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Quality Improvement of Deformed Mesh Models for FEA by Density and Geometry-recovering Phased ODT Smoothing

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ABSTRACT

For realizing efficient product shape design, many dimension-driven mesh deformation methods have been proposed. However, the deformed mesh models often include many distorted elements, and lose the original mesh properties (mesh density and shape approximation accuracy). To solve these problems, we propose a quality improvement method for the deformed tetrahedral mesh models. Our method is based on Optimal Delaunay Triangulation (ODT) smoothing with edge split and edge collapse. In our method, element shape qualities are improved from the boundary to the inside of the mesh model, and original mesh properties are recovered in the deformed mesh models by an additional step in the ODT smoothing. In addition, degenerated and inverted elements are removed by local topological operations. The effectiveness of our method is demonstrated by applications to some simple mesh models.

Keywords: quality improvement, tetrahedral mesh deformation, ODT smoothing.

1. INTRODUCTION

In current product shape design, CAE based on Finite Element Analysis (FEA) becomes absolutely imperative for developing high quality products. In order to find optimal shape parameters, product shapes are first modified by using solid models in the CAD system. Then, meshing is applied to the solid models to generate mesh models, and the product shapes are evaluated by FEA using the mesh models. These processes, the solid modification, meshing, and FEA, are repeated until an optimal product shape is obtained. However, meshing is unstable and requires a high computational cost. Therefore, mesh deformation enable us to perform an efficient product shape design process because it can reduce the frequency of meshing. Especially, dimension-driven mesh deformation is effective for mechanical part design.

Many dimension-driven mesh deformation methods have been proposed [10], [12], [14,15], [18]. However, the resulting deformed mesh models often include many distorted elements, and lose original mesh properties, such as mesh density and shape approximation accuracy. The element shape qualities of deformed mesh models must be improved because distorted elements cause low accuracy and inefficiency of FEA. In addition, deformed mesh models should have the original mesh properties, because changes in them influence the accuracy of FEA and desired results may not be provided. Therefore, for accurate and efficient FEA, a new method which improves element shapes and recovers original mesh properties in the deformed mesh model is required.

Many mesh quality improvement methods have been proposed [2,3], [5–7], [11], [13], [16]. Especially, the methods based on Optimal Delaunay Triangulation (ODT) [2,3], [7], [16] are very effective methods. However, although deformed mesh models are required to have good element shape qualities and original mesh properties for accurate and efficient FEA, ODT-based methods mainly improve element shape qualities, and they cannot recover the original mesh properties.

In this paper, we propose a new quality improvement method for deformed mesh models which does not only improve element shape qualities, but also recovers original mesh properties. Our method is based on ODT smoothing [2] with edge split and edge collapse, and improves mesh qualities from the boundary to the inside. By using our method, mesh density of the original mesh model is recovered in the deformed mesh models based on the target

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mesh density field represented by a regular grid. The acceptable geometric errors are calculated from the original mesh model to keep the shape approximation accuracy in the resulting mesh model. Moreover, degenerated and inverted elements are removed by edge split and edge collapse. In this paper, we handle tetrahedral mesh models and assume that the models consist only of planar or cylindrical surfaces.

2. RELATED WORKS

There are many mesh quality improvement methods [2,3], [5–7], [11], [13], [16]. Especially, Optimal Delaunay Triangulation smoothing (ODT smoothing) [2,3] are effective and efficient for improving element shape quality of the tetrahedral mesh models. In ODT smoothing, vertex repositioning and edge (face) flipping are repeated to minimize a specific energy. In the repositioning step, interior vertices are moved to the average of circumcenters of their neighboring elements.

In original ODT smoothing [2,3], boundary vertices are fixed and only interior vertices are moved. Therefore, the element shape qualities near the boundary of mesh models are not improved. To solve this problem, Tournois et al. [16] proposed Natural Optimal Delaunay Triangulation (NODT). They add a special term to the repositioning equation of ODT smoothing for boundary vertices. In NODT, if the vertices on the boundary (surface of the tetrahedral mesh) move to the outside of the given mesh model, the vertices are repositioned to the boundary of the mesh model. Therefore, the element shape qualities near the boundary of mesh models may not be improved completely. Gao et al. [7] proposed Boundary-Optimized Delaunay Triangulation (B-ODT). In B-ODT, boundary vertices move under the tangent plane constraint and feature preserving constraint. B-ODT can provide better results than NODT. In our method, the element shape qualities are improved from the boundary to the inside, i.e. sharp edges, surface triangles, and tetrahedra, in order to improve shape qualities of all elements. In each improvement phase, standard ODT smoothing in different dimensions is applied without any additional constraints and vertex movements. In addition, the mesh properties before the deformation are recovered by adding new steps for controlling them into the ODT smoothing.

Sometimes mesh models include degenerated elements, such as slivers. When only using ODT smoothing, they often are not removed. In NODT and B-ODT, explicit perturbation and local topological modification are used for improving their quality. Moreover, many researches for improving the degenerated and inverted elements based on vertex movements have been conducted [1], [4], [8,9], [13], [15], [17]. In our research, a simple method for removing the degenerated and inverted elements caused by the mesh deformation is proposed. The method is based on the local topological modification.

