www.cadanda.com Computer-Aided Design AND APPLICATIONS

The Analysis of T-spline Fairness

Wang Aizeng and Zhao Gang

Beihang University, Beijing, China, aizwang@126.com

ABSTRACT

The fairness of curves and surfaces has been used in the shape design of industrial products more and more widely. With the wide applications of T-splines in the surface modeling, this paper analyzes the fairness of T-splines and gives some fairness properties of standard T-splines and semi-standard T-splines. For some special T-splines, this paper proposes a method of comparing the T-spline fairness. The research will contribute to the relevant research of T-spline fairing algorithms, and will contribute to the applications of T-splines in computer-aided manufacturing.

Keywords: T-spline, fairness, surface.
DOI: 10.3722/cadaps.2013.817-825

1 INTRODUCTION

The modeling technology of curves and surfaces, that originates from the craft of laying off adopted in the manufacturing of aircrafts, automotive and marine, etc, is an important research in the computer-aided geometric design and computer-aided manufacturing. Nowadays modeling technology has been widely used in the manufacturing of aircrafts, automotive and marine, etc, and in the topography description and human organs modeling, etc. In the late 1980's, non-uniform rational B-spline method had become an important tool for shape description of curves and surfaces. Now, classical CAD/CAM system usually uses NURBS as common expressions for free curves and surfaces, and widely applies it to the product design such as aircraft, automobiles and ships, etc[1][2]. The surface modeling method based on non-uniform rational B-spline (NURBS) is continuous modeling, but it is not easy to represent complex surfaces. When the topology structure of the surface is much more complex, we often cut and splice multiple B-spline surface patches to construct the complex surface. These B-spline surface patches often have gaps in the surface merging, and it is difficult to ensure geometric continuity. Manual intervention is needed in these gaps during NC machining, hence these gaps affect NC Machining efficiency and quality.

For these shortcomings of the NURBS, in 2003, Sederberg, etc, proposed the T-spline surface [3][4], that is non-uniform rational B-spline surface with T-knots. Compared with the B-spline surface, control grid lines of the T-spline do not have to be run through by allowing the emergence of T-knot, which breaks the limitation that control mesh of the traditional B-spline surface must satisfy rectangular topology. This property brings many significant advantages to the T-spline surface in the surface merging, data compression, and local refinement, etc [3][4]. Since the introduction of T-splines, many scholars have done extensive research for T-spline theory. The research can be mainly divided into two categories:

(1) surface modeling based on T-spline and the relevant theory research;
(2) The conversion between other modeling methods and T-splines;
For the T-spline surface modeling, in 2005, Song gave a free-form deformation method of the weighted T-spline in literature [5]; in 2006, Wang proposed a compression algorithm about control points while maintaining T-spline surfaces unchanged in literature [6]; in 2007, literature [7] presented a T-spline local refinement method based on NURBS; in 2008, Weng proposed an algorithm of embedding watermark on T-spline surfaces in literature [8]. In the same year Sederberg solved the NURBS triming problem using the T-spline [9]. After that, in 2010, Buffa gave a proof of linear independence about T-spline blending functions associated with some particular T-meshes [10]. Then literature [11] did further research on the T-spline blending functions. The analysis methods of combining T-splines and Isogeometric Analysis appeared in recent years [12][13]. In addition to the conversion between other modeling methods and T-splines, in 2005 Wang, etc, proposed an algorithm of mutual conversion between T-splines and level B-splines [14]. In the same year, they also proposed a method of fitting Z-Map models with T-splines[15]. In 2006, Li etc, proposed a method of transforming meshes with arbitrary topology shape into T-spline surfaces automatically [16]. Digital 3D modeling is generally represented by triangular grids, and in 2007 Wang, etc, proposed an algorithm of transforming triangular grids into T-spline surfaces [17]. In 2008, Peng gave a method of transforming arbitrary triangular grid model into the T-spline adaptively in literature [18]. After the T-spline was presented in 2003, T-spline methods develop rapidly in CAD, however, the literatures of T-spline fairness have not been found. As the fairness of curves and surfaces plays an important role in CAD/CAM, this paper has done some relative research for the fairness of T-splines.

