G^2 Continuous Generalized Log-Aesthetic Curves

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ABSTRACT

Log-Aesthetic (LA) curve has been claimed as one of a most promising curve for design purpose. However, LA curve has no shape variable which can be used to control its end curvatures directly. Generalized Log-Aesthetic Curve (GLAC) which is the family of LA curve has ability to control its curvature with an extra shape parameter. This paper highlights on designing two segments of GLACs in the form of C- and S-shapes with curvature continuity using interior point method. Designers will be able to design S-shapes and C-shapes by inputting control points, its direction of travel and estimated end curvatures.

Keywords: spiral, monotonic curvature, Hermite data, aesthetic curves.

1. INTRODUCTION

The success of a product released in the market or the design of an architectural building depends not just on its functionalities but it must be visually pleasing [15]. For example, Museo Guggenheim in Spain is regarded as the most influential modern architecture in the world while Ferrari is famous for its elegant and stunning look.

Product design such as in automotive, aircraft industries, architecture designs & computer animations emphasizes on the usage of world class aesthetic curves such as logarithmic spiral & clothoid (or cornu spiral). The importance of aesthetic shapes to design industrial products leads to the studies of planar aesthetic curves [9,10]. Aesthetic curves have monotonic curvature profile & are regarded as of high aesthetical value. The general function representing aesthetic curves are known as Log-Aesthetic (LA) curve [12]. The formulation of LA curves produces logarithmic spiral, clothoid, Nielsen's spiral & circle involute depending on its Logarithmic Curvature Graph's (LCG) gradient α. Yoshida & Saito [17] proposed a novel method to render LA curves interactively using given endpoints & their respective tangent vectors. Consequently, numerous research papers have been published lately presenting works on LA curves such as [11,16,13,5,14] and reference therein.

However, the standard formulations of LA curves have weakness where its curvature cannot be control directly using its shape variables. The standard form 1 represented by Yoshida & Saito cannot be used to design LA curve with defined end curvatures. Thus, in 2013 Miura et al. [13] used standard form 2 to represent a method to design G^2 continuous LA curves by using 3 pieces of LA segment called LA triplets. It has been successfully used in Rhino 3D CAD systems for automobile design. However, Generalized Log-Aesthetic Curve (GLAC) is the generalization of LA curves which can be used to achieve the desired curvature using the extra shape variable ν[1,2,8] with less effort. This extra shape parameter distinguishes GLAC from LAC and GLAC has the aptitude to fix desired curvature at the origin. GLAC produces LA curves, Generalized Cornu Spiral (GCS), logarithmic spiral, clothoid, Nielsen's spiral & circle involute a [6]. The vitality of GLAC is further elucidated in [7] where they proved that the drawable region of GLAC is wider as compared to LAC in which GLAC provides wider solution in which the LA curves could not. Recent research on GLAC includes the extension of spatial GLAC [3].

This paper proposes G^2 scheme to design S-shape & C-shape GLACs. The result looks promising where the designer may directly input the desired G^2 Hermite data to obtain a GLAC spline with curvature continuity. The next section elaborates on the formulation & its respective algorithms. The final section presents numerical examples for S-shape & C-shape GLACs.

2. G^2 CONTINUOUS GLAC

GLAC is derived through the curve synthesis process where the formulation of a curve is derived from a well-defined curvature function. When the extra shape parameter ν is 0, GLAC becomes LA curves. GLAC has the ability to dictate curvature values at the origin & is suitable for interactive design [8]. Eqn. (1), (2)& (3) presents the curvature, arc length & turning angle function of GLAC respectively. GLAC is arc length parametrized and $\{\Lambda, \alpha, \nu\} \in \mathbb{R}$. are variables that can be used to shape a segment of GLAC (Eqn. (4)).

