Particle Swarm Optimization (PSO) Based Topology Optimization of Part Design with Fuzzy Parameter Tuning

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ABSTRACT

In order to improve the performance of the Particle Swarm Optimization (PSO) scheme on topology optimization of a part design, a fuzzy parameter tuning system is introduced in this study. Because thecorrelation and effect of constraints in PSO based topology optimization are uncertain and not well defined, a fuzzy logic system is used to adjust the constraints in PSO. After setting up the mathematical model, two illustrative examplesrunning in MATLAB environment are used to compare the performance of three topology optimization schemes: the Solid Isotropic Material with Penalization (SIMP) scheme, the standard PSO based topology optimization scheme and the fuzzy logic based PSO topology optimization scheme. It is found that, fuzzy logic based PSO has better search performance and efficiency than the standard PSO.

Keywords: particle swarm optimization (PSO), fuzzy logic, topology optimization, part design.

1. INTRODUCTION

Structural topology optimization is a complicated and multi-objective oriented optimization method used frequently for both structural and mechanical part design. Some researchers and manufacturers have already applied topology optimization in CAD process [6,7]. Briefly speaking, topology optimization attempts to achieve one or multiple objectives subject to several pre-defined constraints. Obviously constraints play a very important role in the optimization process. In traditional mathematical solution, constraints are expressed as strict criteria, however, the optimization algorithm or the optimization problems themselves are usually complicated and uncertain in practice, a clear set of constraints seems not truly reflect practical situation. Take an actual structural optimization problem as an example, the optimization may consider both the material weight constraint and a set of manufacturing constraints. The correlation and effect of these two types of constraints are uncertain and not well defined. A clear set of the effect of these constraints will cause man-made interference to the optimization results. Even under a single constraint, a clear set of constraint is still questionable for evolutionary based optimization algorithm. The reason is that evolutionary algorithms themselves are based on a natural selection process. Strict criteria may throw out potential candidates who slightly violate the pre-defined constraint at the beginning of optimization process. To solve the problem of uncertainty, vagueness, and application dependence of the optimization constraints, fuzzy logic is introduced into the PSO-based topology optimization method.

As a potential topology optimization method, Particle Swarm Optimization (PSO) is an evolutionary algorithm which was first reported by Kennedy and Eberhart [4]. This method attempts to mimic the social behavior of bird flocking. In a previous study by the authors [3], the performance and efficiency of PSO in topology optimization is found to lag behind traditional topology optimization algorithms such as the Solid Isotropic Material with Penalization (SIMP) method [1]. In previous studies, there were several modifications on PSO to improve its performance and efficiency [5,9]. However in traditional PSO and those modified PSO schemes, a pre-defined crispy material weight constraint is normally used. This strictly tightens search space. After introducing fuzzy parameter tuning into PSO, candidate solutions that slightly violate constraints at the early optimization process will be reserved, since they still stand a chance to obtain the optimal solution in subsequent iterations. As a

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result, the search space of PSO will be more flexible, the global search ability of PSO will be improved after using fuzzy parameter tuning. On the other hand, the search efficiency of PSO will be improved simultaneously because knowledge and experience about the PSO process represented as fuzzy membership functions will help PSO to run in an intelligent way and thereby increase the search efficiency.

Some other researchers have already applied fuzzy logic into optimization algorithm. Yuhui Shi and Eberhart [10,11] have adjusted inertia weight of PSO algorithm by a fuzzy system. Yaowen Yang and CheeKiongSoh [8] have applied fuzzy logic into genetic programming. However, there is still no application of fuzzy logic on topology optimization of part design. In this study, the fuzzy tuning of a single constraint PSO problem will be developed mathematically. Two illustrative examples are used to validate the proposed method. By comparison, the PSO with fuzzy parameter tuning is found to have better search performance and higher search efficiency.

