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Application of an Iterative Learning Control Algorithm to Volumetric Error Compensation for CNC Machines

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ABSTRACT

To promote higher-quality machining, this paper presents a software-based method for compensating for volumetric errors. The developed software compensation method modifies CNC part programs by applying previously obtained volumetric error tables to modify the axial motion commands. Furthermore, iterative learning control is employed to repetitively and precisely calculate these modified motion commands and further reduce any newly generated volumetric errors that deviate from the machining contours described by the CNC part programs. Several experiments and simulations were performed on a commercial CNC milling machine to validate the approach developed in this study. The results of the experiments indicate that volumetric errors are significantly reduced by applying the developed software compensation method. The rate of reduction was 77.99% for a tested circular contour and 87.59% for a tested spiral contour; therefore, the feasibility of the developed software compensation method was successfully validated for volumetric error compensation for CNC machines.

Keywords: iterative learning control, volumetric error, error compensation, CNC machines.

1. INTRODUCTION

There are many geometric factors that affect the machining accuracy of CNC machines, such as the straightness and flatness of the guideways, pitch error, and backlash in the motion axes. However, pitch error compensation for each axis, which is generally used in CNC machines, cannot provide any further improvement in the machining accuracy due to the existence of volumetric errors in the machine volume [4],[7],[18],[22,23]. For a machine tool with three orthogonal motion axes, movement along one motion axis would incur six errors, including a position error, vertical straightness error, horizontal straightness error, as well as pitch, yaw, and row errors. The three motion axes would incur three squareness errors if the cutting tool were to move synchronously on the XY, YZ, and XZ planes. Therefore, there are a total of twenty-one errors that constitute the volumetric error in the machine volume of a three-axis machine tool [3],[18]. Conventionally, two types of error are classified and considered in CNC machine applications, namely, linear displacement errors and volumetric errors. A linear displacement error is the positioning error in the axial direction; usually, this error is caused by pitch errors in the leadscrews or linear encoders. A volumetric error is a positioning error in a spatial direction but not necessarily in an axial direction; usually, this type of error is given by a vector and its components are a linear error, vertical straightness, and horizontal straightness [18,19]. Because the measured linear displacement error in the diagonal directions of the machine volume is sensitive to any error parallel and perpendicular to the linear motion axes, a reduction in the linear displacement error cannot completely eliminate the volumetric error. Some international and American standards, such as ISO230-6 and ASME B5.54, further define and describe a verifiable process for determining the volumetric error of CNC machines so that the manufacturers of these tools can follow these standards, evaluate the motion accuracy, and validate the precision of their machines [2,3]. Therefore, to ensure precise machining by CNC machines, both the measurement of, and compensation for, volumetric errors must be carefully considered.

In recent decades, with the development of laser measurement technologies, volumetric error measurement algorithms and devices have been proposed

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and implemented [1],[9,10],[15],[18,19],[20]; moreover, there are many advanced CNC machine controllers that feature functions for compensating for volumetric errors, such as the FANUC Series 15M/16M/18M/18i controllers and the SIEMENS SINUMERIK 840D/840Di/810D/FM-NC controllers. In this study, these compensations are classified as being either hardware compensation or software compensation. Here, "hardware compensation" denotes a compensation algorithm that is directly implemented in a CNC machine controller, allowing the controller to compensate for volumetric errors either in real time or online [4],[5],[8],[14–16]; "software compensation" denotes a compensation algorithm that modifies the motion commands of a CNC machine offline by referring to a previously measured distribution of volumetric errors [6],[9],[11],[13],[17],[20–22]. The hardware compensation approaches generally produce superior results due to their realtime or online processing capabilities; however, the hardware architecture adopted by the CNC machine controllers limits the performance of the applied hardware compensation approaches and therefore few advanced CNC machine controllers take this approach. Because software compensation compensates for volumetric errors offline by modifying the motion commands for a CNC machine, they generally cannot be applied to those CNC machines that must handle a heavy payload or to machining processes involving large cutting forces. Nevertheless, they can be applied to fine cutting and the finish machining of lightweight mechanical parts for which dimensional accuracy is of high importance. Moreover, these approaches are not constrained by the hardware architecture and thus can be directly applied to commercial CNC machine controllers. Machines with conventional CNC controllers that are widely utilized in the manufacture of mechanical parts usually do not support hardware compensation for volumetric errors; the machining quality of such machines is thus limited because pitch error compensation along each linear motion axis cannot suppress errors caused by volumetric factors. To apply volumetric error compensation to machines with conventional CNC controllers and improve their machining quality, a software compensation method was developed as part of this study to compensate for volumetric errors in the machining volume of CNC machine tools.