$$\kappa_{GLAC}(s) = \begin{cases} e^{-\Lambda s} + \nu & \alpha = 0 \\ (\Lambda \alpha s + 1)^{\frac{-1}{\alpha}} + \nu & otherwise \end{cases} \tag{1}$$

$$s_{GLAC}(\rho) = \begin{cases} \frac{1}{\Lambda} \log\left[\frac{1}{\rho^{-1} - \nu}\right] & if\ \alpha = 0 \\ \frac{1}{\Lambda \alpha}((\rho^{-1} - \nu)^{-\alpha} - 1) & otherwise \end{cases} \tag{2}$$

$$\theta_{GLAC}(s) = \begin{cases} \frac{1}{\Lambda}(1 - e^{-\Lambda s}) + \nu s & if\ \alpha = 0 \\ \frac{1}{\Lambda} \log[\Lambda s + 1] + \nu s & if\ \alpha = 1 \\ \frac{1}{\Lambda(\alpha - 1)}((\Lambda \alpha s + 1)^{\frac{\alpha-1}{\alpha}} - 1) + \nu s & otherwise \end{cases} \tag{3}$$

$$C_{GLAC}(s) = \left\{\int_0^s \cos[\theta_{GLAC}(u)]du, \int_0^s \sin[\theta_{GLAC}(u)]du\right\} \tag{4}$$

The overall shapes of GLAC have been identified in [6] where it produces world class aesthetic spirals such as clothoid & logarithmic spiral. The interactive design of GLAC satisfying G^1 Hermite data has been proposed by Gobithaasan et al. [7] where its drawable region has been proven to be much wider than the leading LA curves.

It is vital in product design to achieve G^2 continuity when modelling aesthetic shapes to avoid unwanted jerks and oscillations. In this paper, we propose an algorithm for GLAC satisfying G^2 continuity and generate C & S-shapes. Practitioners will be able to control curvatures at both ends for design purposes and at the same time decide where exactly they want an inflection point to occur (for the case of S-shape). However, the output end curvatures are scaled end curvatures given by user. Interior point methods employed to satisfy the constraints in which suitable values of $\{\theta_d, \Lambda, s\}$ are identified to fit given G^2 Hermite data.

2.1. Function & Bounds for G^1 & G^2 Continuous GLAC

An overall study of GLAC shapes has been studied in detail in order to find proper bounds of GLAC which can be used for design [6]. Hence, we employ the bounds as constraints in the proposed algorithm to satisfy given inputs. Tab. 1 describes the constraints exist for $\{\theta_d, \Lambda, s\}$ when $\nu > 0$ & $-1 < \nu < 0$. These constraints will be used to find the values for free parameters$\{\theta_d, \Lambda, s\}$. Eqn. (5) presents the total winding angle of GLAC in Cartesian plane which is the objective function $f(x)$ in this minimization problem.

$$\begin{aligned} f(x) &= \theta(s) \\ &= \cos^{-1}\left[\frac{Re[C_{GLAC}(s)]}{\sqrt{Re[C_{GLAC}(s)]^2 + Im[C_{GLAC}(s)]^2}}\right] \end{aligned} \tag{5}$$

2.2. Configuration for the Placement of Control Points

The S-shape & C-shape consists of two GLACs ($GLAC_1$& $GLAC_2$) having control points$\{P_0, P_1, P_2, P_3, P_4\}$ joined with G^2 continuity. $GLAC_1$ & $GLAC_2$ are the curve segments with control points $\{P_0, P_1, P_2\}$ & $\{P_2, P_3, P_4\}$ respectively and both segments are joined at P_2 where they possess the same curvature value.

2.2.1. S-shape G^2 continuous GLAC

A S-shape GLAC has an inflection at the joint ($\kappa(s) = 0$). We use $-1 < \nu < 0$ for S-shape of GLAC since it gives upper bound which is the point of inflection [6].

