Reconstruction Method of Trimmed Surfaces Maintaining G^1-Continuity with Adjacent Surfaces

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ABSTRACT

In 3D CAD systems, reconstruction of trimmed surfaces is one of the important research topics. When 3D CAD data is exchanged between different CAD systems, it does not deliver the specifications of CAD systems completely. Therefore, we have to correct the CAD data, such as moving the point in order to fill gaps. Generally, CAD data is expressed by set of trimmed surfaces. Then, we need to modify the trimmed surfaces. It is necessary, however, to modify the trimmed surface shape so as to maintain geometrical consistency of the boundary edges and surfaces in direct modeling, and this is a big restriction. It is effective to apply a new free-form surface to a closed region enclosed with the modified edges because the consistency of a trimmed surface can be maintained. Many shapes with holes or concave shapes are included in CAD data. Moreover, it is necessary to consider maintaining G^1-continuity with adjacent surfaces, but it is difficult to reconstruct the trimmed surface by the conventional surface fitting method.

In this paper, we propose the reconstruction method of trimmed surfaces with maintaining G^1-continuity with adjacent surfaces. In the proposed method, boundary edges are input and surfaces are output. Two surfaces are connected with G^1-continuity by using the control points at the connection section obtained from cross boundary derivatives. In addition, the control points in the region or on the discontinuous boundaries are obtained by approximating sample points, and the proposed method can also be applied to shapes with holes or concave shapes.

Keywords: trimmed surface, surface fitting, notch shape, G^1-continuity.

1. INTRODUCTION

The surface fitting method to N-sided regions is the important fundamental technology in geometric modeling and is used for various contents. For instance, data cannot be delivered completely but it is necessary to correct CAD data in the data exchange between different CAD systems. CAD data is expressed with trimmed surfaces in many cases, and in order to correct the data, the trimmed surfaces must be modified. It is necessary, however, to modify the trimmed surface shape so as to maintain geometrical consistency of the boundary edges and surfaces in direct modeling, and this is a big restriction. It is effective to apply a new free-form surface to a closed region composed of the modified edges because the consistency of a trimmed surface can be maintained. Moreover, the trimmed surface compression method [2] that the authors proposed last year is based on the surface fitting method and we succeeded in compressing trimmed surfaces efficiently. Although the surface fitting method [1] currently used in [2] is applicable to shapes with holes or concave shapes, the method can connect surfaces with G^1-continuity only in one direction or in opposite directions. Many analytic surfaces like cylindrical ones are contained in CAD data like machine parts. In many cases, such surfaces become G^1-continuous with adjacent surfaces in one direction or in opposite directions, and the method [1] can express most of surface shapes. In 3D models with trimmed surfaces shown in Fig. 1 (c), however, there is a surface whose boundary edges are connected with adjacent surface with G^1-continuity. Especially, in 3D

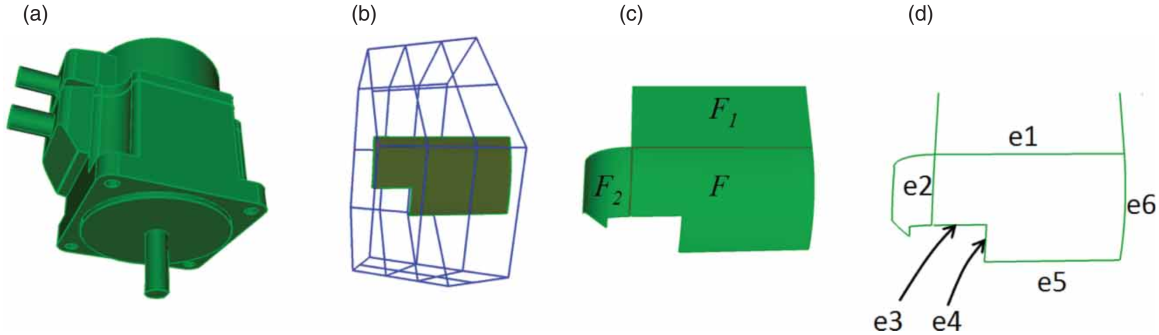

Fig. 1: (a) CAD data, (b) trimmed surface and effective region taken out from (a), (c) concave shape F that has adjacent G^1-continuous surfaces F_1 at the top and F_2 on the left.

models of mechanical parts, a lot of surfaces are G^1-continuous in two adjoining directions. The method [1] is inapplicable to such shapes.