2. PARTICLE SWARM OPTIMIZATION

PSO simulates the behavior of a bird flocking. Each bird in the flock continually processes information of its current position and velocity. When applied to topology optimization problems, the relative densities ρ take the place of positions, and the incremental change of relative densities replaces velocity. After these arrangements, the minimum compliance problem of topology optimization based on PSO could be formulated as:

$$\begin{aligned} &\min \quad c = f^{T}u \\ &\text{s.t.}: \quad Vs \leq \overline{V} \end{aligned} \tag{1}$$

Here c is the compliance of structure, f, and u are the load and displacement matrix of structure respectively. Vs and V bar are practical material volume and pre-defined volume constraint. Assume a flock has p particles, for particle d, the PSO updating schemes for both velocity change and density change are expressed as:

$$\begin{aligned} v_{k+1}^{d} &= wv_{k}^{d} + c_1 r_1 (p_k^d - \rho_k^d) + c_2 r_2 (p_g - \rho_k^d) \\ \rho_{k+1}^{d} &= \rho_k^d + v_{k+1}^{d} \end{aligned} \tag{2}$$

Here k is the time increment; w is an inertia weight; p_g is the best ever densities in the swarm. p_k is the best previous density of particle d at time k. r_1 and r_2 represent uniform random numbers between 0 and 1. Kennedy and Eberhart proposed that $c_1 = c_2 = 2$, in order to allow a mean of 1 [4].

The volume constraint in PSO is achieved by a punishment function as shown in equation 3. In this function λ is a prescribed punishment index which is assigned as 0.02 in the standard PSO [3]. The variable $g_{(\rho)}$ is the difference between current material volume and prescribed material volume:

$$\begin{aligned} &\bar{c} = c_{(\rho)} + \lambda[g_{(\rho)}]^2 \mu_{(g)} \\ &\text{with } \mu_{(g)} = \begin{cases} 0 & \text{if} \quad g_{(\rho)} \leq 0 \\ 1 & \text{if} \quad g_{(\rho)} > 0 \end{cases} \end{aligned} \tag{3}$$

Due to the natural character of numerical method, the result of topology optimization has tendency to obtain checkerboard like structure. Therefore a Checkerboard control equation is used in PSO scheme:

$$\begin{aligned} &\bar{c} = c_{(\rho)} + \sum_{i=1}^{n} x_i [h_{i(\rho)}]^2 \kappa_{i(h_i)} \\ &\text{with } \kappa_{i(h_i)} = \begin{cases} 1 & \text{if} \quad \rho_i \leq \rho_e^{\min} \\ 0 & \text{if} \quad \textit{otherwise} \end{cases} \end{aligned} \tag{4}$$

Where "x_i" is assigned as 2% of structural compliance in this study."h_i" is evaluated as:

$$h_i = \rho_e^{\min} - \rho_i \tag{5}$$

Where "$\rho_e^{\min}$" is the minimum density of elements around the objective element.

As concluded in authors' another paper [3], PSO updates design variables "relative densities" through sharing of information between companions and survival of the fittest rules. Experiments show that, the performance of PSO is weaker than SIMP on topology optimization problem. Therefore the search ability and efficiency of PSO need to be improved. In this study, a fuzzy parameter tuning system is introduced to adjust the volume constraint properly.

3. FUZZY PARAMETER TUNING

The standard PSO scheme uses a pre-defined material volume constraint V bar in equation (1). Due to the drawbacks as mentioned in the introduction section, a fuzzy logic controller is introduced. Equation (1) is modified with fuzzy constraints as below:

$$\begin{aligned} &\min \quad c = f^{T}u \\ &\text{s.t.}: \quad Vs \lesssim \tilde{V} + \tilde{V}^{e} \end{aligned} \tag{6}$$

Here a fuzzy constraint V tilde is used. The symbol tilde is used to indicate that the constraint contains fuzzy information. Ve is the extended tolerance of the fuzzy constraint. Unlike traditional mathematical logic divides object into yes or no (0 or 1), fuzzy logic utilizes membership functions to represent the degree of belonging of object from 0 to 1. Because fuzzy logic could utilize human reasoning to make decisions, the degree of belonging could be described by a series of linguistic terms [12].

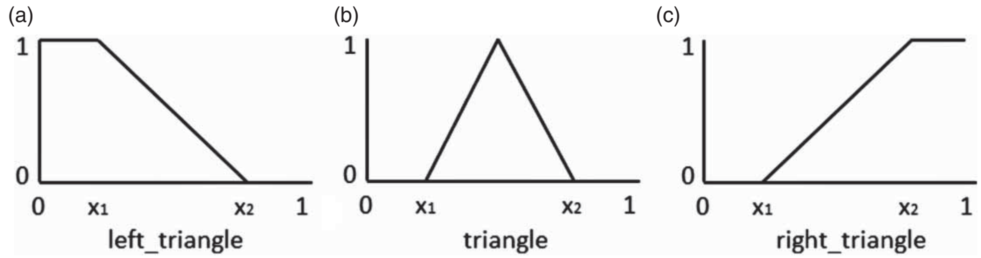

Fig. 1: The illustrative diagrams of three used membership functions.