α	Λ			s	θ_d
	$\nu > 0$ C-shape	$-1 < \nu < 0$ C-shape	S-shape		
$\alpha < 0$	$0 \le \Lambda \le \frac{-1}{\alpha s}$	$0 \le \Lambda \le \frac{(-\nu)^{-\alpha} - 1}{\alpha s}$	$\Lambda = \frac{(-\nu)^{-\alpha} - 1}{\alpha s}$	$s > 0$	$\theta_e < \theta_d \le 2\theta_e$
$\alpha = 0$	$\Lambda \ge 0$	$0 \le \Lambda \le -\frac{\log[-\nu]}{s}$	$\Lambda = -\frac{\log[-\nu]}{s}$	$s > 0$	$\theta_e < \theta_d \le 2\theta_e$
$\alpha > 0$	$\Lambda \ge 0$	$0 \le \Lambda \le \frac{(-\nu)^{-\alpha} - 1}{\alpha s}$	$\Lambda = \frac{(-\nu)^{-\alpha} - 1}{\alpha s}$	$s > 0$	$\theta_e < \theta_d \le 2\theta_e$

Tab. 1: Constraints for G^2 continuous GLAC.

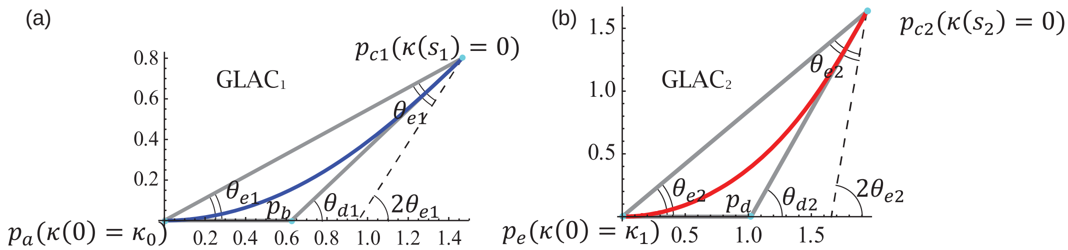

Fig. 1: Configuration for the placement of control points for G^2 data (S-shape): (a) GLAC$_1$& (b) GLAC$_2$.

The inputs are $\{P_0, P_4, \alpha, \theta_{e1}, \theta_{e2}, \kappa_0, \kappa_1\}$ and κ_0 & κ_1 is the curvature at the origin o f both curve segments.

Fig. 1 depicts the configuration for the placement of control points for GLAC$_1$& GLAC$_2$. Fig. 2 describes configuration of GLAC$_1$ & GLAC$_2$ forming S-shape with G^2 continuity. The point P_0 & P_4 is positioned at the origin (denoted as p_a & p_e). The point of the inflection P_2 which is the joint will be placed at θ_{e1} & θ_{e2} (denoted as p_{c1} & p_{c2}). The point P_1 & P_3will be found in the range of $\theta_{e1} < \theta_{d1} \leq 2\theta_{e1}$& $\theta_{e2} < \theta_{d2} \leq 2\theta_{e2}$ on the horizontal axis (denoted as p_b & p_d) to satisfy the condition $\| P_0P_1 \| \leq \| P_1P_2 \|$& $\| P_2P_3 \| \leq \| P_3P_4 \|$ using the interior point method. Note that point P_1 & P_3will have $\kappa(s) = 0$ (point of inflection) as the arc length falls on the upper bound of GLAC. By applying transformations, GLAC$_2$ will be placed such that $p_{c2} = p_{c1}$ & satisfies the tangent continuity. The S-shape is formed and the control points are denoted as $\{P_a, P_b, P_c, P_d, P_e\}$. Finally, the formed S-shape will be scaled and transformed back to original position such that $\{P_a, P_e\} = \{P_0, P_4\}$.

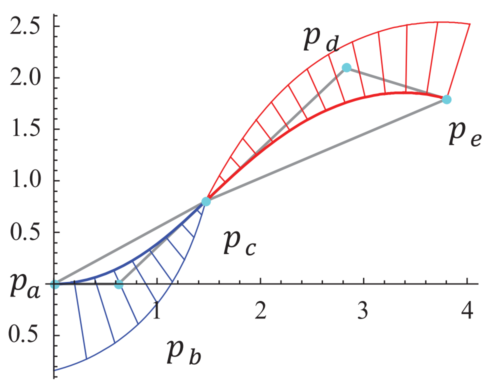

Fig. 2: Configuration for joining GLAC$_1$& GLAC$_2$ forming S-shape.