In this paper, the method [1] is extended and the surface fitting technique is proposed for a closed region where the boundary edges connect with adjacent surfaces with G^1-continuity, as shown in Fig. 1 (c). In the proposed method, boundary edges are input and surfaces are output. Two surfaces are connected with G^1-continuity by using the control points at the connection section obtained from cross boundary derivatives. In addition, the control points in the region or on the discontinuous boundaries e5 and e6, which are shown in the Fig. 1 (d), are obtained by approximating sample points, and the proposed method can also be applied to shapes with holes or concave shapes appearing frequently in CAD data.

2. RELATED WORKS

The surface fitting methods to N-sided closed regions are classified into the ones based on the surface interpolation method [6] and the others based on the N-side filling method [5]. The surface interpolation method interpolates a closed region with free-form surfaces so that the boundary edges of the closed region coincide with the boundary curves of the free-form surface. The N-side filling method generates a quadrilateral trimmed surface that includes a closed region of the boundary edges. This section describes related works of the surface fitting method to the N-sided regions and compares them to these methods.

2.1. Methods Based on the Surface Interpolation Method

The surface interpolation method is the surface fitting method for applying one four-side surface patch, or two or more four-side surface patches. The inner curves are generated based on Catmull-Clark subdivision and an N-sided region is divided into N quadrilateral regions. Then, a surface is interpolated to each of the generated regions. In this method, each patch can be connected with G^1-continuity.

Piegl et al. introduced an interpolation method with the angle tolerance ε to generate smooth surfaces[3]. In his method, it is possible to control the continuity between patches by computing the control points connected between patches with cross boundary derivative coefficient. Yang et al. enhanced the Piegl's method to apply it to rational curve meshes[7]. Moreover, Garcia et al. proposed surface fitting to an arbitrary N-sided region by dividing the region into a star-shaped N-sided patch and quadrilateral patches, which can be controlled using parameter f[4].

2.2. Methods Based on the N-side Filling Method

Tokuyama et al. proposed the N-side filling method [5] that covers an N-sided region with a B-spline surface. A concrete procedure is described below:

1. As shown in Fig. 2 (a), four reference planes are generated outside of the closed region which expressed in blue.

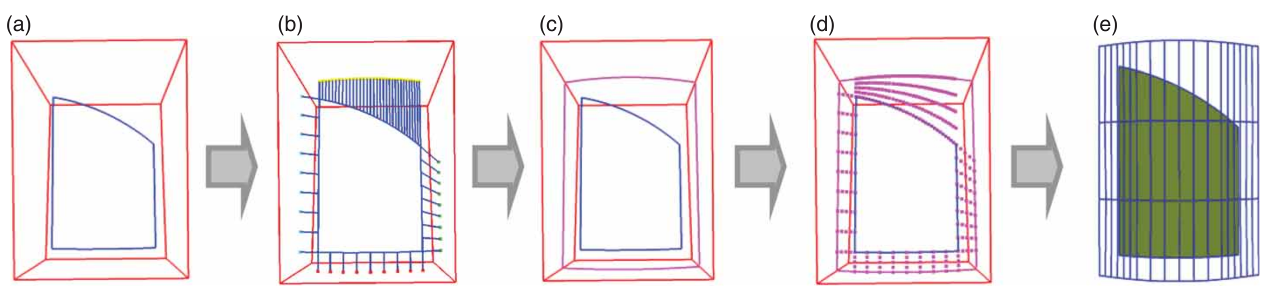

Fig. 2: The procedure of the N-side filling method.

2. As shown in Fig. 2 (b), the cross boundary derivatives are extended to the outside of the closed region, and the intersection points with the reference planes are generated.
3. As shown in purple lines of Fig. 2 (c), the intersection point sets which generated in Step.2 are approximated with B-spline curve, and the boundary curves that cover an N-sided region are generated.
4. As shown in purple points of Fig. 2 (d), the sample points are generated on the line that generated in Step.2
5. As shown in Fig. 2 (e), the control points of the B-spline surface are generated with the boundary curve that generated in Step.3 and the sample points that generated in Step.4 by the least-squares method.