The paper is structured as follows. Section 1 introduces the fairness of curves and surfaces; the definition of the T-spline and the relative properties of T-spline blending functions are introduced in Section 2; the analysis of the T-spline fairness is given in Section 3; Section 4 concludes this paper.

2 THE FAIRNESS OF CURVES AND SURFACES

The fairing of curves and surfaces plays an important role in curve and surface modeling, and with the development of society, the demanding of fairness is much higher. From the perspective of manufacturing and processing, the poor geometric fairness of design products can lead to difficult processing, which increases the processing cost. In addition, the fairness of curves and surfaces has a direct effect on the product quality and its physical properties. Therefore, the fairing of curves and surfaces is an important research in CAD/CAM.

Currently there are many fairing methods of NURBS curves and surfaces, according to the fairing principle these methods can be divided into local fairing and global fairing, etc. The earliest fairing method which is more influential is the B-spline curve fairing method based on knot removal and insertion, that proposed by Farin in 1987[19], and its essence is doing human interaction modification using the curve curvature map to make the curve curvature change more evenly. In 1990, Farin and Sapidis gave an automatic fairing algorithm of B-spline curves [20]. Then some fairing algorithms about energy method appeared subsequently [21][22]. In 2004, Li, etc, proposed an algorithm that finds bad points based on curvature map and fairs the curve by constraints optimization algorithm [23]. With the development of wavelet technology, in 2007, Amati proposed a fairing algorithm of curves based on quasi-uniform B-spline wavelet [24]. In 2009 Wang, etc, proposed a simplification and fairing algorithm of NURBS curves and surfaces based on non-uniform B-spline wavelet [25]. These methods expand the application of wavelet in curves and surfaces fairing and achieve some fairing results.

Compared to the NURBS theory, T-spline is a relatively new surface modeling technology and its relevant theory research needs further improvement. However, there have few literatures about T-spline fairing at present. In particular, if the T-spline will be extended to industry, the fairing of T-splines must be solved. Therefore, this paper gives the theoretical analysis of T-spline fairness, which can promote the relevant research of T-spline fairing algorithm and further make the application of the T-spline modeling technology wider in the digital design and industrial production.

3 T-SPLINES

T-Splines were defined first in 2003 by Sederberg et al., who also showed how to define PB-splines [3]. This work was extended in literature [4] to include definitions of semi-standard and non-standard T-splines. Obviously, the B-spline surface is a special standard T-spline. According to the relative literatures, the mathematical definition of the T-spline surface and the classification relations of the T-spline are given as follows [3][4]. The PB-spline surface defined on the T-mesh is called as the T-spline surface. While the notion of T-splines can extend to any degree, this paper restricts the discussion to cubic T-splines and defaults that the knot repeatability is less than $\deg ree+2$. Then we give the definition of the T-spline and the properties of T-spline blending functions.
The definition of the T-spline is:

$$P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)=\frac{\sum_{i=1}^{n}(x_i,y_i,z_i)B_i(u,v)}{\sum_{i=1}^{n}\omega_i B_i(u,v)} \tag{1}$$

Where

$$P_i=(w_i x_i, w_i y_i, w_i z_i, w_i)$$

the T-spline blending function of the control point P_i is $B_i(u,v)$, $B_i(u,v)$ is given by

$$B_i(u,v)=N[u_i](u)N[v_i](v) \tag{2}$$

Where $N[u_i](u)$ is the B-spline basis function defined on the knot vector

$$\mathbf{u}_i=[u_{i1},u_{i2},u_{i3},u_{i4},u_{i5}]$$

$N[v_i](v)$ is the B-spline basis function defined on the knot vector

$$\mathbf{v}_i=[v_{i1},v_{i2},v_{i3},v_{i4},v_{i5}]$$

$\mathbf{u}_i=[u_{i1},u_{i2},u_{i3},u_{i4},u_{i5}]$ and $\mathbf{v}_i=[v_{i1},v_{i2},v_{i3},v_{i4},v_{i5}]$ are given by the corresponding T-mesh respectively. Some properties of T-spline blending functions are given as follows.