2.2.2. C-shape G^2 continuous GLAC

The joint P_4will have the same curvature (κ_1) and the inputs are $\{P_0, P_4, \alpha, \theta_{e1}, \theta_{e2}, \kappa_0, \kappa_1, \kappa_2\}$. Note that, curvature at the point P_0 & P_4 is κ_0 & κ_2. The condition $\kappa_0 > \kappa_1 > \kappa_2$ is important when selecting values for control the curvature as the curvature decreases as $s > 0$.

Fig. 3 illustrates the configuration for the placement of control points for GLAC$_1$ & GLAC$_2$. Fig. 4 describes the configuration of GLAC$_1$ & GLAC$_2$ forming C-shape with G^2 continuity. For GLAC$_1$ the point P_0 will be placed at the origin (denoted as p_a) meanwhile for GLAC$_2$, P_4 will be placed at θ_{e2} (denoted as p_e). For GLAC$_1$, point P_2 is positioned at θ_{e1} (denoted as p_{c1}) & at origin for GLAC$_2$ (denoted as p_{c2}). Points P_1 & P_3(denoted as p_b & p_d) will be searched in the range of $\theta_{e1} < \theta_{d1} \leq 2\theta_{e1}$ & $\theta_{e2} < \theta_{d2} \leq 2\theta_{e2}$ to satisfy the condition $\| P_0P_1 \| \leq \| P_1P_2 \|$& $\| P_2P_3 \| \leq \| P_3P_4 \|$ using the interior point method. By applying transformations, GLAC$_2$ will be placed such that $p_{c2} = p_{c1}$and satisfies the tangent continuity. Note that, the curvature at the point p_{c2} and $p_{c1}$are the same. At this stage, the C-shape is formed and the control points are denoted as $\{P_a, P_b, P_c, P_d, P_e\}$.

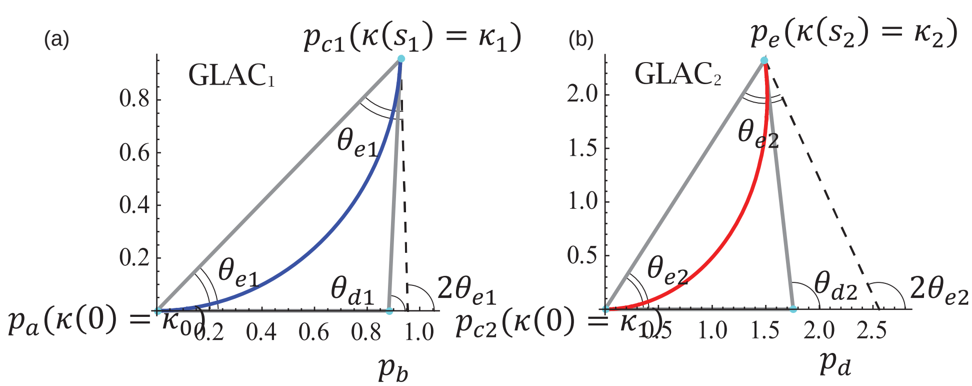

Fig. 3: Configuration for the placement of control points forG^2 data(C-shape): (a) GLAC$_1$& (b) GLAC$_2$.

Finally, the formed C-shape will be scaled and transformed back to original position such that $\{P_a, P_e\} = \{P_0, P_4\}$.

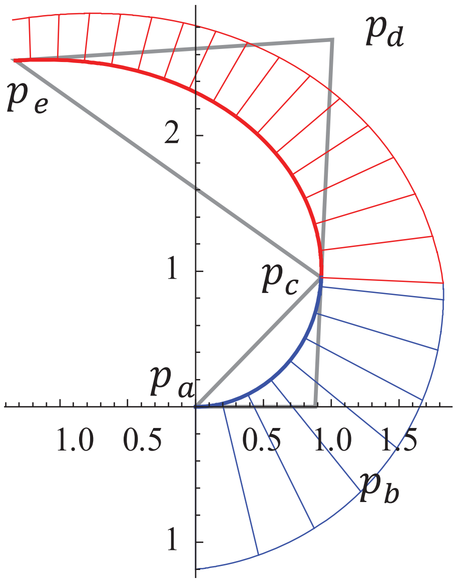

Fig. 4: Configuration for joining $GLAC_1$& $GLAC_2$ forming C-shape.

