



A Design System for 3D Bending Shape of Flexible Printed Circuit in Electronics Devices

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ABSTRACT

Flexible Printed Circuits (FPCs) are often used in electronics devices. It is made on a thin elastic film which is bent in the devices and mounted on circuit boards. In this paper, a system to estimate the 3D bending shape of the FPC is proposed. A 3D path of the central curve of the FPC is first designed to connect the end connectors and then its surface is generated from this curve and the 2D diagram of the FPC. We propose a method based on rectifying developable surfaces to generate the surface. A new method for preserving the curve length is also described. The prototype software is developed to demonstrate some simple design examples.

Keywords: digital mockup, surface modeling, rectifying developable, flexible film.

1. INTRODUCTION

Electromagnetic Interference (EMI) and Compatibility (EMC) are critical properties to be considered in electronics device development. Simulation systems have been extensively used to analyze these properties. We are interested in a problem to generate 3D shape information of components of the electronics device needed for such electromagnetic simulations.

Among such components, Flexible Printed Circuits (FPCs) are increasingly being used in small electronics devices such as mobile phones. Fig. 1 shows a typical FPC used in mobile devices for connecting circuit boards. The FPC is made on a thin film which has wires to transmit signals and electric power, and often includes electronic circuits. After these electronic components are determined, a 2D layout diagram of these components is designed. Then 3D surface of the FPC must be designed by bending the profile on the 2D plane as shown in Fig. 2. As the FPC can be considered non-stretching and non-shrinking in their usual usage state without external forces in the electronics devices, they can be modeled as a developable surface. In addition, since some of the edges of the surface correspond to the terminal ends of the FPC whose mounting positions and orientations are given by the 3D design of the device,

the developable surface must satisfy such boundary conditions.

In this paper, we propose a system for designing 3D bending surfaces of FPCs. As shown in Fig. 2, the input is a 2D flat pattern representing a profile of the FPC and its medial line connecting the terminal ends of the FPC. Then the user can design a 3D path of the FPC by modifying a curve corresponding to this medial line. Thus we call this curve *control curve* of the FPC. Then the developable surface is generated along the control curve. A method of *rectifying developable surface* [2] is applied to generate the surface with the following extensions to our problem:

- 1) Boundary conditions for the control curve are defined in such a way that the terminal ends of the FPC must be mounted to specified connectors of circuit boards (Sec. 3.1).
- 2) During modifications by the designer, the curve length must be preserved. A method is proposed in [2] to modify the parameter range of the curve to give the target length. However, in our case as both end points are fixed, their method cannot be used. A new method based on global optimization for preserving the curve length is developed (Sec. 3.2).



Fig. 1: This photo [6] shows the teardown of an electronic device and several Flexible Printed Circuits (FPCs) (yellow components).

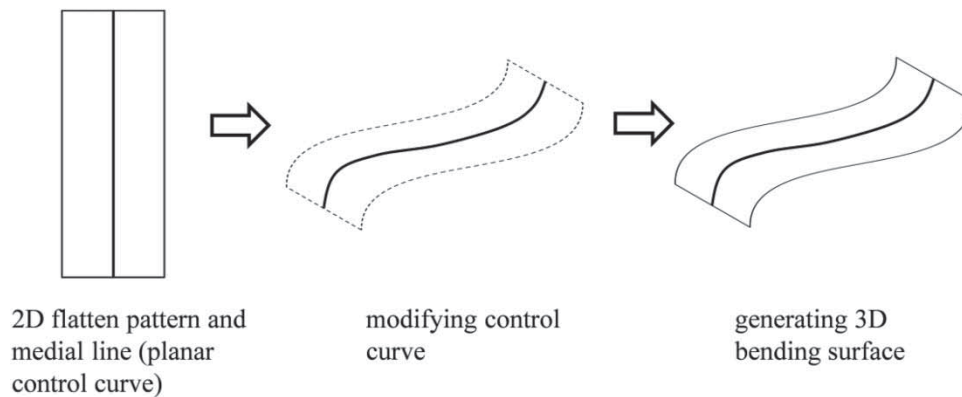


Fig. 2: Design of 3D bending shape of FPC using rectifying developable surface method [2].