2.1 The properties of T-spline blending functions

Figure 1 shows a T-spline blending function $B_i(u,v)$ defined on the knot lines, the T-spline blending functions have some properties as follows.

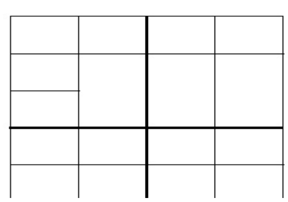

Fig. 1: Knot lines for T-spline blending function.

(1) Non-negative: like the properties of B-spline basis functions, the T-spline blending functions are non-negative.

(2) Compact support: $B_i(u,v)\begin{cases}\geq 0,[u_{i1} \quad u_{i5}]\times[v_{i1} \quad v_{i5}] \\ = 0, otherwise\end{cases}$

(3) Differentiability: the T-spline blending function is infinitely differentiable inside the knot interval and $degree - r$ times differentiable at the knot, where r is knot multiplicity.

(4) Unlike B-splines, T-spline blending functions can not consist of a set of basis functions unless the T-spline satisfies some conditions.

According to the properties of T-spline blending functions, the classification definition of T-splines is given as follows[3][4].

2.2 The classification of T-splines

For any T-spline

$$P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)$$

assume $B_i(u,v)$ corresponds to the i^{th} T-spline control point, then the classification definition of the T-spline is as follows[3][4].

(1) If

$$\sum_{i=1}^{n} B_i(u,v)\equiv 1$$

then we call the T-spline is a standard T-spline.

(2) If there exists a set of weights ω_i not all equal to 1 for which

$$\sum_{i=1}^{n} \omega_i B_i(u,v)\equiv 1$$

then we call the T-spline is a semi-standard T-spline.

(3) If there does not exist a set of weights ω_i for which $\sum_{i=1}^{n} \omega_i B_i(u,v)\equiv 1$, then we call the T-spline is a non-standard T-spline.

From the classification definition of T-splines we know:

{T-splines}={standard T-splines}$\bigcup${semi-standard T-splines}$\bigcup${non-standard T-splines}.

As Figure 2 shows.

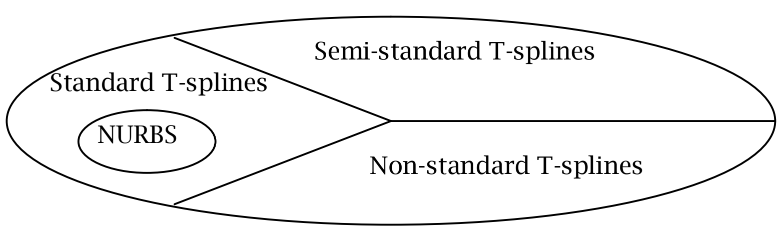

Fig. 2: The classification of T-splines.

Based on the above literatures, from the view of algebraic point this paper gives the mathematical analysis and relevant proof of the T-spline fairness in Section 4.

4 THE ANALYSIS OF T-SPLINE FAIRNESS

So far, we have not found the relevant literatures on the T-spline fairness, to some extent, the problems about the T-spline fairness restrict the application of T-splines in CAD/CAM. This section gives some mathematical properties of T-spline fairness based on relevant literatures. The definition of global fairness of a set of functions is given first as follows.

The global fairness of a set of functions: for a set of functions $\{B_i\}_{i=1}^n$, if there exist $\{\omega_i\}_{i=1}^n$ satisfying $\sum_{i=1}^n \omega_i B_i \equiv 1$, then we call the set of functions $\{B_i\}_{i=1}^n$ has the property of global fairness.

