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# Parameterized Geometric Design of a Generic Form Milling Cutter 

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#### Abstract

This paper presents the design methodology to develop a comprehensive geometric model for a generic form milling cutter. Unlike conventional projected geometry approach, the geometric design of the form milling cutter is modeled here using (a) composite NURBS curve to create the generic profile for the multi-cutting edges and (b) biparametric sweep and planar surfaces to form the rest of the geometry. The methodology is illustrated by varying the dimensional and geometric parameters of the composite model and generating a variety of cutters lying in the same conceptual family of generic form milling cutter. The proposed method is validated by fabricating one of the form milling cutters.


Keywords: parameterized geometric design, form milling cutter, generic profile.
DOI: 10.3722/cadaps.2013.757-765

## 1. INTRODUCTION

The accurate design of cutting tools affects its machining performance significantly in terms of products' shape and form realization. Presently, with the rise in demand for parts with complex shapes (free form surfaces), precise cutting tools are required for better performances; as most of the shapes are still realized by subtractive machining processes. So, there is a need to design cutting tools with complex surfaces in order to machine free-form desired surfaces efficiently, with a few passes and limited setups as well as to obtain high material removal rate. One of the solutions can be precise parameterized geometric design of generic profile form milling cutter. As reported in the literature, lot of work has been done related to the geometric design of single point [6], [7], [10] and multi-point [2], [16] cutting tools but information concerned with the geometric design of generic profile form milling cutter is scarce. Some work, related to development of a mathematical model of form cutting tools, has been done by Hsieh and Tsai [3], Radzevich [9] and Wang et al. [15]. Authors' previous works [4], [5], [8], [11], [12], [13], [14] followed the surface modeling approach to accurately model three-dimensional (3D) geometry of various generic multi-point cutters for downstream applications.

This paper presents a systematic method of parameterized geometric design to directly represent a generic model of a form milling cutter. NURBS curve(s) and sweep surfaces are employed to generate and control the complex shape of cutting flutes of the generic form milling cutter. To demonstrate the technology based on the proposed design approach, one of the form milling cutters is developed and manufactured. The generic definition of the cutter can further be used to design a cutter for similar /
related application(s), analyze the cutting tool using finite element based engineering analysis approach and manufacture it.

## 2. MATHEMATICAL MODEL OF A GENERIC FORM MILLING CUTTER

The generic form milling cutter is a rotary form relieved cutter whose cutting edges are shaped to generate various complex surfaces on work pieces. The three-dimensional model of the cutter is developed keeping in mind that NURBS formulation simplifies shape modifications by changing only a few simple parameters. Multiple profiles are formed on a single tooth of the cutter so as to replace a number of form cutters of various cutting edge profiles i.e. the shape generation that may require multiple cutters can be produced with a single form cutter.

The parameterized geometric design of the teeth of the generic form milling cutter consists of surface patches formed by sweeping the section profile along the boundary curves. Sweep surface can be 1 -rail sweep or 2 -rail sweep [1] accordingly as the section curve is swept along 1 or 2 boundary curves respectively. A NURBS representation is used to define the geometry of cutting edge profile of the cutter teeth. The cutter body consists of surface patches that are either planar or cylindrical in geometry. In the present work, a detailed mathematical model is discussed for a generic tooth. Later, this tooth is placed in the proper position and orientation on the periphery of the cutter body as many times as the number of teeth to complete the model.

### 2.1 Mathematical Model of a Generic Tooth

The generic tooth of the form milling cutter is shown in Fig. 1. It is designed to consists of surface patches labeled $\Sigma_{1}$ to $\Sigma_{5+l+f}$, where $l$ and $f$ are the number of peripheral lands and flank faces of the multiple profile of the generic cutter. These surfaces are modeled as a combination of sweep surfaces. The cutting edge profile (composite sectional curve) of the tooth is defined using NURBS curves.