Take the minimum compliance problem as an example,there are two inputs in this study. The first is relative compliance, which expresses the change degree of objective function "structural compliance". The relative compliance could be obtained from equation 7. Where c_r is the relative compliance, c is the current compliance, gbestval is the global minimum compliance during optimization procedure. And c_{max} is a prescribed maximum compliance. In this study, $c_{max} = 1.2$*gbestval.

$$c_r = \frac{\text{c-gbestval}}{c_{max}\text{-gbestval}} \tag{7}$$

The relative compliance could also be described as "very low""low", "medium", "high" and "very high". The second input material volume also could be described as "very small","small", "medium", "large" and "very large". In original PSO algorithm, there is a punishment index λ to increase structural compliance if material volume is beyond the material volume constraint. In the fuzzy logic controller, there are several rules to decide the magnitude of the punishment index. For example, if the relative compliance is low, and the material volume is small, then the punishment index is small. Another example, if the relative compliance is medium, and the material volume is large, then the punishment index is large, etc. Generally there are five steps for the fuzzy parameter tuning. They are "fuzzification", "integration", "implication", "aggregation" and "defuzzificaiton", Details of each step will be described in the following.

3.1. Fuzzification

According to the input values and definitions of membership functions, the fuzzy input is calculated in this step.In fuzzy logic, these fuzzy inputs are named as "antecedent", meanwhile the fuzzy output is named as "consequent". In this step, the crisp inputs are transformed to fuzzy inputs "antecedent" membership functions. A membership function is a curve, which defines how crisp input is mapped to a membership value between 0 and 1[2]. In this study, three standard membership functions are used, they are left_triangle function, triangle function and right_triangle function. The definition and illustrative diagrams of these three functions are shown below.

$$f_{left\_triangle} = \begin{cases} 1, & \text{if} \quad x < x_1 \\ \dfrac{x_2 - x}{x_2 - x_1}, & \text{if} \quad x_1 \le x \le x_2 \\ 0, & \text{if} \quad x > x_2 \end{cases} \tag{8}$$

$$f_{right\_triangle} = \begin{cases} 1, & \text{if} \quad x < x_1 \\ \dfrac{x - x_1}{x_2 - x_1}, & \text{if} \quad x_1 \le x \le x_2 \\ 0, & \text{if} \quad x > x_2 \end{cases} \tag{9}$$

$$f_{triangle} = \begin{cases} 0, & \text{if} \quad x < x_1 \\ 2\dfrac{x - x_1}{x_2 - x_1}, & \text{if} \quad x_1 \le x \le \dfrac{x_2 + x_1}{2} \\ 2\dfrac{x_2 - x}{x_2 - x_1}, & \text{if} \quad \dfrac{x_2 + x_1}{2} < x \le x_2 \\ 0, & \text{if} \quad x > x_2 \end{cases} \tag{10}$$

As mentioned in this study, there are two inputs "compliance" and "material volume", one output "punishment index". These three variables are described byfive linguistic terms "very small", "small", "medium", "large" and "very large". The left_triangle function is used to describe the linguistic term "very small". The triangle function is to describe terms "small","medium" and "large". The right_triangle function is to describe the term "very large". The sketch map of the membership functions of variables is shown in Figure 2.

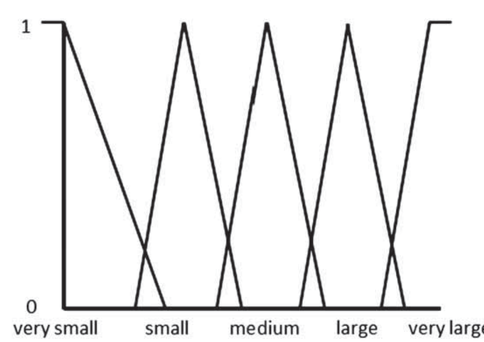

Fig. 2: Sketch map of membership functions of variables.

In the definitions of these five functions, variables x_1 and x_2 should be decided by the experience and prior knowledge. In this study, the configurations of

Variables	Linguistic terms	Membership functions (MF)	Parameters of MFs			
			Lower limit	Upper limit	x_1	x_2
relative compliance	very small	left_triangle	0	1	0	0.06
	small	triangle			0.04	0.2
	medium	triangle			0.16	0.46
	large	triangle			0.42	0.72
	very large	right_triangle			0.68	1
material volume	very small	left_triangle	0.5	0.6	0.5	0.52
	small	triangle			0.51	0.54
	medium	triangle			0.53	0.56
	large	triangle			0.55	0.58
	very large	right_triangle			0.57	0.6
punishment index	very small	left_triangle	0	0.02	0	0.006
	small	triangle			0.005	0.01
	medium	triangle			0.009	0.013
	large	triangle			0.012	0.016
	very large	right_triangle			0.015	0.02

Tab. 1: Configurations for all membership functions in this study.