Property 1: If a set of functions $\{B_i\}_{i=1}^n$ has the property of global fairness, then for the PB-spline $P(s,t)=\sum_{i=1}^n P_i B_i(s,t)$, we can always have $P(s,t)=\sum_{i=1}^n P_i B_i \equiv P_s$ by moving control points $\{P_i\}_{i=1}^n$ continuously.

Property 2: If the T-spline blending functions $\{B_i\}_{i=1}^n$ have the property of global fairness, we can always have the T-spline $P(s,t)=\sum_{i=1}^n P_i B_i \equiv P_s$ by moving control points continuously.

It can be seen that the blending functions of standard T-splines and semi-standard T-splines have the property of global fairness, but the blending functions of non-standard T-splines don't have the property of global fairness. If a set of functions has the property of global fairness, then the corresponding PB-spline can construct smooth surface easier. For PB-splines, we generally choose a set of blending functions that has the property of global fairness.

The fairness analysis of some special T-splines:

(1) For a standard T-spline, it has the smoothest status when $P_1 = P_2 = \cdots = P_i = \cdots = P_n$.

Proof: For a standard T-spline, The corresponding blending functions satisfy

$$\sum_{i=1}^n B_i(u,v) \equiv 1 \tag{3}$$

and when

$$P_1 = P_2 = \cdots = P_i = \cdots = P_n \tag{4}$$

we can get

$$P(u,v)=\sum_{i=1}^n P_i B_i\left(u,v\right) \equiv P_n \tag{5}$$

It is to say the T-spline has the smoothest status when $P_1 = P_2 = \cdots = P_i = \cdots = P_n$. □

(2) For a standard T-spline, this paper assumes that $P_1 = P_2 = \cdots = P_{n-1} = \frac{1}{m} * P_n$, $P_1^a = P_2^a = \cdots = P_{n-1}^a = \frac{1}{n} * P_n^a$, where $P_1 = P_1^a \neq 0$, if $1 < m < n$, then compared $P\left(u,v\right)=\sum_{i=1}^n P_i B_i\left(u,v\right)$ with $P^\alpha\left(u,v\right)=\sum_{i=1}^n P_i^a B_i\left(u,v\right)$, the T-spline surface $P\left(u,v\right)=\sum_{i=1}^n P_i B_i\left(u,v\right)$ has better fairness.

Proof: For a standard T-spline, when

$$P_1 = P_2 = \cdots = P_{n-1} = \frac{1}{m} * P_n \tag{6}$$

then we have the T-spline

$$P(u,v) = \sum_{i=1}^{n} P_i B_i(u,v) = P_1 + \frac{m-1}{m} * P_n B_n(u,v) \tag{7}$$

When

$$P_1^a = P_2^a = \cdots = P_{n-1}^a = \frac{1}{n} * P_n^a \tag{8}$$

then the corresponding T-spline is

$$P^{\alpha}(u,v) = \sum_{i=1}^{n} P_i^{\alpha} B_i(u,v) = P_1^{\alpha} + \frac{n-1}{n} * P_n^{\alpha} B_n(u,v) \tag{9}$$

As

$$0 < \frac{m-1}{m} < \frac{n-1}{n} \tag{10}$$

and

$$P_1 = P_1^a,\ P_n = P_n^a \tag{11}$$

From Equations(7)(9)(10)(11), we can see the T-spline surface $P(u,v) = \sum_{i=1}^{n} P_i B_i(u,v)$ has better fairness compared with the T-spline surface $P^{\alpha}(u,v)$. □

(3) For a semi-standard T-spline, if $\sum_{i=1}^{n} \omega_i B_i(u,v) \equiv 1$, then when $\frac{P_1}{\omega_1} = \frac{P_2}{\omega_2} = \cdots = \frac{P_i}{\omega_i} = \cdots = \frac{P_n}{\omega_n}$, T-spline $P(u,v) = \sum_{i=1}^{n} P_i B_i(u,v)$ has the smoothest status.