### 2.1.1 Sectional geometry of cutter tooth

A frame of reference $\mathrm{C}_{1}\left\{\mathrm{O}_{1}: \mathrm{X}_{1}, \mathrm{Y}_{1}, \mathrm{Z}_{1}\right\}$ is considered at midway along the lateral surface of one of the tooth surface lying on $\mathrm{Z}_{1} \mathrm{X}_{1}$ plane. Fig. 2 shows the composite sectional curve ( $V_{0} \ldots V_{i}$ ) lying on $\mathrm{Z}_{1} \mathrm{X}_{1}$ plane with their control polygons ( $V_{0}, B_{1}, B_{2}, B_{3}, V_{1}, \ldots V_{i-1}, B_{3 i-2}, B_{3 i-1}, B_{3 i}, V_{i}$ ), where $i$ is the total number of surface patches required for peripheral lands and flank surfaces of the generic cutter. Here, the composite sectional curve is a tip-to-tip curve on the outer circle diameter of the cutter on the axial plane. To make the cutter definition generic, the composite sectional curve ( $V_{0} \ldots V_{i}$ ) is represented as a composite NURBS curve with $i(=l+f)$ NURBS segment. Segments $V_{2 m-2} V_{2 m-1}(m=1$ to $l)$ and $V_{2 n-1} V_{2 n}$ ( $n=1$ to $f$ ) of the composite curve corresponds to curve profiles of $l$ peripheral lands and $f$ flank surfaces respectively. These are shown as NURBS curves individually on a two-dimensional projected plane (Fig. 2). $R$ and $2 \theta$ are the cutter's crown radius and crown angle respectively (in case of circular crown type cutter). For straight or tapered type of cutter, L is the length of the cutter.

For the generic cutter, the composite sectional curve $\left(V_{0} \ldots V_{i}\right)$ could be a combination of circular arcs, straight lines and free form curves represented using NURBS curves. The homogeneous coordinates of these vertices may be evaluated with the help of Fig. 2 in terms of various dimensional parameters. Let $D_{c}, D_{R e}$ and $D_{L e}$ be the outer circle diameters at the center, right and left ends of the cutter respectively. Each of the $l+f$ NURBS segments are represented by $b$ (say 5) control vertices. A total of $(l+f) .(b-1)+1$ vertices $\left(V_{0}, B_{1}, B_{2}, B_{3}, V_{1}, \ldots . V_{i-1}, B_{3 i-2}, B_{3 i-1}, B_{3 i}, V_{i}\right)$ are required to define the composite NURBS curve (Fig. 2). The value of these vertices depends on the required design of the form milling cutter (straight, tapered or circular type).

The sectional profile $\left(V_{0} \ldots V_{i}\right)$ along the axial plane of a cutter tooth consists of ' $i$ ' parametric NURBS curves, namely, $p_{1}(t)$ to $p_{i}(t)$ in a two-dimensional plane. As already mentioned, these are parametrically represented using ' $b$ ' number of polygonal control points. The generic definition of the sectional profile in $\mathrm{Z}_{1} \mathrm{X}_{1}$ plane in terms of parameter ' $t$ ' may be represented by,

Computer-Aided Design \& Applications, 10(5), 2013, 757-765
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Fig. 1: Modeling of a generic tooth of the form milling cutter.


Fig. 2: Composite sectional curve with their control polygons in $Z_{1} X_{1}$ (axial) plane.

$$
p_{i}(t)=\left[\begin{array}{llll}
f_{i 1}(t) & 0 & f_{i 3}(t) & 1 \tag{2.1}
\end{array}\right] \text {, for } 0 \leq t \leq t_{\max } .
$$