The authors enhanced the N-side filling method for apply it to the shapes with holes or concave shapes and to maintain G^1-continuity with adjacent surfaces in one direction[1].

2.3. Comparison to Previous Methods and Problems

We compared the surface fitting methods in Tab. 1. The methods based on the surface interpolation method [3,4,6] are excellent in continuity with adjacent surfaces. The methods [3,4,6] consider the continuity between two or more surface patches. However, since these methods divide an N-sided region into two or more regions, it is inapplicable to the shapes with holes or concave shapes. Also, the methods are dependent on the shapes of the boundary edges expressing a closed region, and the division of concave shapes may fail. Since notches appear frequently in the machine parts include shapes with holes or concave shapes are expressed with trimmed surfaces, it is difficult to apply the surface interpolation method.

Surface fitting method	convex	hole or concave	continuity
Surface interpolation[6]	○	×	○
Piegl's method[3]	○	×	○
Yang's method[7]	○	×	○
Garcia's method[4]	○	×	○
N-side filling[5]	○	○	×
Extended N-side filling[1]	○	○	△

Tab. 1: Comparison of surface fitting methods: "Circle" for success, "cross" for failure, and "triangle" for conditional success.

On the other hand, the methods based on the N-side filling method described in section 2.2 are applicable to the shapes with holes or concave shapes. Moreover, the method [1] realizes connection with an adjacent surface with G^1-continuity in only one direction. If, however, the number of the boundary edges connected with adjacent surfaces with G^1-continuity increases, the number of the sample points will decrease, and the control points will be unstable.

In this paper, we enhance the method [1] and propose the surface fitting method with maintaining G^1-continuity in all directions.

3. SURFACE FITTING METHOD

The proposed method unites the advantages of the N-side filling method [5] and the surface interpolation method [6]. In a section where two surfaces are connected with G^1-continuity, the cross boundary derivatives are calculated based on the basis patch method used in the surface interpolation method to generate the control points. The control points in a surface are generated by approximating sample points by using the N-side filling method. The concept of our method is shown in Fig. 3. The blue markers show the control points calculated by the N-side filling method and the red markers show the ones calculated by the surface interpolation method. As shown in Fig. 3, by uniting the control points obtained by the two methods, then it is possible to generate surfaces that are G^1-continuous with adjacent surfaces of concave shape or with holes. Moreover, by using the sample point generation method independent of the number of G^1-continuous boundary edges, it is possible to generate a surface surrounded by surfaces in all directions connecting with G^1-continuity. Our surface fitting method proposed in this paper is executed in the following three steps:

1. Four B-spline curves are generated in the outside of a closed region. The generated curves will be the boundary curves of a B-spline surface.
2. In the inside of the four boundary curves generated in Step.1, sample points are calculated.
3. The control points of a B-spline surface are calculated by approximating the sample points generated in Step.2 by the least-squares method. After that, two surfaces are connected with G^1-continuity by calculating the control points with using the surface interpolation method.

3.1. Method to Generate Boundary Curves that Cover a Closed Region

This section describes the method of generating the boundary curves of a B-spline surface. As shown in Fig. 4 (a), we consider surface fitting for a region F that has adjacent G^1-continuous surfaces F_1 and F_2. Since the boundary edges are connected with adjacent surfaces with G^1-continuity, the conventional