Proof: As the T-spline blending functions corresponding to a semi-standard T-spline satisfy

$$\sum_{i=1}^{n} \omega_i B_i(u,v) \equiv 1 \tag{12}$$

When

$$\frac{P_1}{\omega_1} = \frac{P_2}{\omega_2} = \cdots = \frac{P_i}{\omega_i} = \cdots = \frac{P_n}{\omega_n} \tag{13}$$

we have

$$\sum_{i=1}^{n} P_i B_i(u,v) = \frac{P_n}{\omega_n} \tag{14}$$

It is to say that the T-spline has the smoothest status. □

(4) For a semi-standard T-spline satisfying $\sum_{i=1}^{n} \omega_i B_i(u,v) \equiv 1$, this paper assumes $\frac{P_1}{\omega_1} = \frac{P_2}{\omega_2} = \cdots = \frac{P_{n-1}}{\omega_{n-1}} = \frac{1}{m} * \frac{P_n}{\omega_n}$, $\frac{P_1^a}{\omega_1} = \frac{P_2^a}{\omega_2} = \cdots = \frac{P_{n-1}^a}{\omega_{n-1}} = \frac{1}{n} \frac{P_n^a}{\omega_n}$. where $P_1 = P_1^a \neq 0$, $1 < m < n$, then

compared the T-spline surface $P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)$ with $P(u,v)=\sum_{i=1}^{n} P_i^a B_i(u,v)$, the T-spline surface $P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)$ has better fairness.

Proof: For a semi-standard T-spline, when

$$\frac{P_1}{\omega_1}=\frac{P_2}{\omega_2}=\cdots=\frac{P_{n-1}}{\omega_{n-1}}=\frac{1}{m}*\frac{P_n}{\omega_n} \tag{15}$$

we can get

$$P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)=\frac{P_1}{\omega_1}+\frac{m-1}{m}*\frac{P_n}{\omega_n}B_n(u,v) \tag{16}$$

and when

$$\frac{P_1^a}{\omega_1}=\frac{P_2^a}{\omega_2}=\cdots=\frac{P_{n-1}^a}{\omega_{n-1}}=\frac{1}{n}\frac{P_n^a}{\omega_n} \tag{17}$$

the T-spline is

$$P^\alpha(u,v)=\sum_{i=1}^{n} P_i^\alpha B_i(u,v)=\frac{P_1^\alpha}{\omega_1}+\frac{n-1}{n}*\frac{P_n^\alpha}{\omega_n}B_n(u,v) \tag{18}$$

As $0<\frac{m-1}{m}<\frac{n-1}{n}$ and $P_1=P_1^a$, from Equations(16)(18), we can get the T-spline surface $P(u,v)=\sum_{i=1}^{n} P_i B_i(u,v)$ has better fairness compared with the T-spline surface $P^\alpha(u,v)$. □

Property 3: For a standard T-spline and a semi-standard T-spline, we can make the original T-spline into a point by moving its control points continuously.

Property 4: For a non-standard T-spline, if its T-spline blending functions are linearly independent, then no matter how to move its control points continuously, it is difficult to make the original surface into a point.

5 EXAMPLES

This section applies the result of the above section to several T-splines. Figure 3 shows the example of a standard T-spline, the original T-spline 1 which contains 26 control points. Let $B^0=\left[B_1^0(u,v) \quad B_2^0(u,v) \quad \cdots \quad B_{26}^0(u,v)\right]^T$ be blending functions of the original T-spline 1, then $\sum_{i=1}^{26} B_i^0(u,v)=1$. According to the result of the above section, it can be seen that the blending functions of T-spline 1 have the property of global fairness, which means we can make the T-spline 1 surface into a point by moving its control points continuously.