The equations of each NURBS segment are evolved as:
Case 1: When $\mathrm{b}=5$ and order of the curve $=4$, then

$$
p_{\mathrm{i}}(t)=\frac{\left\{\begin{array}{l}
V_{i-1} h_{4 i-4}\left(1-3 t+3 t^{2}-t^{3}\right)+B_{3 i-2} h_{4 i-3}\left(3 t-4.5 t^{2}+1.75 t^{3}\right)+B_{3 i-1} h_{4 i-2}\left(1.5 t^{2}-t^{3}\right)  \tag{2.2}\\
+B_{3 i} h_{4 i-1}\left(0.25 t^{3}\right)
\end{array}\right\}}{h_{4 i-4}\left(1-3 t+3 t^{2}-t^{3}\right)+h_{4 i-3}\left(3 t-4.5 t^{2}+1.75 t^{3}\right)+h_{4 i-2}\left(1.5 t^{2}-t^{3}\right)+h_{4 i-1}\left(0.25 t^{3}\right)},
$$

for $0 \leq t \leq 1,1 \leq i \leq(l+f)$ and

$$
p_{\mathrm{i}}(t)=\frac{\left\{\begin{array}{l}
B_{3 i-2} h_{4 i-3}\left(2-3 t+1.5 t^{2}-0.25 t^{3}\right)+B_{3 i-1} h_{4 i-2}\left(-2+5 t-3 t^{2}+0.5 t^{3}\right)  \tag{2.3}\\
+B_{3 i} h_{4 i-1}\left(2-6 t+6 t^{2}-1.75 t^{3}\right)+V_{i} h_{4 i}\left(-1+3 t-3 t^{2}+t^{3}\right)
\end{array}\right\}}{\left\{\begin{array}{l}
h_{4 i-3}\left(2-3 t+1.5 t^{2}-0.25 t^{3}\right)+h_{4 i-2}\left(-2+5 t-3 t^{2}+0.5 t^{3}\right) \\
+h_{4 i-1}\left(2-6 t+6 t^{2}-1.75 t^{3}\right)+h_{4 i}\left(-1+3 t-3 t^{2}+t^{3}\right)
\end{array}\right\}},
$$

for $1 \leq t \leq 2,1 \leq i \leq(l+f)$.
Case 2: When $b=3$ and order of the curve $=3$, then

$$
\begin{equation*}
p_{\mathrm{i}}(t)=\frac{V_{i-1} h_{2 i-2}(1-t)^{2}+B_{i} h_{2 i-1}(1-t) 2 t+V_{i} h_{2 i} t^{2}}{h_{2 i-2}(1-t)^{2}+h_{2 i-1}(1-t) 2 t+h_{2 i} t^{2}}, \text { for } 0 \leq t \leq 1 \text { and } 1 \leq i \leq(l+f) \tag{2.4}
\end{equation*}
$$

where $V_{x i}{ }^{\prime} s, \ldots, B_{x i}{ }^{\prime} s \ldots$ are the polygonal control points and $h_{x i}{ }^{\prime} s$ are the weights at the corresponding control points.

### 2.1.2 Cross-sectional geometry of cutting tooth

To model the cross-sectional profile of a tooth, a bottom-to-bottom curve on the root circle diameter, from one end of the flute of the cutter to the second flute is preferred. The composite cross-sectional curve, $V_{0} \ldots V_{9}$ (Fig. 3) is made up of ten segments with $V_{0} V_{1}, V_{2} V_{3}, V_{4} V_{5}$ and $V_{7} V_{8}$ as straight lines, $V_{0} V_{3}$ and $V_{9} V_{8}$ as virtual straight lines, segments $V_{1} V_{2}$ and $V_{5} V_{6} V_{7}$ as circular arcs of radius $r_{f}$ with centers at $c_{1}$ and $c_{1}{ }^{\prime}$ respectively, $V_{0} V_{8}$ as a circular arc of diameter $d_{r e}$ with centre at $C$ and the curve $V_{3} V_{4} V_{9}$ as per the desired form of the flank or relieving surface. The relieving surface profile may be employed as logarithmic / Archimedean spiral or straight line profiles.