A ruled surface is generated by moving a straight line along some curve. The ruled surfaces are classified into developable surfaces and non-developable surfaces. Developable surfaces are extensively studied in computer aided geometric design. For instance, developable surfaces can be modeled as Bézier surfaces [1,3]. However, to our knowledge, little work has been done on generating developable surfaces from a single curve except [2] in which paper bending is studied. Paper is the most typical example of developable surfaces. Representing behavior of paper and its use for modeling have been studied also in other places. Kergosien Y. et. al. [7] represented paper shape by its boundary and a mapping between boundary points, and a shape is generated by using simple physical simulation. Thus it is not an intuitive way for designing paper shapes. Frey W. H. [5] proposed a method to design a developable surface from a given closed curve, which is difficult to compute and not intuitive to be used in our problem.

2. OVERVIEW OF RECTIFYING DEVELOPABLE SURFACE

In this section, we give an overview of an algorithm proposed in [2] to generate a developable surface which is called a *rectifying developable surface* of a given geodesic curve. For more details, refer to this paper [2]. Suppose that a curve $C(s)$ with an arc length parameter is a geodesic on a developable surface S . We assume that $C(s)$ is at least C^3 having up to the third order derivative.

A developable surface can be described as a locus of a line called *ruling* along the curve. Fig. 3 illustrates the geometrical relation of the ruling and geodesic curve. A ruling at a point on the curve $C(s)$ extends in the direction $R(s)$ given by normalizing the vector $r(s)$ defined by the following equation:

$$r(s) = \tau(s)T(s) + \kappa(s)B(s), \quad (2.1)$$

where $\kappa(s)$ and $\tau(s)$ are the curvature and torsion of the curve respectively; $T(s)$ and $B(s)$ are the tangent

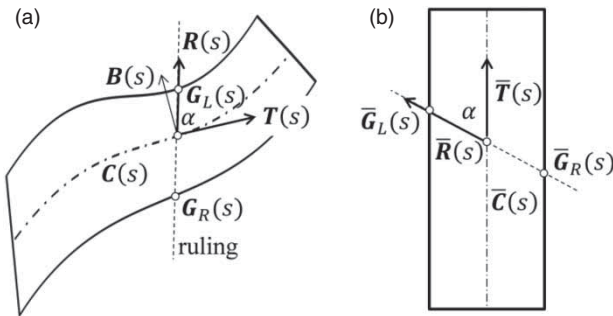


Fig. 3: Rectifying developable surface [2]: (a) A surface in the 3D space and (b) its flat pattern on the plane.

and binormal vectors of the curve respectively. Then S can be defined by the following equation:

$$S(s, u) = C(s) + uR(s), \quad (2.2)$$

where u is the surface parameter in the ruling direction.

As S is a developable surface, there exists an isometric mapping from S to a plane. Under this mapping the geodesic curve C is mapped to the medial line \bar{C} on the plane as shown in Fig. 4. In other words, the plane is bent into the surface S with the mapping defined by C . A ruling R is also mapped to a line in the \bar{R} direction on the plane. As the angle α between the tangent and ruling is preserved in the mapping, the direction \bar{R} of can be easily computed by using α .

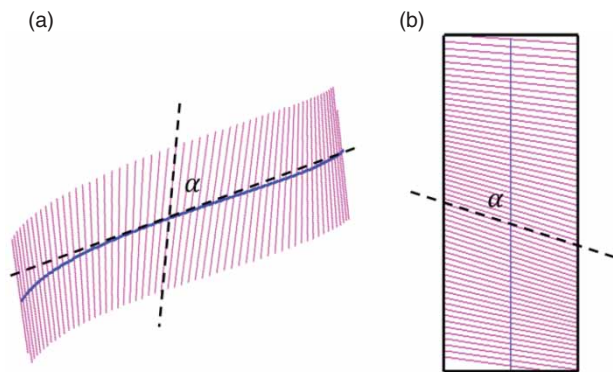


Fig. 4: (a) Geodesic curve and locus of rulings on rectifying developable surface. (b) Unfolded pattern of rulings on plane.