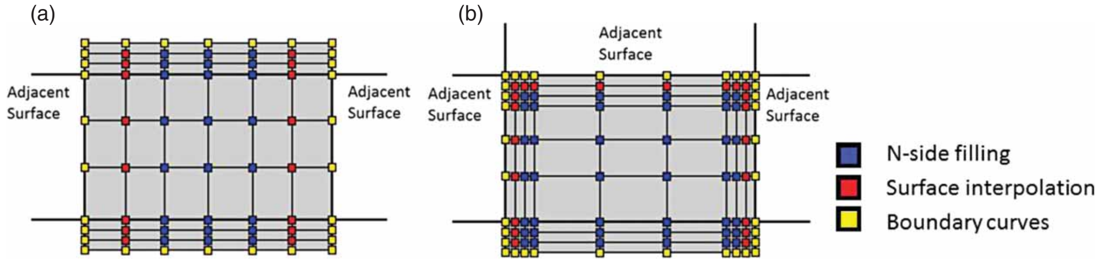

Fig. 3: Concept of surface fitting for a closed region: (a) Surface fitting to a closed region that has adjacent G^1-continuous surfaces in the left and right sides, (b) surface fitting to a closed region that has adjacent G^1-continuous surfaces in the left, right and upper sides.

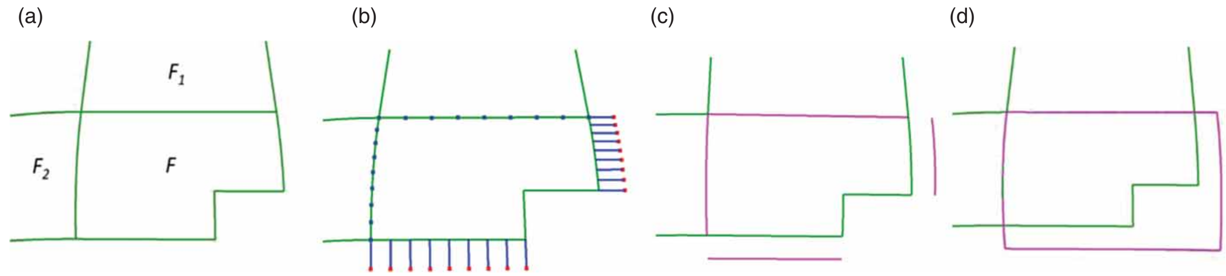

Fig. 4: Boundary curve generation: (a) Surface fitting to region F that has adjacent G^1-continuous surfaces in left and upper sides, (b) Point set generation, (c) Four boundary curves are generated by approximating the point set by B-spline curves. (d) The four B-spline curves are connected, and the boundary curves surrounding the closed region are generated.

method[1] is inapplicable. If the boundary edges are G^1-continuous with the adjacent surfaces, as shown in the blue points in Fig. 4 (b), sample points are generated on the boundary edges and the edges are approximated by B-spline curves. If the boundary edges are not G^1-continuous with the adjacent surfaces, as shown in the red points in Fig. 4 (b), the cross boundary derivatives are generated in the outside of the region, and the intersection points with reference planes that include a closed region are generated. Then, as shown in Fig. 4 (c), each set of the generated intersection points are approximated by a B-spline curve. After that, the B-spline curve is extended to the tangential direction as shown in cyan lines of Fig. 5, and the intersection points between the curve and the reference plane are generated as shown in blue points of Fig. 5. The average of the obtained intersection points are calculated as shown in green point of Fig. 5, and the obtained center point is added to the set of the intersection point. Then, the connected B-spline curve is generated by re-fitting the set of the intersection points. The connected B-spline curves are shown in purple lines of Fig. 4(d). The generated four-sided region is the boundary curves of a B-spline surface.

3.2. Method to Generate General-purpose Sample Points

In method [1], when a boundary edge is connected with an adjacent surface with G^1-continuity, the sample points are generated only on the boundary edge.

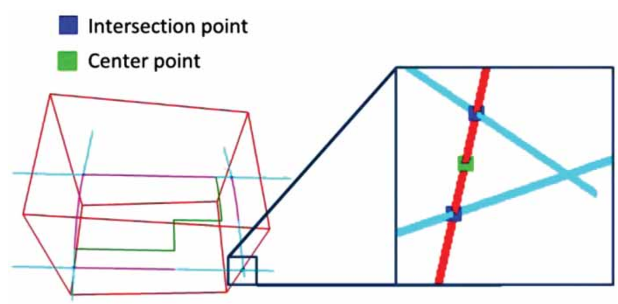

Fig. 5: Connection of the boundary curves.