In modeling the cross-sectional profile (Fig. 3) of a single tooth in a two-dimensional plane, the input parameters are (i) width of the rake face ( $l_{1}$ ), (ii) 3D angles obtained to form face ( $\gamma_{1}$ ) and land $\left(\gamma_{2}\right)$ about Z axis, (iii) radii of fillet $\left(r_{f}\left(=r_{r f}=r_{l f}\right)\right)$, (iv) outer circle diameter at the end of the cutter $\left(D_{e}\left(=D_{R e}=D_{L e}\right)\right)$, (v) the root circle diameter at one of the cutting end of the cutter ( $d_{r e}$ ) and (vi) number of flutes $(N)$. The position vectors of vertices $V_{0}$ to $V_{9}$ can be evolved parametrically in terms of various 3D rotational angles and other dimensional parameters.

The cross-sectional profile $\left(V_{0} \ldots V_{9}\right)$ of a single tooth at one end of the cutter is composed of ten parametric curve (real / virtual) segments defined in terms of parameter s , from $\mathrm{p}_{1}(s)$ to $\mathrm{p}_{10}(s)$. The curve segments $\mathrm{p}_{1}(s), \mathrm{p}_{3}(s), \mathrm{p}_{4}(s), \mathrm{p}_{6}(s), \mathrm{p}_{7}(s)$ and $\mathrm{p}_{9}(s)$ are straight lines joining vertices $V_{0} V_{3}, V_{9} V_{8}, V_{7} V_{8}, V_{4} V_{5}, V_{0} V_{1}$ and $V_{2} V_{3}$ respectively, while, the curve $\mathrm{p}_{2}(s)$ is the profile of the relieving surface governed by vertices $V_{3} V_{4} V_{9}$. The curves $\mathrm{p}_{5}(s), \mathrm{p}_{8}(s)$ and $\mathrm{p}_{10}(s)$ are circular arcs of radii $r_{f}, r_{f}$ and $d_{r e} / 2$ respectively between $V_{5} V_{6} V_{7}, V_{1} V_{2}$ and $V_{0} V_{8}$. The equation of these curve segments are parametrically evolved as,

$$
\begin{align*}
& p_{1}(s)=\left[\begin{array}{llll}
\frac{d_{r e}}{2}+s\left(\frac{D_{e}}{2}-\frac{d_{r e}}{2}\right) & 0 & 0 & 1
\end{array}\right]  \tag{2.5}\\
& p_{3}(s)=\left[\begin{array}{lll}
r_{v 9} \cos \psi+s \cos \psi\left(\frac{d_{r e}}{2}-r_{v 9}\right) \quad r_{v 9} \sin \psi+s \sin \psi\left(\frac{d_{r e}}{2}-r_{v 9}\right) & 0 & 1
\end{array}\right] \text {, where } 0 \leq s \leq 1 \text {. }  \tag{2.6}\\
& p_{5}(s)=\left[\begin{array}{llll}
c_{1 x}^{\prime}+r_{f} \cos s & c_{1 y}^{\prime}-r_{f} \sin s & 0 & 1
\end{array}\right] \text {, where } \frac{\pi}{2}-\left(\tan ^{-1} m_{v 5 v 4}\right) \leq \mathrm{s} \leq \pi-\left(\tan ^{-1} m_{v 7 c^{\prime} 1}\right) \text {, }  \tag{2.7}\\
& m_{v 5 v 4} \text { and } m_{v 7 c^{\prime} 1} \text { are the slopes of line segments } V_{5} V_{4} \text { and } V_{7} c_{1}^{\prime} \text { respectively. } \\
& p_{6}(s)=\left\lfloor r_{v 4} \cos \phi+s\left(V_{5 x}-r_{v 4} \cos \phi\right) \quad r_{v 4} \sin \phi+s\left(V_{5 y}-r_{v 4} \sin \phi\right) \quad 0 \quad 1\right\rfloor \text {, where } 0 \leq s \leq 1 \text {. }  \tag{2.8}\\
& p_{8}(s)=\left[\begin{array}{llll}
c_{1 x}+r_{f} \cos s & c_{1 y}-r_{f} \sin s & 0 & 1
\end{array}\right] \text {, where } 2 \pi-\left(\tan ^{-1} m_{c 1 v 1}\right) \leq s \leq\left(\frac{3 \pi}{2}+\gamma_{1}\right) \tag{2.9}
\end{align*}
$$