3. QUALITY IMPROVEMENT OF DEFORMED MESH MODELS FOR FEA BY DENSITY AND GEOMETRY-RECOVERING PHASED ODT SMOOTHING

3.1. Overview of Our Method

Our method consists of following four steps as shown in Fig. 1.

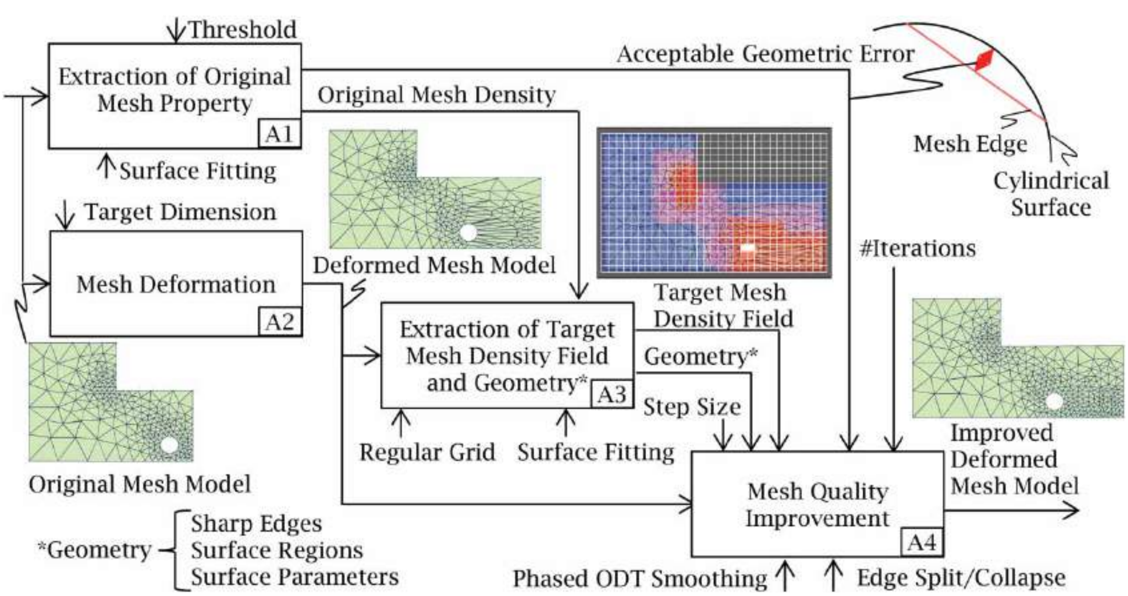

Fig. 1: Overview of our method.

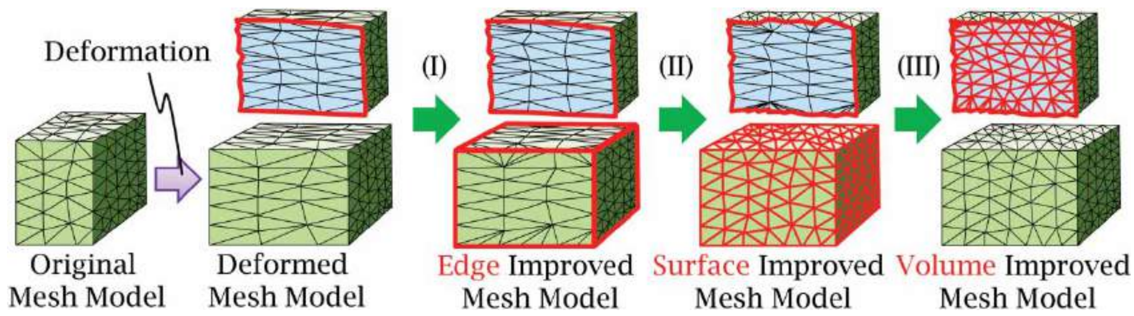

Fig. 2: Our method improves mesh quality from the boundary to the inside.

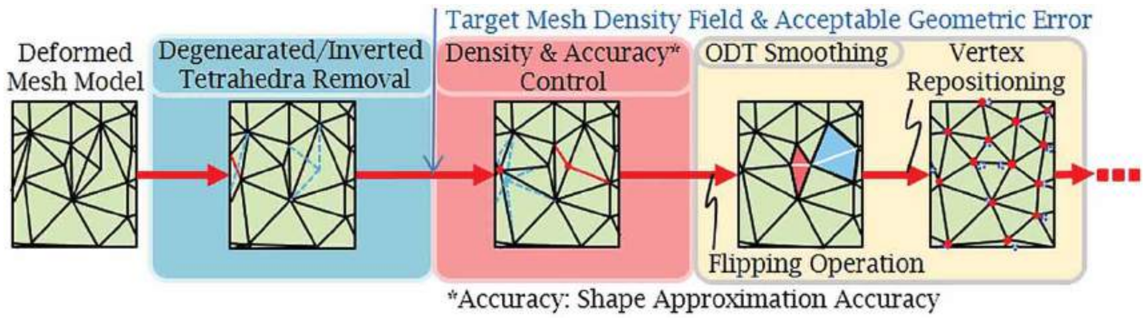

Fig. 3: The improvement process in each phase of the phased ODT smoothing.