Imagine bending a piece of rectangle paper on the plane with $\bar{C}(s)$. Each line $\bar{R}(s)$ intersects with the rectangle at two boundary points $\bar{G}_R(s)$ and $\bar{G}_L(s)$. The boundary of the bent rectangle can be represented by their corresponding $G_R(s)$ and $G_L(s)$ on the surface. They are given by

$$G_R(s) = C(s) + u_R R(s) \text{ and } G_L(s) = C(s) - u_L R(s), \quad (2.3)$$

where u_R and u_L are lengths of the segments between $\bar{C}(s)$ and $\bar{G}_R(s)$ and $\bar{G}_L(s)$ respectively. These lengths are also preserved by the mapping. Note that the above formalization can be also made for a geodesic curve with general parameterization.

3. METHODS

3.1. Fixing Surface Boundaries

We use B-spline curve $\bar{C}(t), (0 \leq t \leq 1)$ of degree 4 with $K + 1$ control points $\{P_0, P_1, \dots, P_K\}$ to represent the control curve of a FPC [4]. As shown in Fig. 5, the boundary conditions of the surface are given as follows:

1. Two end positions of (t) : C_0 and C_1 .
2. Tangent vectors of the surface at C_0 and C_1 : T_0 and T_1
3. Normal vectors of the surface at C_0 and C_1 : N_0 and N_1
4. Ruling vectors of the surface at C_0 and C_1 : R_0 and R_1

$C_0, C_1, T_0, T_1, N_0, N_1, R_0$ and R_1 are actually derived from mounting conditions of the surfaces of the two end of the FPC. These conditions can be satisfied by positioning four control points near to the ends of $C(t)$. In the following, we derive equations only for the control points at one end $C(0)$ of the curve, but those for the other end $C(1)$ can be derived in the same manner. We also set such a condition so that T, N and R are orthogonal, however this condition is not so restrictive for practical FPC designs.

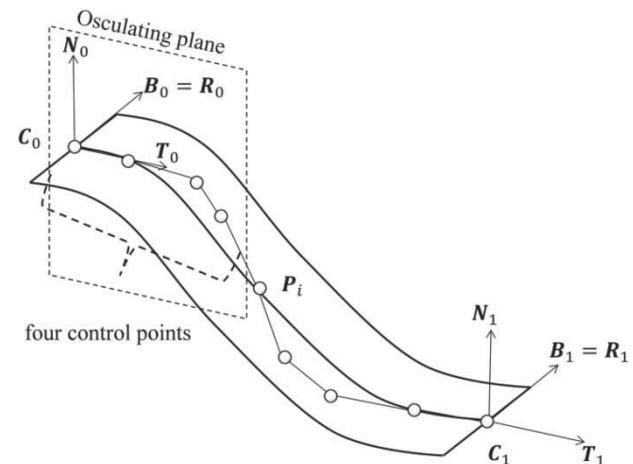


Fig. 5: Boundary conditions for control curve of surface.

By assuming the knot of $C(t)$ at parameters 0 and 1 to have multiplicity of 4, the end positions of $C(t)$, namely $C(0)$, is equal to C_0 . So the first condition can

be satisfied by positioning the end control point \mathbf{P}_0 by the following equation.

$$\mathbf{P}_0 = \mathbf{C}_0. \quad (3.1)$$

Secondly, the tangent vectors at \mathbf{B}_0 are parallel to $\mathbf{P}_0\mathbf{P}_1$, thus \mathbf{P}_1 is positioned according to the following equation to satisfy the second condition:

$$\mathbf{P}_1 = \mathbf{P}_0 + \mu\mathbf{T}_0, \quad (3.2)$$

where μ is a positive constant.

The third condition is used to constrain \mathbf{P}_2 on a plane. The normal vector \mathbf{N}_0 is perpendicular to the tangent vector \mathbf{T}_0 and lies in the plane passing through three control points \mathbf{P}_0 , \mathbf{P}_1 and \mathbf{P}_2 . So \mathbf{P}_2 must be on the osculating plane of the curve at \mathbf{C}_0 .