The developed software compensation method modifies CNC part programs based on the relationship between volumetric errors and motion along the axes of movement, and thus can be applied to conventional CNC machines. The design concept on which the software compensation method is based involves calculating the volumetric error caused by movement along the axes and applying compensation to the axial motion commands by applying the calculated volumetric error. The volumetric error of the CNC machine must be carefully measured in advance and an interpreter and interpolator are required to interpret the NC codes in the part programs and to perform fine interpolation for motion along the axes of movement. CNC machine simulation is employed in this study to generate precise motion commands to which compensation has been applied using the calculated volumetric error. The precisely compensated motion commands are used instead of the original ones to provide motion that is more precise. In this way, the original CNC part programs are modified. In contrast to existing approaches, not only can the developed software compensation method be applied directly to conventional CNC machines that do not support volumetric error compensation, but it also utilizes the concept of iterative learning control to more precisely calculate motion commands by referring to the obtained volumetric error. Iterative learning control (ILC) is a methodology that was originally used to address the problem of transient response in control systems that operate repetitively [12]. Because the ILC algorithm is computationally efficient and it is simple to implement, the ILC algorithm was selected and employed in this study to repetitively and precisely calculate motion commands. Because a modified motion command would induce new volumetric errors that cannot be neglected when a machine tool moves in response to that command, the volumetric compensation performance is limited, if no consideration is given to the newly generated volumetric errors that could accumulate when the machine tool moves sequentially according to the modified CNC part programs. Therefore, in this study, ILC is employed to repetitively and precisely calculate motion commands so as to significantly reduce the volumetric errors that would lead to deviations from the planned machining contours described by the CNC part program. The developed software compensation method was implemented with a LEADWELL MCV-OP CNC milling machine, and an OPTODYNE MCV-500 LDDM instrument was used to measure the volumetric error. To test the developed software compensation method, a circular contour on the X-Y plane and a spiral contour in the machine volume were divided into several parts to generate motion command points. The experimental results indicate that the developed software compensation method significantly reduces the volumetric errors during axial motion of the CNC machine, with a reduction of 77.99% for the circular contour and 87.59% for the spiral contour. Thus, the experimental results validate the developed software compensation method for volumetric error compensation of CNC machines and also demonstrate the feasibility of this method as a means of significantly improving the spatial motion accuracy of CNC machines.

This paper is organized as follows. Section 2 describes the software compensation method that compensates for volumetric errors by modifying part programs. Here, the volumetric errors are measured using a MCV-500 LDDM instrument developed by

OPTODYNE. Section 3 details the iterative learning control algorithm that was applied to the development of the software compensation method in this study to further improve the spatial motion accuracy of CNC machines. Section 4 presents the results of several experiments and simulations that validate the developed software compensation method. Section 5 concludes this paper.

2. VOLUMETRIC ERROR SOFTWARE COMPENSATION

In the application of software compensation, the definition of the coordinate system affects the computation of the compensation rules. Therefore, a part program coordinate system and machine coordinate system should be clearly defined in any software compensation. A part program coordinate system is defined in a working draft. Generally, operatives follow engineering graphics when defining a part program system and programming cutting commands. A machine coordinate system is the actual coordinate system that precisely expresses the motion of a CNC machine. The home position is the origin of the machine coordinate system. It is normal for different machine manufacturers to set the home position differently. However, any coordinate system with its origin at the home position can be defined as being a machine coordinate system. Generally, a part program coordinate does not necessarily consist of machine coordinates, except in the case of a homing operation. A homing operation causes the tip of a cutting tool to return to the home position and also resets all the coordinate systems. Because of the difference in the coordinate systems, the relationship between the reference position for error measurement and the start position of machining must be clearly defined in the software compensation. In the application of software compensation, the reference position and start position are both in machine coordinates to reflect the actual position in the machine volume. The reference position is usually that position which is referenced during error measurement. The start position is that position at which the machining process begins. Therefore, the machine coordinates of a part program are compounded from the start position and part program coordinates. In Fig. 1., the black triangle, black star, and black dot denote the reference position, start position, and home position, respectively. The light gray area is the machine volume. The white area is the measurement volume. The blue area corresponds to the workspace volume. The area outside the white area but inside the light gray area contains unpredictable errors. The home position, start position, and reference position can be any point within the machine volume. However, the home position and reference position are both fixed after measuring the volumetric errors.

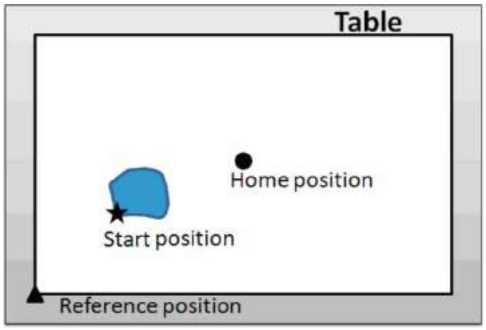


Fig. 1: Relationship between home position, start position, and reference position in a machine volume.