(1) Mesh density, geometry (such as sharp edges, surface regions, and surface parameters), and acceptable geometric errors of original mesh model are extracted (A1).
(2) The mesh model is deformed by a dimension-driven mesh deformation method (A2).
(3) Target mesh density field and geometry of deformed mesh models are extracted using original mesh properties (A3).
(4) Element shape qualities are improved by phased quality improvement based on ODT smoothing (A4). In the smoothing, degenerated and inverted elements are removed by edge split and edge collapse at first. Then original mesh properties are recovered by edge split and edge collapse with target mesh density field and acceptable geometric errors. Finally, element shape qualities are improved by ODT smoothing.

Our method has four features as shown in Fig. 2 and Fig. 3.

- All element shapes are progressively improved from the boundary to the inside (sharp edges, surface triangles, and tetrahedra in that order) by ODT smoothing (Fig. 2).
- To recover mesh density of the original mesh model in the deformed mesh model, the target mesh density field represented by a regular grid is generated, and edge split and edge collapse according to the target mesh density field are performed during ODT smoothing (Fig. 3).
- To recover shape approximation accuracy of the original mesh model, acceptable geometric error is calculated from the original mesh model, and edge split is applied during ODT smoothing depending on the acceptable geometric error (Fig. 3).
- Degenerated and inverted tetrahedra are removed by the combination of edge split and edge collapse (Fig. 3).

In the following sections, methods for generating the target mesh density and extracting geometry, including definition of geometric error, are outlined. Secondly, ODT smoothing is explained, and ways of applying it to the sharp edges, the surface triangles, and the tetrahedra are shown. Then, a method for recovering original mesh properties by edge split and edge collapse is described. Finally, methods for degenerated and inverted tetrahedra removal are presented.

In our research, stretch is used as an element shape quality measure. The stretch of tetrahedron $st(\tau)$ is calculated by Eqn. (3.1):

$$st(\tau) = \frac{6\sqrt{6}V(\tau)}{\{\max_{e\in E_\tau} l(e)\}S(\tau)}, \tag{3.1}$$

where $V(\tau)$ is the signed volume of τ, $S(\tau)$ the surface area of τ, E_τ a set of edges of τ, and $l(e)$ is the length of edge e. Stretch becomes 1 for regular tetrahedron,

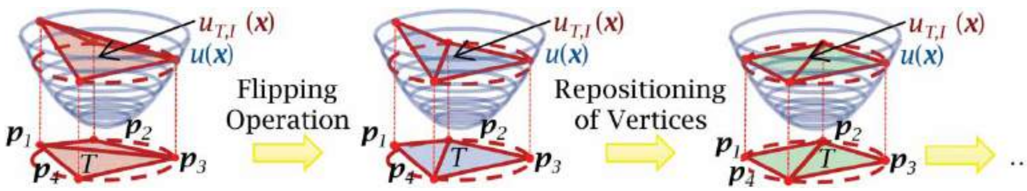

Fig. 4: ODT smoothing improves element shape qualities by two steps.

0 for degenerated tetrahedron, and a negative value for inverted tetrahedron.

3.2. Extraction of Target Mesh Density Field

For recovering the mesh density of the original mesh model in the deformed mesh model, a target mesh density field is extracted. At first, mesh density at each vertex of the original mesh model is calculated as the average of reciprocals of incident edge lengths. Secondly, the mesh model is deformed using a dimension-driven mesh deformation method. Then, a regular grid including the deformed mesh model is generated. Next, mesh density of each grid point g is estimated by barycentric interpolation of the mesh density of the deformed mesh model using Eqn. (3.2):

$$\rho'_g := \frac{1}{|\tau_j|} \sum_{i \in \tau_j} \left|\tau_j^i\right| \rho_i, \tag{3.2}$$

where τ_j is a tetrahedron which includes g, $|\tau_j|$ the volume of τ_j, τ_j^i a tetrahedron which is obtained by substituting the vertex i with g on τ_j, and ρ_i mesh density at vertex i. Finally, mesh density of each grid cell is calculated by taking the average of the nonzero mesh densities of its corner grid points.

3.3. Geometry Extraction

Sharp edges, surface regions, and their surface parameters are extracted as follows. At first, the sharp mesh edges are found by the thresholding of dihedral angles. Secondly, the vertices having more than two incident-sharp mesh edges are identified as a corner point. Then, a sharp edge is defined as the connected sequence of sharp mesh edges between two corner points or that shaping a loop. Next, a set of triangles surrounded by sharp edges is identified as a surface region, and each region is classified into either a planar or cylindrical region by surface fitting. At the same time, surface parameters are extracted. Finally, each sharp edge is classified into either a straight line segment, a circular arc, or a circle, according to the types of neighboring surface regions. These processes are applied to both the original mesh model and the deformed mesh model.

In cylindrical surface regions, the geometric error of each mesh edge is defined as the shortest distance from the midpoint of the edge to the fitted cylinder. In each cylindrical surface region of the original mesh model, acceptable geometric error is calculated as the maximum geometric error of the region.