and $m_{c 1 v 1}$ is the slope of line segment $c_{1} V_{1}$.


Fig. 3: Half view of the composite cross-sectional curve in XY plane.

$$
\begin{gather*}
p_{9}(s)=\left[\begin{array}{llll}
\frac{D_{e}}{2}-l_{1} \cos \gamma_{1}(1-s) & l_{1} \sin \gamma_{1}(1-s) & 0 & 1
\end{array}\right], \text { where } 0 \leq s \leq 1 .  \tag{2.10}\\
p_{10}(s)=\left[\begin{array}{llll}
\frac{d_{r e}}{2} \cos s & \frac{d_{r e}}{2} \sin s & 0 & 1
\end{array}\right], \text { where }(2 \pi-\psi) \leq s \leq 2 \pi \text { and } \psi=\frac{2 \pi}{\mathrm{~N}} . \tag{2.11}
\end{gather*}
$$

$p_{4}(s)$ and $p_{7}(s)$ are enveloped by the virtual curves $p_{3}(s)$ and $p_{1}(s)$ respectively.

### 2.1.3 Cutter tooth surface patches

As mentioned earlier, surface patches labeled $\Sigma_{1}$ to $\Sigma_{5+l+f}$ (as shown in Fig. 1) represents a cutter tooth of the generic form milling cutter. The surface patches $\Sigma_{2 \mathrm{i}}(i \in 1$ tol $)$ and $\Sigma_{2 j+1}(j \in 1$ tof $)$ are modeled as 1rail sweep surfaces. These surfaces are formed while sweeping the profile curve ( $V_{0} \ldots V_{i}($ for $i \in l+f)$ ) lying on $\mathrm{Z}_{1} \mathrm{X}_{1}$ plane along a boundary curve ( $p_{2}(s)$ between vertices $\mathrm{V}_{3} \mathrm{~V}_{4} \mathrm{~V}_{9}$ ) lying on XY plane. These are parametrically defined as,

$$
\begin{equation*}
\mathbf{p}_{\mathrm{i}}(t, s)=p_{\mathrm{i}}(t) \cdot\left[\mathrm{T}_{s}\right] \tag{2.12}
\end{equation*}
$$

Here, $\left[\mathrm{T}_{s}\right.$ ] is the transformation matrix meant for sweeping along logarithmic spiral, it is given as,

$$
T_{s}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\frac{\mathrm{D}_{\mathrm{e}}}{2} e^{s \tan \gamma_{2}} \cos s & \frac{\mathrm{D}_{\mathrm{e}}}{2} e^{s \tan \gamma_{2}} \sin s & 0 & 1
\end{array}\right],
$$

where $(2 \pi-\psi) \leq s \leq 2 \pi, \psi=\frac{2 \pi}{N}$ and $\gamma_{2}$ is the constant relief angle.
The sweeping operation $p_{2 i-1}(t)$.[T $\left.T_{s}\right]$ models peripheral lands $\left(\Sigma_{2 i}(i \in 1\right.$ to $l)$ ), while, flank surfaces $\left(\Sigma_{2 j+1}(j \in 1\right.$ to $\left.f)\right)$ are modeled by performing the operations $p_{2 i}(t) .\left[\mathrm{T}_{s}\right](i \in 1$ to $f)$. The end surface $\left(\Sigma_{4+l+f}\right)$ is a linear lofted surface generated by joining corresponding points on two space curves ( $p_{10}(s)$ and $p_{2}(s)$ lying on XY plane) explicitly. The surface patch $\Sigma_{5+l+f}$ is formed by translating the surface patch $\Sigma_{4+l+f}$ by $d_{(5+l+f) 3}$ along $Z$ axis. The distance $d_{(5+l+f) 3}=2 R \sin \theta$ for circular crown type form milling cutter and for various other profiles (straight or tapered type), $d_{(5+l+f) 3}=$ length of the cutter.