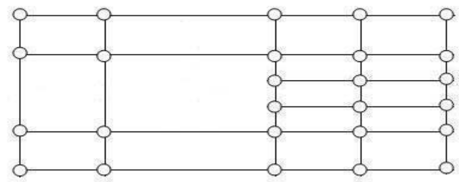

Fig. 3: T-spline 1.

Figure 4 is another example about T-splines, where the T-spline 2 contains 28 control points without repeated knots. Let $B^0 = \left[B_1^0(u,v) \quad B_2^0(u,v) \quad \cdots \quad B_{28}^0(u,v) \right]^T$ be the blending functions of the T-spline 2, the T-spline 2 is a non-standard T-spline and its T-spline blending functions are linearly independent. The blending functions of T-spline 2 satisfy $\forall \omega_i$, $\sum_{i=1}^{28} \omega_i B_i^0(u,v) \neq 1$. By the result of the above section we can get the blending functions of T-spline 2 don't have the property of global fairness. That means no matter how to move the control points of T-spline 2 continuously, it is difficult to make the T-spline 2 surface into a point.

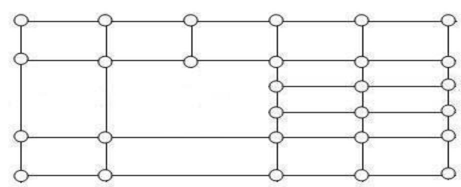

Fig. 4: T-spline 2.

6 CONCLUSIONS

This paper gives some mathematical analysis of the T-spline fairness and proposes a method of comparing the fairness for some special T-splines. The T-spline fairness plays an important role in T-spline surface modeling and digital manufacturing, so the results in this paper have a significant effect on the surface modeling and digital manufacturing, and it will increase the T-spline application in industry.

Acknowledgements
This work is supported by the National Nature Science Foundation of China (Project Number 61170198).

REFERENCES

[1] Zhang J.: Research on Generating Subdivision Surface and Applying to Surface Modeling. Zhejiang University, Hangzhou, P.R.China, Department of Mathematics, 2003.
[2] Piegl L, Tiller W. The NURBS Book, Springer Press, 1997.