3.4. Quality Improvement by Phased ODT Smoothing

3.4.1. ODT smoothing [2]

ODT smoothing [2] is one of the mesh improvement methods which minimizes an error function defined by Eqn. (3.3):

$$E(T) = \int_{\Omega} \left|u_{T,I}(\boldsymbol{x}) - u(\boldsymbol{x})\right| \rho(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}, \tag{3.3}$$

where T is a triangulation of the given vertex set $\mathrm{P} = \{p_i\}$, $\Omega \subset \mathbb{R}^n$ a convex domain of P, $u(\boldsymbol{x}) = \|\boldsymbol{x}\|^2$, $u_{T,I}(\boldsymbol{x})$ the piecewise linear interpolation of $u(\boldsymbol{x})$ by triangulation T, and ρ is a given density function defined on Ω. In ODT smoothing, repositioning of vertices and flipping operations are repeated in order to minimize $E(T)$ (Fig. 4). In repositioning, new positions of vertices are calculated using Eqn. (3.4):

$$\mathbf{x}'_i = (1-\alpha)\mathbf{x}_i + \frac{\alpha}{|\omega_i|} \sum_{\tau_j \in \omega_i} \boldsymbol{c}_j, \tag{3.4}$$

where $\boldsymbol{x}_i$ is the position of vertex i before repositioning, $\boldsymbol{x}'_i$ the new position of vertex i after repositioning, ω_i a set of elements which includes i, $\boldsymbol{c}_j$ position of the circumcenter of element τ_j, and α is step size $(0 < \alpha \leq 1.0)$. The flipping operation is applied so that all elements satisfy the Delaunay condition.

ODT smoothing is efficient and effective for improving element shape qualities. However, ODT smoothing allows only interior vertices to move, it cannot improve element shape qualities near the boundary of mesh models. In order to improve them, we propose phased ODT smoothing which improves the mesh quality from the boundary to the inside (Fig. 2). In our method, ODT smoothing is applied in three phases: sharp edges improvement, surface triangles improvement, and tetrahedra improvement. In the following sections, we explain ways of applying ODT smoothing in each phase.

3.4.2. Sharp Edges Improvement

For straight line segments, the repositioning of vertices by ODT smoothing is done by using Eqn. (3.4), where midpoints of edges are used as $\boldsymbol{c}_j$ (stated exactly, this smoothing is called CPT smoothing [2]). On the other hand, the circles, or circular arcs, shrink when their vertices are moved by Eqn. (3.4). Therefore, for the circles, or circular arcs, the vertex repositioning is performed on the 1D parameter space based on the central angle. As shown in Fig. 5, each vertex position on the circle (or circular arc) is parameterized by its central angle θ, and the new central angle is calculated by Eqn. (3.4). The vertex moves onto the circle, or the circular arc, corresponding to the new calculated central angle. In this phase, flipping operations are not applied.

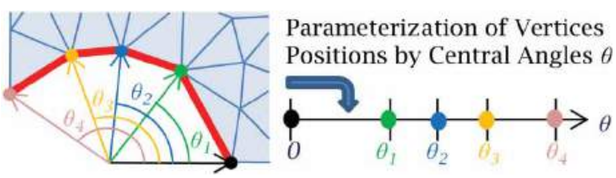

Fig. 5: ODT smoothing is applied on the 1D parameter space.

3.4.3. *Surface Triangles Improvement*

In the surface triangles improvement, for planar regions, 2D ODT smoothing is done by Eqn. (3.4) where circumcenters of surface triangles are used as $\boldsymbol{c}_j$. In order to avoid the generation of distorted triangles near the region boundary, barycenters are used as $\boldsymbol{c}_j$ in Eqn. (3.4) for triangles that have at least one vertex on the region boundary instead of their circumcenters.

On the other hand, cylindrical regions shrink by the ODT smoothing. Therefore, for cylindrical regions, the vertex repositioning is performed on 2D parameter space given by the surface development as the following steps (Fig. 6). At First, circumcenters (or barycenters) $\{\boldsymbol{c}_j\}$ of neighboring surface triangles of vertex i and $\boldsymbol{x}_i$ are mapped onto the $r\theta$-z plane that is a development of the cylinder. The mapped $\{\boldsymbol{c}_j\}$ and $\boldsymbol{x}_i$ are denoted by $\{\hat{\boldsymbol{c}}_j\}$ and $\hat{\boldsymbol{x}}_i$. Then, $\boldsymbol{x}'_i$ is calculated by Eqn. (3.3) using $\{\hat{\boldsymbol{c}}_j\}$ and $\hat{\boldsymbol{x}}_i$. Finally, the new position of vertex i is derived from $\boldsymbol{x}'_i$ by inverse mapping of the first step.

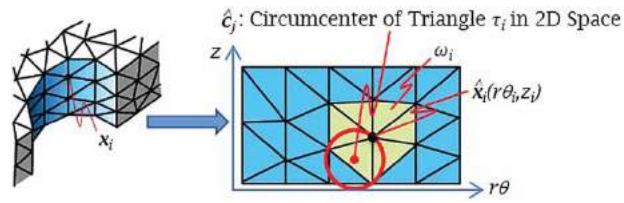

Fig. 6: ODT smoothing is applied on 2D parameter space for the cylindrical surface.

In the surface triangles improvement, edge flipping is applied to obtain a Delaunay triangulation. In our method, to preserve the topological consistency of tetrahedra, the edge flipping is realized by the combination of edge split and edge collapse, as shown in Fig. 7. In edge split, an edge is divided in two edges and a vertex is inserted (Fig. 7(a)). On the other hand, in edge collapse, an edge is collapsed and its two end points are merged into a new vertex (Fig. 7(b)). In edge flipping, at first, the edge is split. Then a new edge whose two vertices are on the surface is collapsed. If degenerated or inverted elements would occur by these operations, flipping is not applied to that edge. Therefore, some triangles do not satisfy the Delaunay condition. Barycenter is used as $\boldsymbol{c}_j$ in Eqn. (3.4) for such triangles instead of their circumcenters.