The surface patches $\Sigma_{1}, \Sigma_{2+l+f}$ and $\Sigma_{3+l+f}$ are modeled as a 2-rail sweep surface [1]. The surface patch $\Sigma_{1}$ and partial surface of $\Sigma_{3+l+f}\left(\Sigma_{3+l+f}\right.$ joins $\mathrm{N}^{\text {th }}$ tooth to $(\mathrm{N}+1)^{\text {th }}$ tooth) is modeled by sweeping the composite cross-sectional curve $p(u)$ (composite curve $V_{1} V_{2} V_{3}: p_{8}(s)$ and $p_{9}(s)$ ) lying on XY plane along two boundary curves $p_{12}(t)$ and $p_{13}(t)$. Shown in Fig. 4 is the typical description of sweep surface patch. $p_{12}(t)$ is the virtual outer crown (circular arc) $V_{0} V_{11}$ lying in the axial sectional plane $\mathrm{Z}_{1} \mathrm{X}_{1} \cdot p_{13}(t)$ is the inner crown of the form milling cutter in axial sectional plane $\mathrm{Z}_{1} \mathrm{X}_{1}$ joining the vertex $V_{1}$ of the cross-sectional profile of the cutter at one end to the corresponding vertices at the middle section and at the opposite end of the cutter.

Hence, the surface patch $\Sigma_{1}$ and partial surface patch of $\Sigma_{3+l+f}$ is mathematically defined as,

$$
\begin{equation*}
\mathbf{p}_{1,3+l+f}(u, v)=f(v)[p(u)-p(0) \alpha(u)-p(1) \beta(u)] \cdot \operatorname{Sweep}(\eta, \theta(v))+\left[\alpha(u) p_{12}(v)-\beta(u) p_{13}(v)\right], \tag{2.13}
\end{equation*}
$$

for $0 \leq u, v \leq 1$, where $f(v)=\left(p_{12}(v)-p_{13}(v)\right) /(p(0)-p(1))$
$\eta, \theta(v)$ : intersection angles on section plane and base plane respectively
$\alpha(u)=1-3 s^{2}+2 s^{3} ; \beta(u)=3 s^{2}-2 s^{3}$ : blending functions and

$$
\text { Sweep }(\eta, \theta(v))=\left[R_{z, \eta}\right]\left[R_{x,-90}\right]\left[R_{z, \theta(v)}\right]
$$

$$
=\left[\begin{array}{cccc}
\cos \eta \cos \theta(v) & \cos \eta \sin \theta(v) & -\sin \eta & 0 \\
-\sin \eta \cos \theta(v) & -\sin \eta \sin \theta(v) & -\cos \eta & 0 \\
-\sin \theta(v) & \cos \theta(v) & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

Surface patch $\Sigma_{2+l+f}$ and partial surface patch of $\Sigma_{3+l+f}\left(\Sigma_{3+l+f} \text { joins } \mathrm{N}^{\text {th }} \text { tooth to ( } \mathrm{N}-1\right)^{\text {th }}$ tooth) is a 2-rail sweep surface. A similar approach is adopted for these surfaces. The composite cross-sectional curve $p(u)$ (composite curve $V_{4} V_{5} V_{6} V_{7}: p_{6}(s)$ and $\left.p_{5}(s)\right)$ lies on XY plane along two boundary curves $p_{14}(t)$ and $p_{15}(t)$. $p_{14}(t)$ is the virtual outer crown (circular arc) joining the vertex $V_{4}$ of the crosssectional profile of the cutter at one end to the corresponding vertices at the middle section and at the opposite end of the cutter. $p_{15}(t)$ is the inner crown of the generic form milling cutter and is formed while rotating the curve $p_{13}(t)$ by angle $\psi$ in an anti-clockwise direction about origin 0 .