Rule No.	abbreviated formula	Rule No.	abbreviated formula	Rule No.	abbreviated formula
1	[1 1] [1]	11	[3 1] [2]	21	[5 1] [4]
2	[1 2] [1]	12	[3 2] [3]	22	[5 2] [5]
3	[1 3] [2]	13	[3 3] [3]	23	[5 3] [5]
4	[1 4] [3]	14	[3 4] [4]	24	[5 4] [5]
5	[1 5] [4]	15	[3 5] [5]	25	[5 5] [5]
6	[2 1] [1]	16	[4 1] [3]		
7	[2 2] [2]	17	[4 2] [4]		
8	[2 3] [3]	18	[4 3] [4]		
9	[2 4] [4]	19	[4 4] [5]		
10	[2 5] [5]	20	[4 5] [5]		

Tab. 2: 25 fuzzy rules in abbreviated formula.

membership functions are listed in table 1. According to the inputs, the antecedents could be obtained by using these membership functions.

3.2. Integration

If there are more than two inputs such as "compliance" and "material volume", the integration step is needed to integrate antecedents of these two inputs. For example:

if structural compliance == small,
AND material volume == small,
then punishment index = small.

In this rule, the logical operation "AND" is the integration operator, used to integrate antecedents of compliance and volume.

3.3. Implication

In this step, the consequent is reshaped by the integration result of antecedents.The consequent is the fuzzy output "punishment index"described by five membership functions. The "implication" step is implemented for each predefined rules. In this study, there are 25 rules. For simplicity, the rules are expressed in abbreviated formula. For example rule 1 is expressed as follow:

rule 1: if (compliance is very small) and (volume is very small) then (punishment is very small)

Here linguistic term "very small" is defined as "1", "small" is indicated as "2", "medium" is "3", "large" is "4" and "very large" is defined as "5". Then rule 1 could be expressed as:

[1 1] [1]

According to this principle, the abbreviated formula of 25 fuzzy rules are listed in table 2.

After implication, the result is the reshaped fuzzy output for each rule.

3.4. Aggregation

Because the final conclusion is decided by all the rules, therefore this step combines all fuzzy sets representing outputs of each rule into one fuzzy set. In this study the "maximum" method is used to obtain the aggregated fuzzy set.This method integrates the results by choosing the highest value part of each fuzzy set into one fuzzy set.

3.5. Defuzzification

Because fuzzy result is meaningless for practical control, this step attempts to obtain the crisp output from the fuzzy output (the aggregated output). In this study, the standard defuzzification method, centroid method, is used to obtain the crisp result. This method returns the center of area of the fuzzy output.

By using this fuzzy logic controller in PSO, the risk of premature solutions and loss of potential solutions in traditional PSO could be reduced. Meanwhile because human reasoning and experience are adopted in the membership functions, PSO with fuzzy tuning could search the optimal solutions in an intelligent and efficient way.

4. ILLUSTRATIVE EXAMPLES

The following are two examples to compare the performance among three topology optimization schemes. The first is the most mature topology optimization method, called "Solid Isotropic Material with Penalization" (SIMP) method. The second method is the standard PSO based topology optimization scheme. The last one is the PSO with fuzzy parameter tuning. The examples are two typical part design problems. The first example is shown in Figure 3, the design domain is a square material; the left side of this block is fixed in horizontal direction. The lower right corner of the block is fixed in vertical direction.

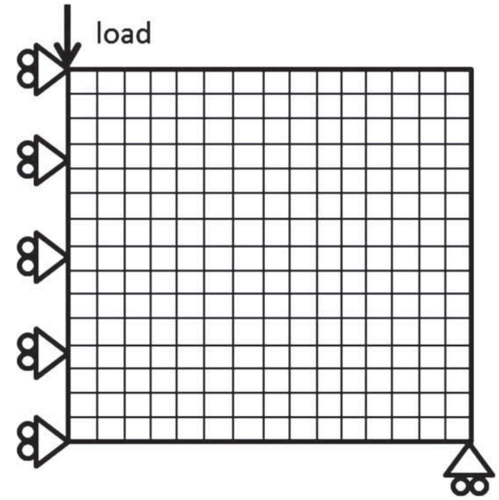

Fig. 3: The meshes and load & boundary conditions of example 1.