3.4.4. *Tetrahedra Improvement*

In the tetrahedra improvement, original 3D ODT smoothing is used. Therefore, interior vertices are moved by using Eqn. (3.4) where circumcenters of tetrahedra are used as $\boldsymbol{c}_j$. In order to avoid the generation of distorted tetrahedra near the surface, barycenters are used as $\boldsymbol{c}_j$ in Eqn. (3.4) for tetrahedra that have at least one vertex on the surface instead of their circumcenters. Also, if the tetrahedra do not satisfy the Delaunay condition, their barycenters are used as $\boldsymbol{c}_j$ in the Eqn. (3.4) instead of their circumcenters.

In the tetrahedra improvement, *Flipping 2-3*, *Flipping 3-2*, and *Flipping 4-4* [6] (Fig. 8) are applied so that the tetrahedra satisfy the Delaunay condition. In Flipping 2-3, two neighboring tetrahedra sharing a triangle become three neighboring tetrahedra sharing an edge. Flipping 3-2 is an inverse operation of Flipping 2-3. In Flipping 4-4, an edge shared by four tetrahedra which form an octahedron is swapped. These

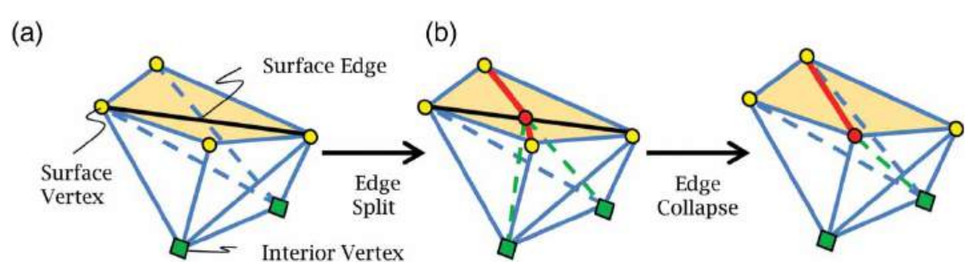

Fig. 7: Edge flipping is done by combination of edge split and edge collapse.

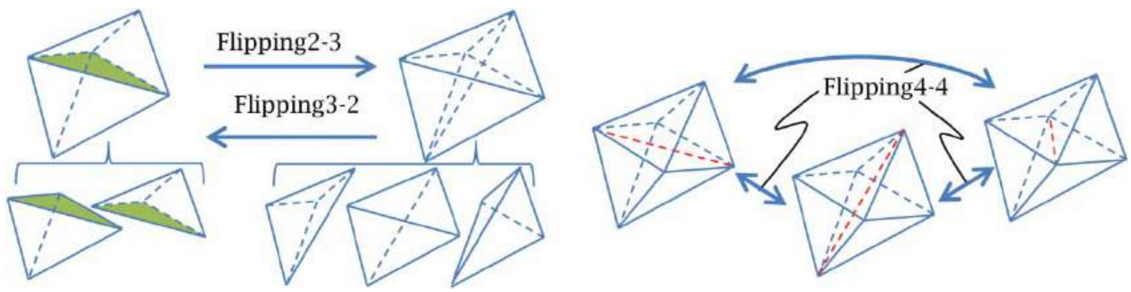

Fig. 8: Three flipping operations are done in the tetrahedra improvement phase.

flipping operations are applied to triangles or edges which satisfy the following three conditions.

- Some neighboring tetrahedra do not satisfy the Delaunay condition.
- All resulting tetrahedra satisfy the Delaunay condition after the flipping operation.
- After the flipping operation, the minimum quality of the resulting tetrahedra is higher than the one before the flipping operation.

3.5. Recovering of Original Mesh Properties and Bad Elements Removal

3.5.1. Recovering of Original Mesh Properties

In order to recover the original mesh properties in the deformed mesh models, edge split and edge collapse are applied (Fig. 3). In order to recover the shape approximation accuracy of the original mesh model, at first, surface mesh edges which have a larger geometric error than the acceptable geometric error of the cylindrical surface are split by edge split. The vertices generated by the edge split are moved onto the fitted cylinder. Thus, the shape approximation accuracy of the deformed mesh becomes similar to one of the original mesh models.

To recover mesh density of the original mesh model, if $\rho_e < \gamma\rho'_e$, edge e is split, and if $\rho_e > \delta\rho'_e$, edge e is collapsed where ρ_e is the reciprocal of the length of edge e, and ρ'_e is the mesh density of the grid cell of the target mesh density field which includes the midpoint of the edge e. γ and δ are constants (and in our experiments, γ and δ are set to 2/3 and 3/2, respectively). After edge split and edge collapse, the new vertex which is generated by these operations is moved onto the fitted cylinder if edge e is on the cylindrical surface. In order to preserve shape of mesh model, edge collapse is not applied to the following edges; (a) the edge connecting two corner vertices, (b) the edge which is not lying on the sharp edges and connects two vertices on the sharp edges, and (c) interior edge connecting two vertices on the surface.