### 2.2 Mathematical Model of the Cutter Body

The surface patches that join all the surface patches of the cutter teeth and completes the body of the form milling cutter are mentioned as surface patches of the cutter body. The core cutter body depends upon the type of form milling cutter; (a) end mill type (b) disc type. For an end mill type cutter, it is quite similar to the body of generic end mill cutter as described in [14]. In disc type cutter, these surface patches are six in number, identified as (i) right body end surface patch ( $\Sigma_{50}$ ), (ii) left body end surface patch ( $\Sigma_{51}$ ), (iii) bore surface patch ( $\Sigma_{52}$ ) and (iv) three surface patches ( $\Sigma_{53}, \Sigma_{54}$ and $\Sigma_{55}$ ) making the keyway [12]. These patches are either planar or cylindrical in geometry and accordingly expressed parametrically.

## 3. IMPLEMENTATION

This section presents the implementation of proposed methodology to model the geometry of various form milling cutters (belonging to the same conceptual family) from a set of specified variables. The parameters (3D rotational angles and other dimensional parameters) with their relevant data needed

(a)

(b)

(c)

Fig. 4: Description of a sweep surface patch (a) Section view (b) Plan view (c) 3D view.
to completely describe the cutters are shown in Table 1 . The dimensional parameters adopted in this work follow the basic tool design principles. As the primary objective of the modeling is to obtain the valid geometry of the tool meant to achieve the desired cutting surface, the constraints are incorporated in the mathematical model of the cutter developed. To model a complex cutter conveniently, an interactive tool design interface has been developed. This design tool helps to render the proposed geometry of the cutter in a commercial CAD environment and validates the parameterized geometric design of the cutter. The rendered cutters for the parameters given in Table 1 are shown with the help of Fig. 5. These cutters are generated through the design tool interface and rendered in CATIA V5 environment. This validates the proposed modeling approach. The work does not include any additional data verification system to check the integrity of the model, but it can be incorporated if proper software is developed for a commercial CAD system.

The cutting edge profile of the cutter is controlled by the user dependent polygonal control points for respective NURBS curves. By varying the geometric and dimensional parameters of the proposed cutter model, form cutters like straight, circular and tapered profile cutters are generated. These cutters can be effectively used for generating multiple sculptured surfaces, multiple radii fillets or circular profiles as observed in various aircraft and automobile components, etc.

## 4. TECHNOLOGY DEMONSTRATION

This section depicts one of the advantages of 3D modeling of cutting tools over conventional approach by unfolding one of the possible downstream applications that can be performed, once a valid 3D cutter model is available. Here, the fabrication of one of the form milling cutter is performed using additive manufacturing (AM) process. For the purpose of physical visualization and design verification,

Tab. 1: Geometric (dimensional) parameters of various form milling cutters.