2.3. The Algorithm for G^2 Continuous GLAC

Algorithm 1 presents the path to generate desired S-shape & C-shape of G^2 continuous GLAC. Observe that in GLAC, we can obtain shapes with flexible curvature at origin by manipulating the shape variable ν. To control the curvature at the other end, we use interior point method by adding Eqn. (1) as one of its constraints.

2.3.1. Algorithm for G^2GLAC: S-shape & C-shape

Algorithm 1

REMARK: Endpoints $\{P_0(X_0, Y_0), P_4(X_4 Y_4)\}$ are given with a preferred $\alpha, \theta_{e1}, \theta_{e2}, \kappa_0$ & κ_1 (additionally κ_2 for C-shape). Let $P_0(X_0, Y_0)$ & $P_4(X_4 Y_4)$ be the start point of interactive $GLAC_1$ & $GLAC_2$ for S-shape. For C-shape, $P_0(X_0, Y_0)$ & $P_4(X_4 Y_4)$ will be the starting point & endpoint of interactive $GLAC_1$ & $GLAC_2$. $\{\theta_{d1}, \Lambda_1, s_1\}$ & $\{\theta_{d2}, \Lambda_2, s_2\}$ are searched through the primal dual interior point method correspond to θ_{e1} & θ_{e2} which gives p_{c1} & p_{c2} with $\kappa(s) = 0$ for S-shape. For C-shape, p_{c1} & p_e will be searched satisfying $\kappa(s) = \kappa_1$ & $\kappa(s) = \kappa_2$. S-shape & C-shape of GLAC segments is formed such that it satisfies the tangent & curvature continuity at the joint via transformation. The curve is then plotted corresponding to the original control points via transformation.

INPUT: $P_0, P_4, \alpha, \theta_{e1}, \theta_{e2}, \kappa_0, \kappa_1$ (additionally κ_2 for C-shape).

OUTPUT: AS shape or C-shape of GLAC segment.

BEGIN

- Step 1 Transform the input points such that:
  - i. P_0 is at (0,0)& set as $p_0(x_0, y_0)$,
  - ii. Place P_4 at first quadrant & set as $p_4(x_4, y_4)$.
- Step 2 Calculate: $\nu_1 \leftarrow (\kappa_0 - 1)$ & $\nu_2 \leftarrow (\kappa_1 - 1)$
- Step 3 Calculate total winding angle from origin: $\theta_{p_4} \leftarrow \cos^{-1}[\frac{x_4}{\sqrt{x_4^2+y_4^2}}]$.
- Step 4 Define: Eqn. (1), Eqn. (3), Eqn. (4) & Eqn. (5).
- Step 5 Set constraints such as: Range for $\{\theta_d, \Lambda, s\}$, $\theta_{GLAC}(s) = \theta_d$ & $\theta(s) = \theta_e$ (additionally $\kappa_{GLAC}(s) = \kappa_1$ & $\kappa_{GLAC}(s) = \kappa_2$ for C-shape) (Refer Tab.1).
- Step 6 Search $\{\theta_{d1}, \Lambda_1, s_1\}$ & $\{\theta_{d2}, \Lambda_2, s_2\}$ via primal dual interior point method.
- Step 7 Determine:
  - i. $\{p_a(x_a, y_a), p_{c1}(x_{c1}, y_{c1})\}$ & $\{p_e(x_e, y_e), p_{c2}(x_{c2}, y_{c2})\}$ (or $\{p_{c2}(x_{c2}, y_{c2}), p_e(x_e, y_e)\}$ for C- shape),
  - ii. $p_b(x_b, y_b)$ & $p_d(x_d, y_d)$ via tangent line.
- Step 8 Transform $GLAC_2$ segment such that $p_{c2} = p_{c1}$ & satisfies the tangent continuity with p_{c1}. Set the current points as $\{P_a, P_b, P_c, P_d, P_e\}$.
- Step 9 Calculate:
  - i. Total winding angle from origin: $\theta_{P_e} \leftarrow \cos^{-1}[\frac{X_e}{\sqrt{X_e^2+Y_e^2}}]$,
  - ii. Scaling factor: $r \leftarrow \frac{\sqrt{(x_0-x_4)^2+(y_0-y_4)}^2}{\sqrt{(x_a-x_e)^2+(y_a-y_e)^2}}$.
- Step 10 Scale to r & transform the whole curve by using θ_{p_4} & θ_{P_e} such that $P_e = p_4$.
- Step 11 Transform the whole curve back to original position.
- Step 12 **OUTPUT.**