For the fourth condition, as the ruling vector \mathbf{R}_0 is dependent on the four control points, \mathbf{P}_0 , \mathbf{P}_1 , \mathbf{P}_2 , and \mathbf{P}_3 and should be perpendicular to both the tangent vector and normal vector, thus \mathbf{P}_3 must be on the same osculating plane.

In summary, these four boundary conditions can be satisfied by locating \mathbf{P}_0 and \mathbf{P}_1 using equations (3.1) and (3.2), and by constraining \mathbf{P}_2 and \mathbf{P}_3 to be on the osculating plane.

3.2. Curve Length Preservation

It is important to preserve the length of the control curve. Let L_0 be the length of the medial line $\bar{\mathbf{C}}(s)$ of the 2D profile the FPC. We sample sufficiently many $M + 1$ points $\{\mathbf{Q}_k, k = 0, 1, \dots, M\}$ on the curve $\mathbf{P}(t)$ by evenly sampling the parameter domain and compute its approximate length $L = \sum_{k=0}^{M-1} D_k$ where $D_k = \|\mathbf{Q}_{k+1} - \mathbf{Q}_k\|$. We minimize the difference between the squares of L and L_0 .

$$F = L^2 - L_0^2 = \left(\sum_{k=0}^{M-1} D_k \right)^2 - L_0^2 \quad (3.3)$$

With well-known relationship between arithmetic mean and root mean square we obtain:

$$\frac{1}{M} \sum_{k=0}^{M-1} D_k^2 = \left(\frac{1}{M} \sum_{k=0}^{M-1} D_k \right)^2 + \sigma^2 \quad (3.4)$$

$$\therefore \left(\sum_{k=0}^{M-1} D_k \right)^2 = M \sum_{k=0}^{M-1} D_k^2 - M^2 \sigma^2 \quad (3.5)$$

where $\sigma^2 = \frac{1}{M} \sum_{k=0}^{M-1} (D_k - \bar{D}_k)^2$ and \bar{D}_k is the arithmetic mean of D_k . Then equation (3.3) can be rewritten as

$$F = M \sum_{k=0}^{M-1} D_k^2 - M^2 \sigma^2 - L_0^2. \quad (3.6)$$

F can be regarded as a function $F(\mathbf{P})$ of the control points $\mathbf{P} = (\mathbf{P}_0, \mathbf{P}_1, \dots, \mathbf{P}_K)$. We minimize it by using a gradient method:

$$\mathbf{P}^{j+1} = \mathbf{P}^j - \lambda \frac{F(\mathbf{P}^j) \nabla F(\mathbf{P}^j)}{\|F(\mathbf{P}^j)\|^2} \quad (3.7)$$

$$\nabla F(\mathbf{P}) \equiv \left(\frac{\partial F(\mathbf{P})}{\partial \mathbf{P}_0}, \frac{\partial F(\mathbf{P})}{\partial \mathbf{P}_1}, \dots, \frac{\partial F(\mathbf{P})}{\partial \mathbf{P}_K} \right)^T \quad (3.8)$$

λ is an acceleration parameter and \mathbf{P}^j is the control points at the j -th iteration. By starting from the initial control point \mathbf{P}^0 of the initial control curve, we repeat the above iteration and stop when $F(\mathbf{P})$ becomes below the threshold ε . As introduced in the previous section, some control points on the boundaries are constrained and thus fixed in this minimization. We set $M^2 \sigma^2$ to be constant when computing $\nabla F(\mathbf{P})$. Thanks to this setting, $F(\mathbf{P})$ becomes a quadratic function of \mathbf{P} whose gradient can be easily computed.