A load is added at the upper left corner vertically. This design domain is discretized by a 20*20 meshes.

After optimization of example 1, the results of these three methods are shown in Figure 4. The results of SIMP and standard PSO have volume constraint as 50 percent. The result of fuzzy logic based PSO has a fuzzy tuning volume constraint as 50.017 percent. As shown from Figure 4, the result of PSO with fuzzy parameter tuning (Figure 4c) is slightly better than the result of standard PSO (Figure 4b). This is because the global search ability of fuzzy PSO is more flexible than standard PSO. However, the result of PSO with fuzzy parameter tuning is still not as good as the result from SIMP due to the evolutionary character of PSO.

The detailed indexes of optimization procedures for these three methods in example 1 are listed in table 3. As shown, fuzzy PSO has the lowest structural compliance; meanwhile the iteration number and

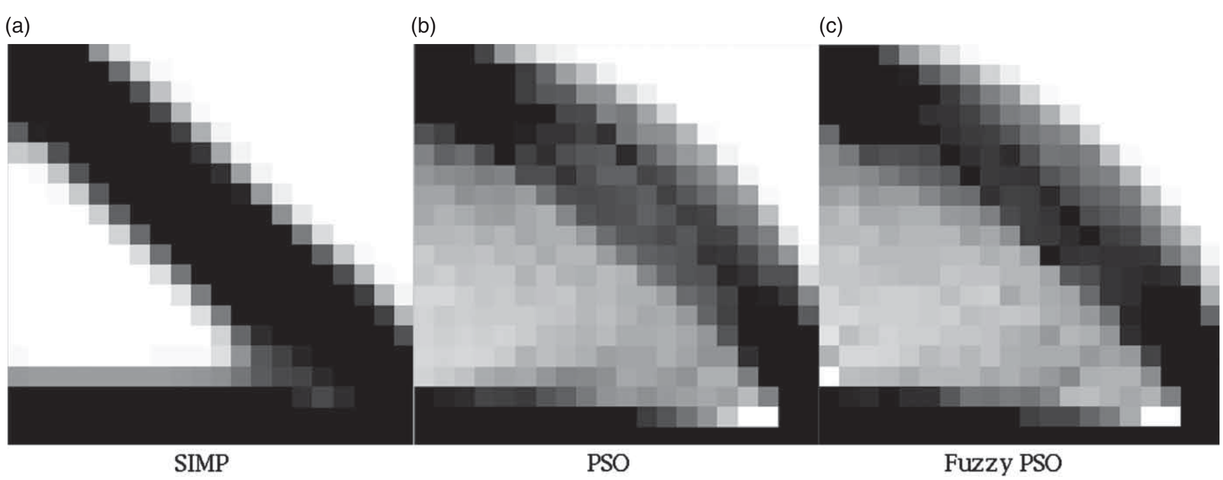

Fig. 4: Example 1 optimization results by (a) SIMP scheme, (b) standard PSO scheme and (c) PSO scheme with fuzzy parameter tuning.

Opitmizaiton method	Structural compliance	Iteration number	Optimizaiton time (seconds)
SIMP	24.81	36	15
Standard PSO	21.74	792	660
Fuzzy PSO	20.38	553	590

Tab. 3: Indexs of opitmizaiont procedure for thetested methods in example 1.

optimization time of fuzzy PSO are both improved compared to standard PSO. The result validates the search efficiency of fuzzy logic based PSO is improved compared to the standard one. Because this example has simple structure, SIMP has very fast convergence rate. For more complicated structure, fuzzy PSO will exhibit its advantages.

Next, the design domain and boundary conditions of example 2 are shown in Figure 5. The left side of the design domain is fixed. A load is added vertically at the middle point of the right side. This design domain is discretized by a 20*20 meshes.

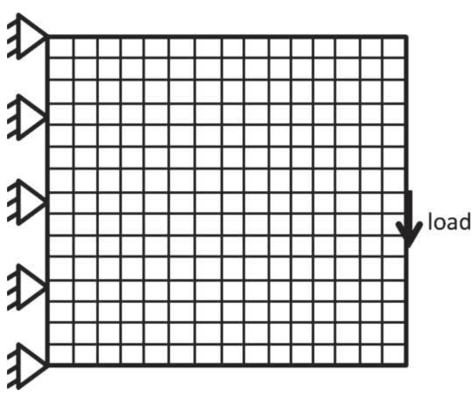

Fig. 5: The meshes and load & boundary conditions of example 2.