The position of new vertex after edge collapse is decided according to the following rules; (a) if an endpoint of the edge is a corner vertex, the new vertex position is the one of the corner vertex, (b) if an endpoint of the edge which is not lying on the sharp edges is a boundary vertex, the new vertex position is the one of the boundary vertex, (c) if an endpoint of the interior edge is a surface vertex, the new vertex position is the one of the surface vertex, and (d) if other edges are collapsed, their endpoints merge into their midpoints.

3.5.2. Degenerated Tetrahedral Elements Removal

In our method, degenerated elements, e.g. sliver, cap, and spade are removed by the combination of edge split and edge collapse (Fig. 9). These elements have four vertices near one plane. In the sliver and cap, their projected vertices on to a plane form a quadrangle and a triangle having one interior vertex, respectively (Fig. 9(a) and (b)). Spade is a tetrahedron whose three vertices are near a line (Fig. 9(c)).

After deformation, mesh models often have degenerated elements, and they often neighbor each other. Existing methods [4], [9], [17] may be useful for improving them, but we introduce a simple method for removing them. In our problem setting, we do not have to consider the mesh properties in this process because they are improved in the later smoothing process. Therefore, the degenerated tetrahedra are removed by a simple combination of edge split and edge collapse in our method. As shown in Fig. 9(a), for slivers, edge split is first applied to two edges which cross each other. Then, a new short edge is collapsed. As a result, a sliver becomes four triangles. As shown in Fig. 9(b) and (c), for caps and spades, edge split is applied to an edge which is shared by the largest triangle and the smallest triangle of the tetrahedron. Then, a new short edge is collapsed. By this operation, a cap or a spade becomes two triangles. In edge collapse, if an endpoint of the edge is a surface vertex, the new vertex position is the one of the surface vertex in order to preserve shape of mesh models.

3.5.3. Inverted Tetrahedral Elements Removal

After deformation, mesh models may have some inverted elements, and they are often stretched thin. Although untangling methods have been proposed [1], [8], [13], [15] for removing or modifying them, we also introduce a simple removal method of inverted

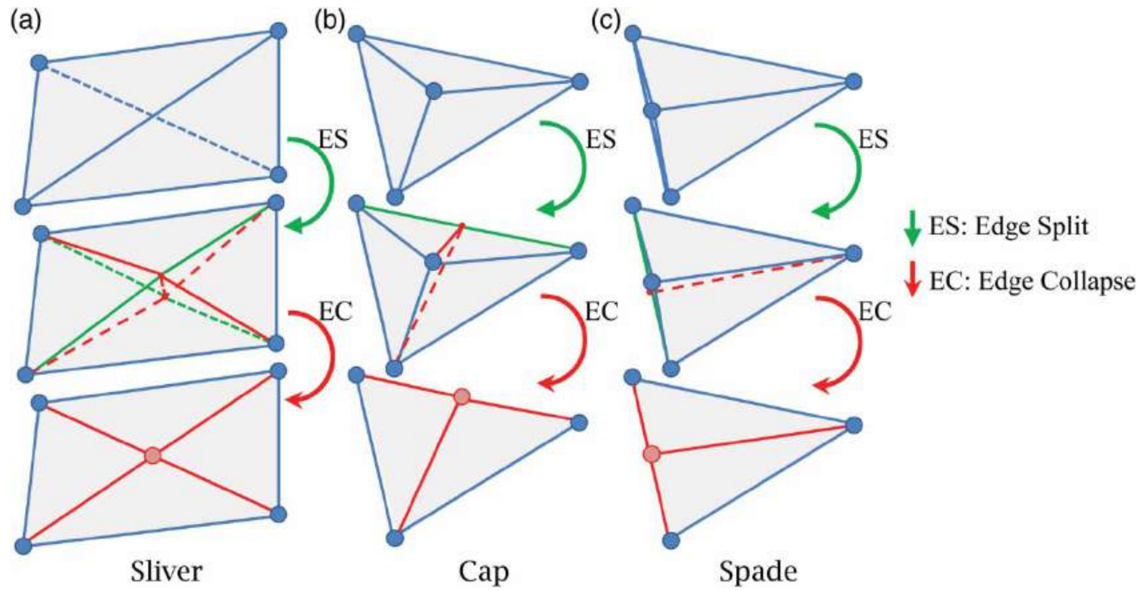

Fig. 9: Degenerated elements are removed by the 2-steps operation.

tetrahedral elements caused by the deformation. Because we do not have to consider the mesh properties as mentioned in section 3.5.2, half edge collapse is adopted for removing inverted tetrahedra. In this operation, all tetrahedra which have a negative value of stretch are found, and half edge collapse is applied to an edge of the tetrahedron which has the minimum stretch until the minimum stretch becomes more than 0. The collapsed edge and the new vertex position are selected so that minimum stretch of tetrahedra neighboring the new vertex is maximized after edge collapse.

4. RESULTS AND EVALUATIONS

In this section, we show three results of our method. For the first and second examples, the effects of element shape quality improvement and recovering the original mesh properties are shown using two simple deformed mesh models. Then, the result of the application of our method to a simple model which has a non-uniform mesh density, some cylindrical surfaces, and some inverted elements is shown and compared with the one of original ODT smoothing [2]. In our experiments, the Takano's method [15] is used as a dimension-driven mesh deformation.