4. EXAMPLES OF DESIGN PROCESSES OF FPCS USING PROPOSED SYSTEM

In this section we show some examples of a design process for a FPC. Here we assume the simplest type of the FPC having a long rectangular shape in the 2D diagram. The control curve is located in the center. First, as shown in Fig. 6 (a), we input the boundary conditions to adjust four control points from the ends. These control points are fixed during the design process. In Fig. 6 (b) we design the curve by moving the control points. In Fig. 6 (b) the curve length is adjusted to the length given by the 2D diagram of the FPC. During the design process, these two procedures are repeated at an interactive rate. In this particular example, the error of the curve length is about 10^{-7} for the true length of 1.8 and $\varepsilon = 10^{-8}$. We consider the curve length preservation is sufficiently precise.

After generating the curve, FPC surface is generated as its rectifying developable surface. A polygonal surface is generated by connecting segments of rulings computed at the same interval of the curve parameter. We skip sampling where the curvature vanishes.

Fig. 7 shows design examples with places of different heights that can be connected on a FPC, places of different connecting directions, and places at orthogonal orientation. The user modifies or adds the control points to edit the curve and every time the control points are changed the curve's length is adjusted. In these examples, about 10 to 20 control points are used to represent the curve and it takes about 200 msec to recalculate the shape by moving the control points on a standard desktop PC. About 80% of the computation is spent on the length preservation. It allows the user to interactively design FPC surfaces on 3D view.

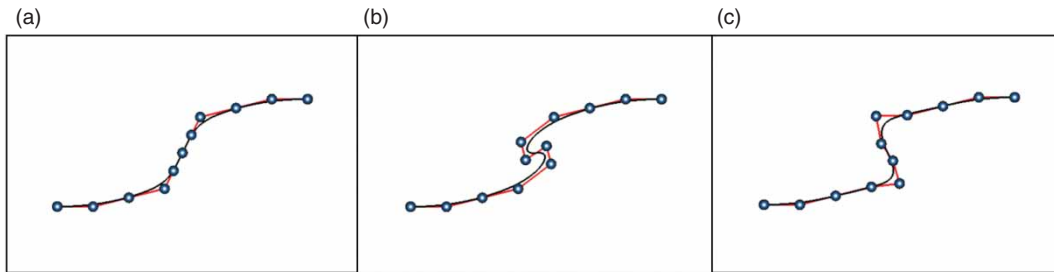


Fig. 6: (a) Initial control curve. (b) Modified control curve. (c) Curve with adjusted length. The circles represent control points.

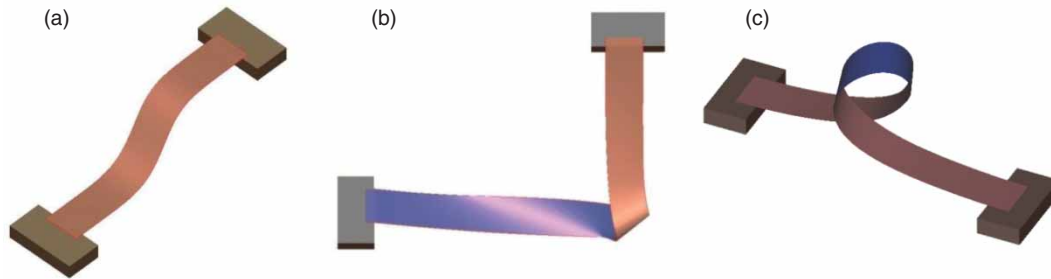


Fig. 7: Examples of FPC surface generation using our prototype system. (a) FPC connecting two places of different heights. (b) FPC folded by 90 degree. (c) FPC of a looped path.

5. CONCLUSIONS

A design system is proposed for 3D bending shapes of FPCs in electronics devices. The 3D bending shape of a flat FPC is generated as a rectifying developable surface using a control curve and the profile shape of the FPC. To facilitate the design process, two kinds of functionalities are implemented; boundary condition setting and curve length preservation. Some simple examples are demonstrated using the proposed system. It is shown that the surface generation can be made at interactive rates with sufficiently accurate length preservation.

As future work, we have to extend the boundary conditions for dealing with general ruling directions. We also need to consider FPCs with more complex profile shapes, particularly those with branching structures. In such cases, the recognition of the skeleton structure of the FPC is considered the most difficult problem. It is also important to develop a method for generating special folding patterns which are often used in electronics devices.

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