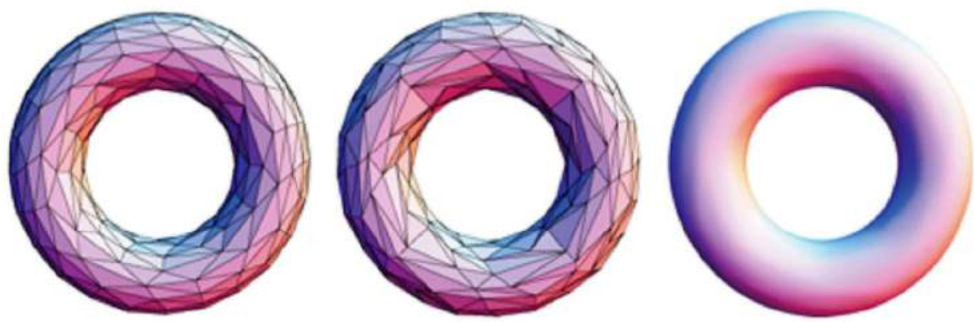


Fig. 8: Input mesh, simplified result by the proposed method, and the resultant subdivision surface.

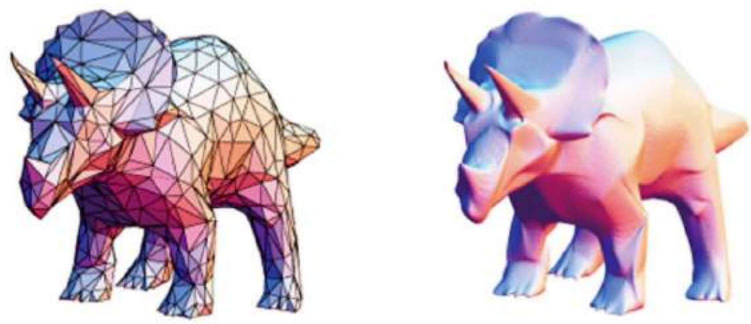


Fig. 9: Simplified result by only QEM, and the resultant subdivision surface.

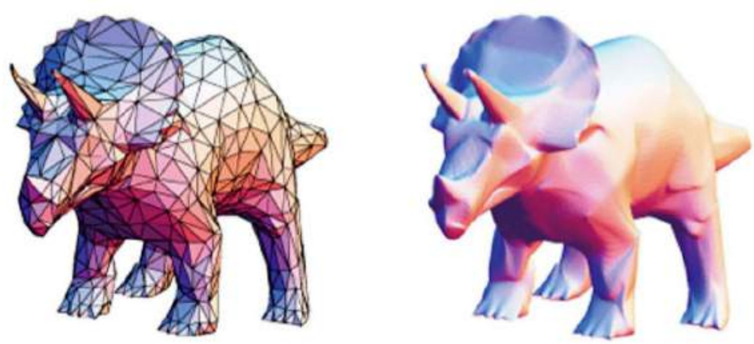


Fig. 10: Simplified result by the proposed method, and the resultant subdivision surface.

6 CONCLUSIONS

In this paper, a mesh simplification scheme in which the mesh distortion check and feature preservation are implemented has been presented. The mesh distortion check can avoid generating a low-quality surface caused by greatly distorted 1-neighbor meshes of a vertex. Feature extraction can preserve the designated geometrically notable features in the initial mesh. Two examples have demonstrated how these schemes work, proving the effectiveness of the proposed method. Future work will be on further effective geometrical classification based on an appropriate feature metric, and a selection of suitable threshold values for distortion check and feature extraction.

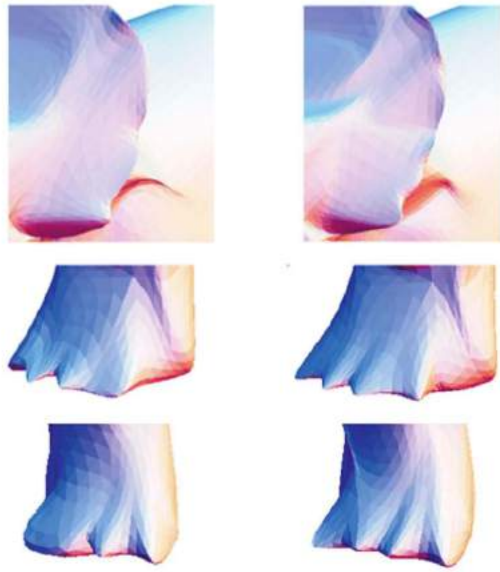


Fig. 11: Detailed comparison of the results by only QEM (left) and result by the proposed method (right). The magnified pictures of the frill (top row), the fingers (middle), and the toes (bottom) are shown.

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