At first, the effectiveness of our method to improve deformed non-uniform density mesh models is shown. In order to evaluate mesh density of each mesh model, a mesh density error $\varepsilon_\rho(e)$ at each edge e is calculated by Eqn. (4.1):

$$\varepsilon_\rho(e) = \frac{|\rho_e - \rho'_e|}{\rho'_e}, \tag{4.1}$$

Fig. 10 shows the original mesh model (a), the deformed mesh model by changing a distance between two parallel planes (DP) (b), and the mesh model improved by our method (c). Cross section views and stretch histograms are also shown in Fig. 10, and Tab. 1 shows the stretches and density errors in the each mesh model. These results show that our method could improve the element shape qualities of the deformed mesh models while recovering the mesh density of the original mesh model in the deformed mesh model.

Fig. 11 shows the original mesh model (a), the deformed mesh model changing a radius of a cylinder (RC) (b), the mesh model improved by our method (c), and each stretch histogram. Tab. 2 shows the stretches and geometric errors in each mesh model. These results show that our method could provide a deformed mesh model which has shape approximation accuracy similar to that of the original mesh model and good element shape qualities.

Finally, the results of original ODT smoothing [2] and our method are compared. Fig. 12 shows the original mesh model (a), the deformed mesh model by changing DP, RC, and the position of a hole (PH) (b), the mesh model improved by the original ODT smoothing (c), and the mesh model improved by our method (d). Each cross section and stretch histogram are also shown in Fig. 12, and Tab. 3 shows stretches, density errors, geometric errors, and the number of inverted elements in each mesh model. The deformed mesh model (Fig. 12 (c)) has some inverted elements, and many bad tetrahedra, which have a lower stretch than 0.1.

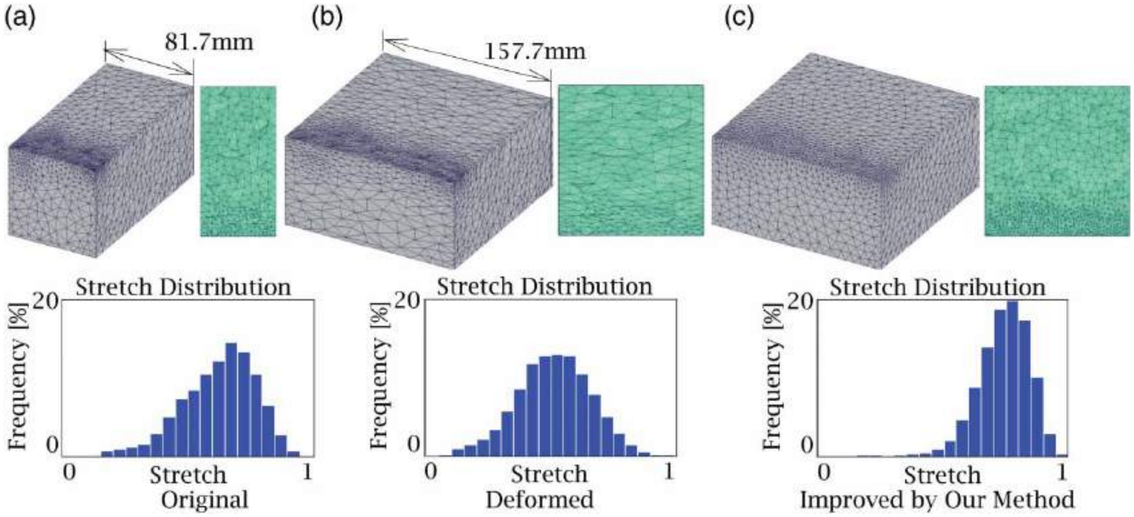

Fig. 10: Result of the improvement of the deformed mesh model (changing DP).

		Stretch		*Density Error*	
Mesh Model	*#Tetra*	Ave	Min	Ave	Max
(a) Original	19,248	0.625	0.121	–	–
(b) Deformed		0.511	0.078	0.293	4.076
(c) Improved by Our Method	22,416	0.743	0.164	0.140	0.970

Tab. 1: Mesh quality of each mesh model (changing DP).

By using original ODT smoothing, inverted elements are not removed and bad elements exist. On the other hand, our method can remove these elements, and minimum stretch becomes 0.190 after improvement. In addition, the density error and geometric error get better by our method. It means that our method can improve element shape qualities while recovering the original mesh properties in the deformed mesh model. The calculation time of the mesh quality improvement was 37 [sec] using a standard PC (CPU: Core i7 2.8GHz, RAM: 8GB).

		Stretch		*Geometric Error*	
Mesh Model	*#Tetra*	Ave	Min	Ave	Max
(a) Original	6,138	0.689	0.488	0.126	0.378
(b) Deformed		0.625	0.015	0.226	0.680
(c) Improved by Our Method	4,948	0.733	0.034	0.084	0.283

Tab. 2: Mesh quality of each mesh model (changing RC).