END

3. NUMERICAL EXAMPLES

Fig. 5 & Fig. 6 shows various examples of S-shape & C-shape of G^2 continuous GLACs along with its control points. The inputs of the figures are shown in Tab. 2 & Tab. 3. To note, the constraints are easily satisfied during the parameter searching and the existence of solutions is proven to be far wider than the LA curves [7]. Although the final end curvatures change due to scaling, the G^2 continuity is achieved successfully.

Practitioners will be able to adapt GLAC easily for aesthetic design as it is feasible to achieve G^2 continuous GLAC. This aptitude helps practitioners to set estimated curvatures for the joining of two pieces of GLAC segment. Moreover, obtaining an inflection with GLAC is not difficult either. A condition for an inflection to occur in GLAC has been identified [6] and it helps to obtain S-shapes easily.

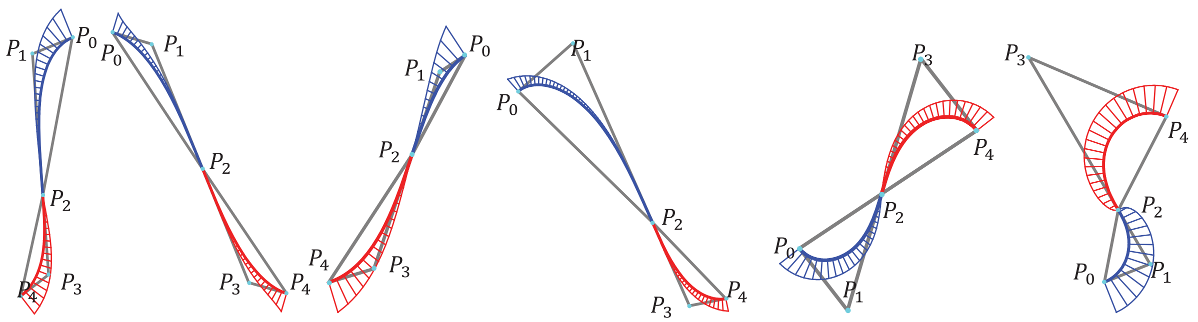

Fig. 5: S-shape of GLAC with G^2 continuity.

α	Input				Output		
	$\{P_0, P_4\}$	$\{\theta_{e1}, \theta_{e2}\}$	$\{\kappa_0, \kappa_1\}$	$\{\nu_1, \nu_2\}$	$\{\theta_{d1}, \theta_{d2}\}$	$\{\Lambda_1, \Lambda_2\}$	$\{s_1, s_2\}$
0	{(2,3),(1,−2)}	{1,0.7}	{0.98,0.72}	{−0.02,−0.28}	{1.25,0.98}	{0.71,0.36}	{5.46,3.46}
2	{(−1,2),(1,−1)}	{0.7,0.7}	{0.8,0.76}	{−0.2,−0.24}	{0.89,0.9}	{1.78,1.32}	{6.72,6.16}
0.5	{(2,4),(−1,−1)}	{0.5,0.7}	{0.84,0.76}	{−0.16,−0.24}	{0.66,0.95}	{1.07,0.54}	{2.78,3.82}
1	{(−2,4),(−1,3)}	{1.5,1}	{0.89,0.7}	{−0.11,−0.3}	{1.87,1.35}	{0.7,0.37}	{11.54,6.25}
−1	{(1,3),(4,5)}	{1.5,1.5}	{0.94,0.81}	{−0.06,−0.19}	{2.19,2.19}	{0.2,0.14}	{4.67,5.42}
−3	{(1,−3),(4,5)}	{1,1.5}	{0.95,0.85}	{−0.05,−0.15}	{1.72,2.5}	{0.13,0.07}	{2.46,4.16}

Tab. 2: Details of input & output for 5 (from left to right).