Due to this, if the number of boundary edges connected with the adjacent surfaces with G^1-continuity increases, the number of sample points decreases and the control points of the B-spline surface calculated by the least-squares method are unstable. In our method, first, a B-spline surface that covers a closed region is generated by the N-side filling method as shown in Fig. 6 (a). Then, as shown in red points of Fig. 6 (b), the points on the generated surface existing inside the closed region are acquired as sample points. For a notch section, as shown in green points of Fig. 6 (b), the cross boundary derivatives are generated outside of the region, and the sample points are generated based on offset curves. The points on the line segments generated in Fig. 4 (b) are acquired as sample points. In the case of the shape with holes, the error near the hole is kept from becoming large by adding the points on the boundary edges

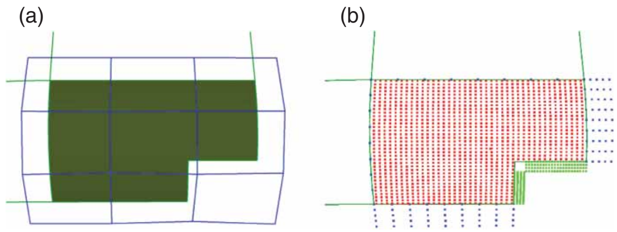

Fig. 6: Sample point generation: (a) A B-spline surface is generated covering the region F by N-side filling method, (b) the sample points are generated in the outside of the closed region based on the tangent planes of the boundary edges and the points on the generated surface existing inside the closed region are acquired as sample points.

representing a hole as the sample points. The sample points can be generated without depending on the number of boundary edges connected with adjacent surfaces with G^1-continuity.

3.3. Generating Surfaces and Control Points for Connecting two Surfaces

This section describes how to generate G^1-continuous surfaces. As shown in Fig. 7 (a), the cross vectors of the boundary edge between two surfaces are calculated based on the surface interpolation method. The obtained cross vectors serve as condition for connecting two surfaces by G^1-continuous. Then, as the red arrows in Fig. 7 (b) show, knots are inserted in the parameters corresponding to the corner points of the boundary edges that connect the adjacent surfaces with G^1-continuity, and the boundary curves are divided. When the boundary edges connect adjacent surfaces with G^1-continuity in two adjoining directions, the conditions used as G^1-continuous cannot be fulfilled near the corner portion with a B-spline surface. Therefore, knots are inserted near the corner [3](at parameters 0.05 and 0.95) as the blue arrows show in Fig. 7 (b). This discontinuous section with adjacent surfaces in a corner is narrowed. After that, the cross vectors connected to each of the divided boundary curves are calculated based on the basis patch method and the control points, which connect two surfaces with G^1-continuity, are obtained as shown in Fig. 8 (a).

The control points of the B-spline surface are calculated by approximating the sample points generated in section 3.2 using the least-squares method. Moreover, the section connected with adjacent surfaces with G^1-continuity is restrained by using the control points generated in this section 3.3, and a B-spline surface that connects adjacent surfaces with G^1-continuity is generated.

3.4. Evaluation of the Generated Surface

This section describes the evaluation method of the generated B-spline surface. To verify the accuracy of the generated surface, the distance between the generated surface and the source surface retained by the trimmed surface is measured. The points on the source surface are projected to the generated surface and the shortest distance is measured. In addition, to verify whether the source boundary edges lie on the generated surface within the tolerance or not, the points on the boundary edges are projected to the generated surface and the shortest distance is measured. Moreover, to find the relative error, the ratio of the bounding box size and the maximum distance are calculated. In this paper, when ratio is smaller than 1%, it is assumed that a shape is approximated in good accuracy.

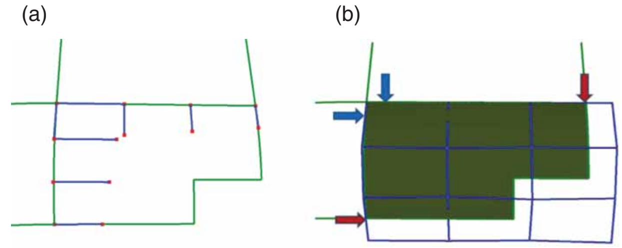

Fig. 7: Boundary curve division: (a) Cross vectors of the boundary edge of the two surfaces are calculated based on the surface interpolation method, (b) before connecting two surfaces, as the red and blue arrows show, knots are inserted and the boundary curves that connect the adjacent surfaces with G^1-continuity are divided.