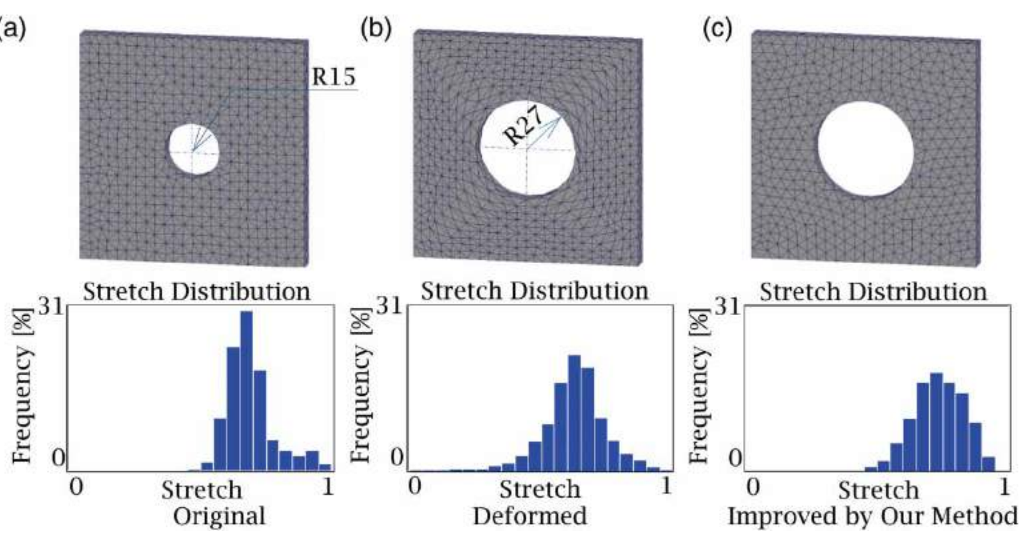

Fig. 11: Result of the improvement of the deformed mesh model (changing RC).

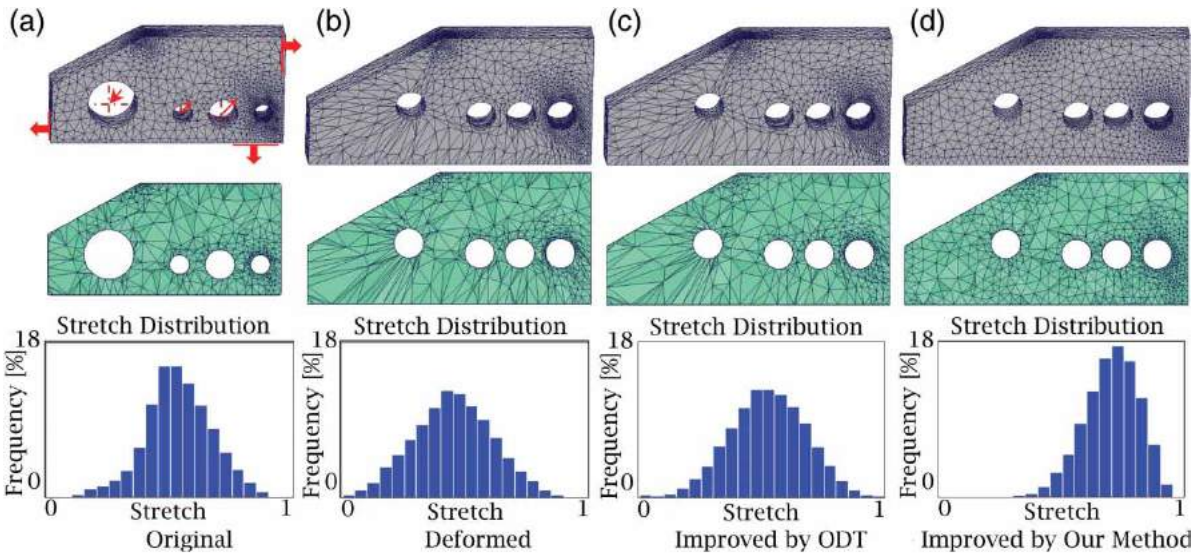

Fig. 12: Result of the improvement of the deformed mesh model (changing DP, RC, and PH).

Mesh Model	*#Tetra*	*Stretch* Ave	*Stretch* Min	*Density Error* Ave	*Density Error* Max	*Geometric Error* Ave	*Geometric Error* Max	*# Inverted Tetra*
(a) Original	13,238	0.534	0.080	–	–	0.100	0.270	0
(b) Deformed		0.450	−0.216	0.311	5.990	0.121	0.359	19
(c) Improved by ODT Smoothing	12,754	0.522	−0.281	0.295	18.257	0.121	0.359	19
(c) Improved by Our Method	18,282	0.700	0.190	0.156	1.713	0.067	0.391	0

Tab. 3: Mesh quality of each mesh model (changing DP, RC, and PH).

5. CONCLUSIONS

In this paper, we proposed a quality improvement method for tetrahedral mesh models deformed by dimension-driven mesh deformation methods. In our method, element shape qualities are improved from the boundary to the inside by phased ODT smoothing. Also edge split and edge collapse based on the target mesh density field and the acceptable geometric error are combined with ODT smoothing in order to recover the original mesh properties. In addition, degenerated and inverted elements are removed by edge split and edge collapse. The experiments showed that our method can improve element shape qualities, recover the original mesh properties, and remove some degenerated and inverted elements. Our future work will focus on extending our method for improving mesh models, including free-form surfaces. In addition, we will compare results of our phased ODT smoothing with other smoothing methods, such as NODT [16] or B-ODT [7], and remeshing methods.

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