After optimization of example 2, the results of these three methods are shown in Figure 6. The results of SIMP and standard PSO have volume constraint as 50 percent. The result of fuzzy logic based PSO has a fuzzy tuning volume constraint as 50.025 percent. For example 2, PSO with fuzzy parameter tuning (Figure 6c) still has better result than standard PSO (Figure 6b). However, compared to the result of SIMP (Figure 6a), the result of fuzzy PSO is worse. The results of these two examples could conclude that, fuzzy tuning could relax the “hard” constraints at early iterations of PSO algorithm; therefore PSO with fuzzy tuning will have better global search ability. However, premature convergence still exists in fuzzy PSO. To solve this problem, the authors will attempt to use the method of moving asymptotes (MMA) as the constraint method, and also use fuzzy logic to adjust it in the future works.

The data of optimization procedures of example 2 are listed in table 4. Consistently, the result of fuzzy has the smallest compliance. The search efficiency of PSO is still much worse than SIMP.

Opitmizaiton method	Structural compliance	Iteration number	Optimizaiton time (seconds)
SIMP	15.06	48	18
Standard PSO	12.65	1680	1500
Fuzzy PSO	12.43	1425	1320

Tab. 4: Indexes of opitmizaiont procedure for the tested methods in example 2.

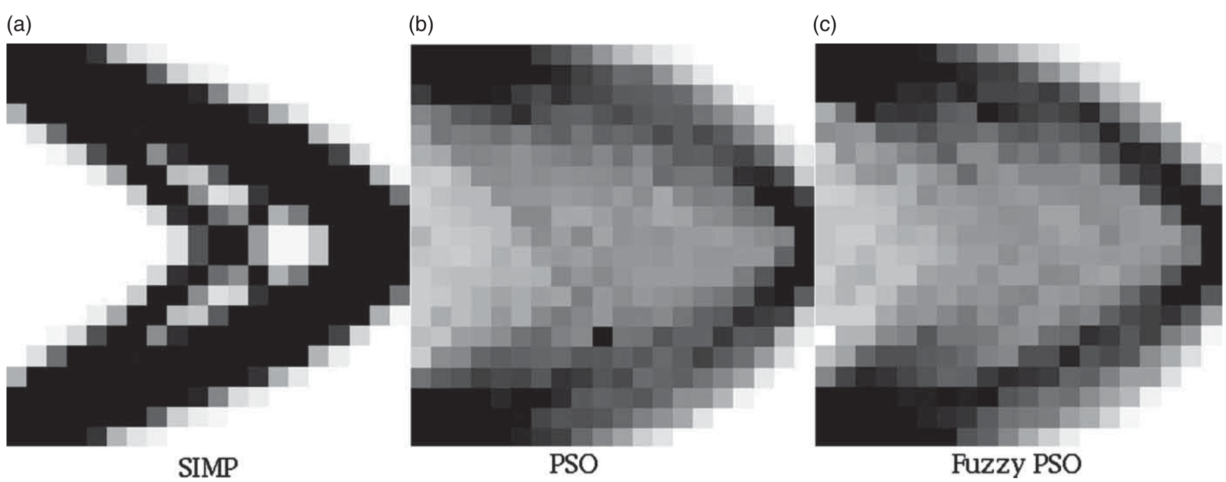

Fig. 6: Example 2 optimization results by (a) SIMP scheme, (b) standard PSO scheme and (c) PSO scheme with fuzzy parameter tuning.

5. CONCLUSIONS

This study attempted to utilize fuzzy logic to improve the performance of PSO based topology optimization. For this purpose, the volume constraint in PSO is adjusted by a fuzzy parameter tuning system. After comparison by sample studies, the fuzzy logic is validated to be feasible and effective in PSO-based topology optimization. It is concluded the fuzzy tuning constraint could relax the "hard" constraint at the early stage of the PSO algorithm. Therefore the PSO based topology optimization with fuzzy parameter tuning has better search ability and higher search efficiency than that of standard PSO.However, the problem of premature convergence of PSO still exists. In the future, the authors will attempt to apply the method of moving asymptotes (MMA) with fuzzy tuning into PSO. Meanwhile, the authors will also attempt to utilize fuzzy logic to adjust the checkerboard control system in the PSO scheme.

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