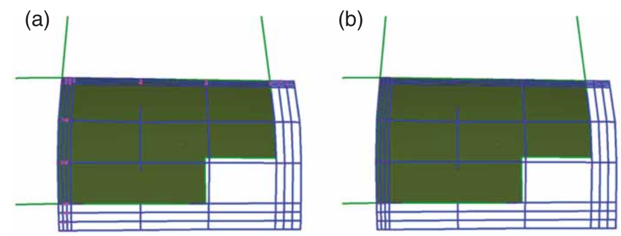

Fig. 8: Connection between two surfaces: (a) The control points of the surface are generated to connect two surfaces, (b) two surfaces are connected with G^1-continuity by restraining the control points of the connection section with adjacent surfaces.

4. EXPERIMENTAL RESULT

In our method, it is necessary to apply the surface fitting method to two closed regions to be connect with G^1-continuity. Then, our method was applied to all closed region obtained from CAD data and the practicality was verified. Our method was also applied to the closed regions that have adjacent G^1-continuous surfaces in two directions, three directions and all directions. Then, the generated surfaces were evaluated with the evaluation method described in section 3.4.

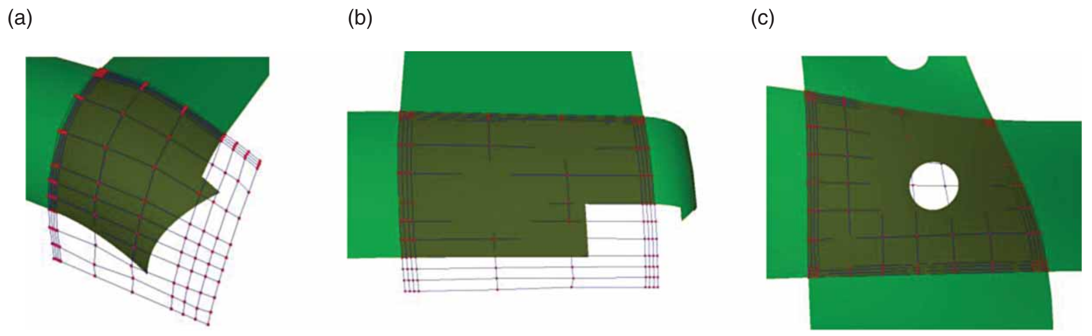

Fig. 9: Result of surface generation

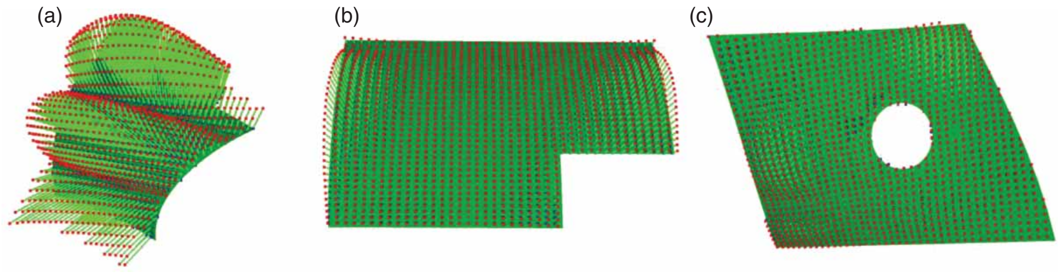

Fig. 10: Result of surface evaluation: Distances between the generated surface and the source surface are calculated.

The control points of the generated surface are shown in Fig. 9. The distances between the generated surface and the source surface are shown in Fig. 10. The blue markers in Fig. 10 represent the points on the generated surface to which the points on the source surface are projected. The red markers in Fig. 10 are obtained by extending the blue markers in each normal direction of the corresponding tangent plane. The lengths between the blue markers and the red ones show the distances between the generated surface and the source surface multiplied by twenty five. The number of control points, sample points and the size of the bounding box are shown in Tab. 2. The error evaluation of the generated surface is shown in Tab. 3, in which *Avg.*, *Max* and *Ratio* are shown as described in section 3.4. *Avg.* indicates the average error margin value obtained by averaging the distances between the generated surface and the source surface, *Max* indicates the maximum error margin value representing the maximum distance between the generated surface and the source surface, and *Ratio* indicates the ratio of the bounding box size and the maximum distance. We can find that shapes are approximated in good accuracy.