[3] Sederberg, T. W.; Zheng, J.; Bakenov, A.; Nasri, A.: T-slines and T-NURCCs, ACM Transaction on Graphics, 22(3), 2003, 477-484. [doi:10.1145/882262.882295]
[4] Sederberg,T.W.; Cardon,D.L.; Zheng, J.;,Lyche, T.: T-spline simplification and local refinement, ACM Transactions on Graphics, 23(3), 2004, 276-283. [doi:10.1145/1015706.1015715]
[5] Song, W.; Yang, X.: Free-form deformation with weighted T-spline, The Visual Computer, 21(3), 2005, 139-151. [doi:10.1007/s00371-004-0277-8]
[6] Wang, Y.; Zheng, J.: Control point removal algorithms for T-spline surfaces, Geometric Modeling and Processing, 4077, 2006, 385-396. [doi:10.1007/11802914_27]
[7] Edward, B.; Sebastien, C.: Beyond NURBS: enhancement of local refinement through T-splines, in proceedings of SPIE, Volume: 6670, 2007, 66700J-66700J-10. [doi:10.1117/12.735584]
[8] Weng, B.; Pan, R.; Yao, Z.; Yang, S.: Watermarking T-Spline Surfaces, 2008 11 IEEE International Conference on Communication Technology Proceedings. [doi:10.1109/ICCT.2008.4716246]
[9] Sederberg, T. W.; Finnigan, G. T.; Li, X.; Lin, H.; Ipson, H.: Watertight trimmed NURBS, ACM Transaction on Graphics, 27(3), 2008, 1-8. [doi:10.1145/1399504.1360678]
[10] Buffa, A.; Cho, D.; Sangalli, G.: Linear independence of the T-spline blending functions associated with some particular T-meshes, Comput. Methods Appl. Mech, Engrg, 199(23-24), 2010, 1437-1445. [doi:10.1016/j.cma.2009.12.004]
[11] Wang, A.; Zhao, G.: The Analysis of T-spline Blending Functions Linear Independence. Computer Aided Design and Applications, 8(5), 2011, 735-745. [doi:10.3722/cadaps.2011.735-745]
[12] Beirao da Veiga, L.; Buffa, A.; Cho, D.; Sangalli, G.: IsoGeometric analysis using T-splines on two-patch geometries, Computer Methods in Applied Mechanics and Engineering, 200 (21-22), 2011, 1787-1803. [doi:10.1016/j.cma.2011.02.005]
[13] Nguyen-Thanh, N.; Nguyen-Xuan, H.; Bordas, S.P.A.; Rabczuk, T.: Isogeometric analysis using polynomial splines over hierarchical T-meshes for two-dimensional elastic solids, Computer Methods in Applied Mechanics and Engineering, 200 (21-22), 2011, 1892-1908. [doi:10.1016/j.cma.2011.01.018]
[14] Wang, Y.; Zheng, J.; Seah, H. S.: Conversion between T-splines and hierarchical B-splines, Proceedings of the Eighth IASTED International Conference COMPUTER GRAPHICS AND IMAGING, August 15-17, 2005, Honolulu, Hawaii, USA, 2005, 8-13.
[15] Zheng, J.; Wang, Y.;,Seah, H. S.: Adaptive T-spline Surface Fitting to Z-Map Models, GRAPHITE, 405-411,2005[doi:10.1145/1101389.1101468]
[16] Li, W.; Ray, N.;,Levy, B.: Automatic and Interactive Mesh to T-Spline Conversion, SGP '06 Proceedings of the fourth Eurographics symposium on Geometry processing, 2006, 191-200.
[17] Wang, Y.; Zheng, J.: Adaptive T-spline Surface Approximation of Triangular Meshes, IEEE, 2007, 1-5. [doi:10.1109/ICICS.2007.4449775]
[18] Peng, X.; Tang, Y.: Adaptive Reconstruction of T-Spline Surfaces of Arbitrary Topological Type, CAID, 2008 IEEE 9th International Conference. 599-603. [doi:10.1109/CAIDCD.2008.4730639]
[19] Farin,G.: Fairing Cubic B-spline Curves, Computer Aided Geometric Design, 4(1/2), 1987, 91-103. [doi:10.1016/0167-8396(87)90027-6]
[20] Farin, G.; Sapidis, N.: Automatic fairing algorithm for b-spline curves, Computer-Aided Design 22(2), 1990, 121-129. [doi:10.1016/0010-4485(90)90006-X]
[21] Terzopoulos, D.; Platt, J.; Barr, A.; Fleischer, K.: Elastically Deformable Models, ACM Computer Graphics, 21(4), 1987, 205-213. [doi:10.1145/37401.37427]
[22] Celniker, G.; Gossard, D.: Deformable Curve and Surface Finite-elements for Free-form Shape Design, ACM Computer Graphics, 25(4), 1991, 257-266. [doi:10.1145/127719.122746]
[23] Li, W.; Xu, S.; Zheng, J.; Zhao, G.: Target Curvature driven fairing algorithm for Planar cubic B-Spline Curves, Computer Aided Geometric Design, 21, 2004, 499-513. [doi:10.1016/j.cagd.2004.03.004]
[24] Amati, G.: A multi-level filtering approach for fairing planar cubic B-spline curves, Computer Aided Geometric Design, 24,2007, 53-66. [doi:10.1016/j.cagd.2006.09.004]
[25] Wang, W.; Zhang, Y.: Wavelets-based NURBS simplification and fairing, Comput. Methods Appl. Mech, Engrg, 199(5/8), 2010, 290-300. [doi:10.1016/j.cma.2009.04.003]