α	Input				Output		
	$\{P_0, P_4\}$	$\{\theta_{e1}, \theta_{e2}\}$	$\{\kappa_0, \kappa_1, \kappa_2\}$	$\{\nu_1, \nu_2\}$	$\{\theta_{d1}, \theta_{d2}\}$	$\{\Lambda_1, \Lambda_2\}$	$\{s_1, s_2\}$
0	{(−2,4),(1,3)}	{1,0.8}	{1.1,0.7,0.2}	{0.1,−0.3}	{1.85,1.33}	{0.24,0.21}	{2.09,3.15}
0.3	{(0,4),(−1,3)}	{1,0.8}	{1.2,0.9,0.3}	{0.2,−0.1}	{1.9,1.35}	{0.2,0.42}	{1.83,2.49}
1	{(0,4),(−1,3)}	{0.4,1}	{1.05,0.85,0.11}	{0.05,−0.15}	{0.77,1.5}	{0.3,0.61}	{0.81,4.65}
2	{(1,2),(−1,2)}	{1,0.5}	{1.15,0.75,0.2}	{0.15,−0.25}	{1.86,0.82}	{0.42,0.88}	{2.06,2.22}
−0.5	{(−1,−3),(1,2)}	{0.5,0.5}	{1.22,0.74,0.29}	{0.22,−0.26}	{0.92,0.86}	{0.58,0.29}	{0.95,1.72}
−3	{(3,3),(3,−2)}	{0.6,0.8}	{1.03,0.88,0.3}	{0.03,−0.12}	{1.16,1.4}	{0.1,0.14}	{1.21,2.11}

Tab. 3: Details of input & output for Fig. 6 (from left to right).

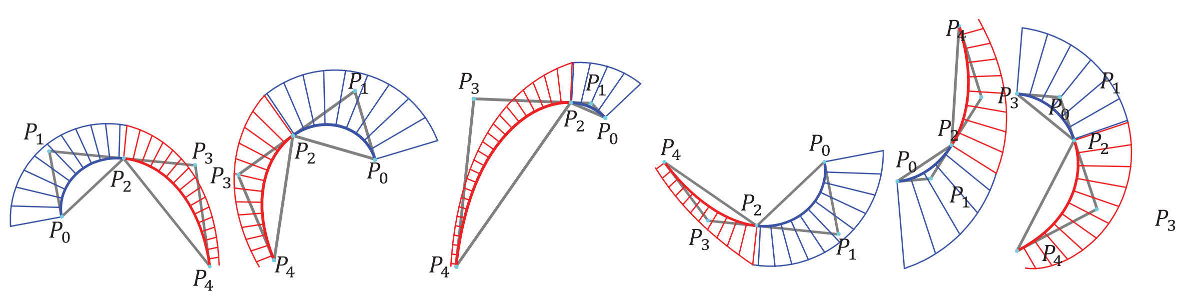

Fig. 6: C-shape of GLAC with G^2 continuity.

4. CONCLUSION AND FUTURE WORK

GLAC produces well-known aesthetic curves that can be utilized for aesthetic product design. GLAC has been proven far more capable of achieving the solutions compared to leading LA curves due to its flexibility [7]. LA curves have less degree of freedom as compared to GLAC & are troublesome when practitioners want to control the curvature. Hence, this paper proposes an algorithm to generate GLAC segment which satisfies G^2 Hermite data to a certain extend. The extra shape parameter that distinguishes GLAC from LA curves helps to estimate the end curvatures which enable practitioners to join curves with G^2 continuity regardless of the scaling effect of final end curvatures. One of our future works includes redefining the curvature function in such a way that curvature continuous Log Aesthetic Spline (LAS) can be created with minimal effort.

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