| Dimensional Parameters | Value (mm) |  |  |
| :--- | :---: | :---: | :---: |
|  | Cutter 1 | Cutter 2 | Cutter 3 |
| Outer circle diameter of the cutter at the centre, $\left(D_{c}\right)$ | 56 | - | 80 |
| Outer circle diameter of the cutter at the right end, $\left(D_{R e}\right)$ | 56 | 25 | 56 |
| Outer circle diameter of the cutter at the left end, $\left(D_{L e}\right)$ | 56 | 80 | 56 |
| Root circle diameter at the centre, $\left(d_{r c}\right)$ | 40 | - | 52.5 |
| Root circle diameter at the right end, $\left(d_{r e}\right)$ | 40 | 16 | 40 |
| Root circle diameter at the left end, $\left(d_{l e}\right)$ | 40 | 52.5 | 40 |
| Bore diameter, $(d)$ | 25 | - | 25 |
| Shank diameter, $(d)$ | - | 32 | - |
| Fillet radius at the centre, $\left(R_{f}\right)$ | 1.0 | - | 2.0 |
| Fillet radius at the right end, $\left(r_{r f}\right)$ | 1.0 | 0.6 | 1.0 |
| Fillet radius at the left end, $\left(r_{l f}\right)$ | 1.0 | 2.0 | 1.0 |
| Length of rake face at centre, $\left(L_{f}\right)$ | 6.8 | - | 11.8 |
| Length of rake face at the right end, $\left(l_{r 1}\right)$ | 6.8 | 3.6 | 6.8 |
| Length of rake face at the left end, $\left(l_{l 1}\right)$ | 6.8 | 11.8 | 6.8 |
| Width of the cutter, $(L)$ | 40 | 60 | 62 |
| Number of inserts, $(N)$ | 10 | 10 | 10 |
| Rotational Angles |  |  |  |
| Radial rake angle, $\left(\gamma_{1}\right)$ | 3 | 3 | 3 |
| Radial relief angle, $(\alpha)$ | 5 | 5 | 5 |
| Crowning angle $(\theta)$ | - | - | 46 |

Computer-Aided Design \& Applications, 10(5), 2013, 757-765
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the designed circular type form milling cutter is manufactured using AM process. The 3D CAD model of the form cutter (as shown in Fig. 5(c)) developed by the proposed methodology is imported in one of the commercial CAD software. The surface model of the cutter is converted into solid model and the .STL file format of the cutter is exported to AM machine. The circular type form milling cutter as shown in Fig. 5(c) is manufactured with Fused Deposition Modeling (FDM) technique (using Stratasys 400 mc system) and shown with the help of Fig 6(a). Here, the material used to manufacture the cutter is Acrylonitrile Butadiene Styrene (ABS-M30) with soluble support material as SR-30.

Further, the same 3D based definition of the novel circular form milling cutter is used to generate G-codes using CNC machining software so as to fabricate it using 5 -axis CNC optical profile grinder (Fig. 6(b)). This cutter has been manufactured at the works of one of the cutter manufacturers. The cutter is manufactured with High Speed Steel (HSS) as the tool material. The present example is just a demonstration of the developed technology and not a detailed exercise on machining or refinement of manufacturing methodology of the cutter. Design verification of the manufactured cutter can also be done by performing the force and torque measurement for the designed cutter.

## 5. CONCLUSIONS

The geometric modeling of cutting tools is an important aspect for design and manufacturing engineers from the viewpoint of shape realization. Once an accurate biparametric surface model of the customized cutting tool is evolved it can be used for various downstream technological applications. Although there exists many 2D based representation schemes and a few 3D based geometric modeling


Fig. 5: Rendering of form milling cutters (a) Cutter 1 (straight type), (b) Cutter 2 (tapered type), (c) Cutter 3 (circular type).


Fig. 6: Circular type form milling cutter fabricated through (a) additive manufacturing process (b) 5axis CNC optical profile grinder.
schema of cutting tools, but there is a need to develop generic parametric models for various families of cutters in order to cover the entire domain of cutting tools. In this manuscript, 3D based parameterized geometric design of generic form milling cutter in terms of NURBS curves and 1-rail, 2rail sweep surfaces has been illustrated. This modeling schema enables profiles of tooth surfaces to be interactively controlled by manipulating the parameters of control polygons and sweep trajectories. Further, a variety of cutters belonging to the same family of form milling cutters are evolved with the help of the generic definition. To physically visualize the cutter, circular type form milling cutter is fabricated. The proposed modeling paradigm opens up avenues to conveniently design and modify the geometry of various other customized cutters as per the application.

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