Object	Number of control points	Number of sample points	Size of Bounding box
(*a*)	100	1567	22.975304
(*b*)	130	1686	39.17714
(*c*)	169	1347	466.449816

Tab. 2: The number of control points, sample points and the size of bounding box.

Moreover, in order to verify the continuity with adjacent surfaces, the normal vectors on the boundary edges of the generated surface are calculated and shown in Fig. 11. The normal vectors on the boundary

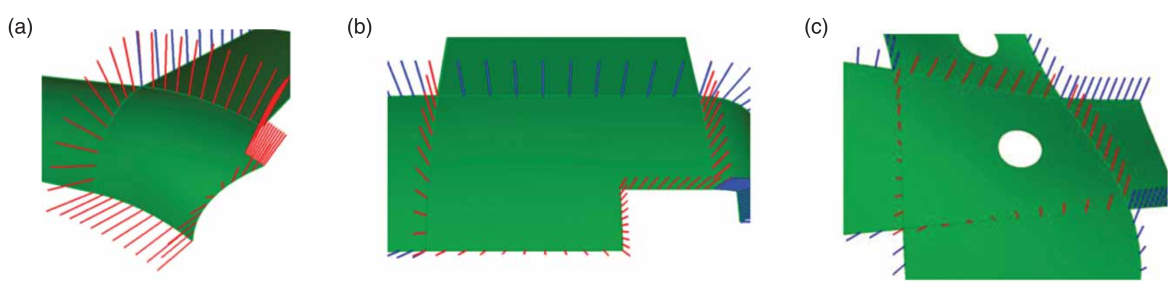

Fig. 11: Verification of the continuity with adjacent surfaces: The normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges.

Object	Evaluation Object	Avg.	Max	Ratio
(*a*)	*Trimmed surface*	0.037484	0.149445	0.696026 %
	Boundary edges	0.016383	0.090798	0.422883 %
(*b*)	*Trimmed surface*	0.037274	0.100441	0.256377%
	Boundary edges	0.024421	0.109491	0.279477 %
(*c*)	*Trimmed surface*	0.071918	0.337261	0.072304 %
	Boundary edges	0.895156	2.447500	0.524708 %

Tab. 3: Error evaluation.

edges of the generated surface are shown in red, and those on the boundary edges of the adjacent surfaces are shown in blue. Since the normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges, we can find that two surfaces are connected with G^1-continuity.

5. CONCLUSION AND FUTURE WORKS

5.1. Conclusion

In this paper, we proposed the reconstruction method of trimmed surfaces with maintaining G^1-continuity with adjacent surfaces. Our method is applicable to the shapes with holes or concave shapes. Moreover, our method is independent of the number of the boundary edges that connect the adjacent surfaces with G^1-continuity by using general-purpose sample points generating method. That is, our method is also applicable to a region surrounded by surfaces in all directions connecting with G^1-continuity. Since our method generates a surface from boundary edge information, it is applicable to various applications. For instance, by including our method in the trimmed surface compression method [2] that the authors proposed, more efficient compression can be attained.

5.2. Future Works

In our method, four boundary curves that include a closed region are generated and the trimmed surface is reconstructed by computing surface control points through approximation of sample points. Since surface control points are computed through approximation, our method is applicable even if a surface contains two or more holes or it is concave. However, the boundary curves cannot be generated to a shape with jagged boundary edges. It is because line segments extended to the outside of the region based on the cross boundary derivatives are intersecting or twisted, and the intersection points between the line segments and the bounding box cannot be calculated correctly. Therefore, it is necessary to consider the general-purpose boundary curve generation method, that is, it is necessary to consider the boundary curve generation method independent of the boundary edge shape expressing a closed region.

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