Moving a machine tool would incur volumetric errors and the results of the motion commands would not actually be the intended positions. As shown in Fig. 2., the motion command obtained from the NC code in an applied part program causes a CNC machine tool to move from position P to position T. Here, the vectors are represented using a machine coordinate system. $\overrightarrow{RP}$ and $\overrightarrow{RT}$ denote the vectors from position R to position P and from position R to position T, respectively. $\vec{R}$ denotes the reference position. $\vec{T}$ and $\vec{P}$ denote the vectors from the reference position to positions T and P, respectively. Therefore, the relationship between all the represented vectors is given by $\overrightarrow{PT}=\overrightarrow{RT}-\overrightarrow{RP}$, $\overrightarrow{RP}=\vec{P}-\vec{R}$, and $\overrightarrow{RT}=\vec{T}-\vec{R}$.

Then, the calculation of and compensation for volumetric errors are applied, using the following steps, when the machine tool moves from position P to position T:

(a) Compute the represented vectors $\overrightarrow{RP}$ and $\overrightarrow{RT}$ for given positions $\vec{R}$, $\vec{P}$, and $\vec{T}$, which can be determined from the NC code in the applied part program.
(b) Load the volumetric error table by referring to the positions of point P and point T, and the direction of motion $\overrightarrow{PT}$.
(c) Estimate the actual position of point E as represented with vector $\vec{E}$ by referring to the length of vector $\overrightarrow{PT}$ and the loaded volumetric error table; then, compute the represented vector $\overrightarrow{PE}$, which is the vector from point P to point E.

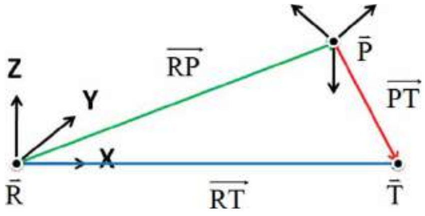


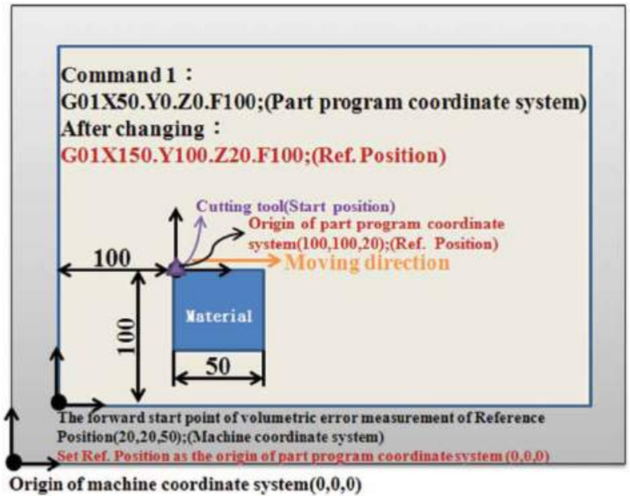
Fig. 2: Vector representations of machine tool movement from position P to position T.

(d) Compute the volumetric error vector $\hat{D}$ from $\hat{D} = \overrightarrow{PT} - \overrightarrow{PE}$.
(e) Modify the NC code in the applied part program, which describes the command for instigating motion from position P to position T, by referring to the modified motion command $(\overrightarrow{PT} + \hat{D})$.

In the software compensation algorithm, the forward and backward volumetric error tables must be loaded prior to computation. Normally, the motion commands are set in the part program coordinate system for the workpiece. To enable accurate computation using the software compensation method, the motion commands must set the origin of the part program coordinate system as a reference position. Fig. 3 shows an example of setting a coordinate system and modifying the NC code in the applied part program using the software compensation method. Square material, 50 mm long, is placed in the CNC machine and the origin of the part program coordinate system is set to the top-left of the material, as usual. The first cutting command starts a straight line cut from the origin of the part program coordinate system. The G-code for command 1 is G01 X50.Y0.Z0. F100. After changing the coordinate system with software compensation, command 1 would change to G01 X150. Y100. Z20. F100. However, even when using this software compensation method, new volumetric errors arise as a result of the execution of the compensated motion commands. Because the modified motion command can induce new volumetric errors when the machine tool moves according to the modified NC code, the volumetric error compensation performance is thus limited when using the software compensation method, as it does not consider these new volumetric errors. To further improve the volumetric accuracy, an iterative learning control (ILC) algorithm is employed to repetitively and precisely calculate motion commands and to determine the modified motion commands, which minimize the volumetric errors.

3. SOFTWARE COMPENSATION USING AN ITERATIVE LEARNING CONTROL ALGORITHM

Iterative learning control (ILC) [12] is a methodology that sets out to address the problem of transient response performance for systems that operate



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Command 1：
G01X50.Y0.Z0.F100;(Part program coordinate system)
After changing：
G01X150.Y100.Z20.F100;(Ref. Position)
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Fig. 3: Setting of coordinate system and modification of NC code.

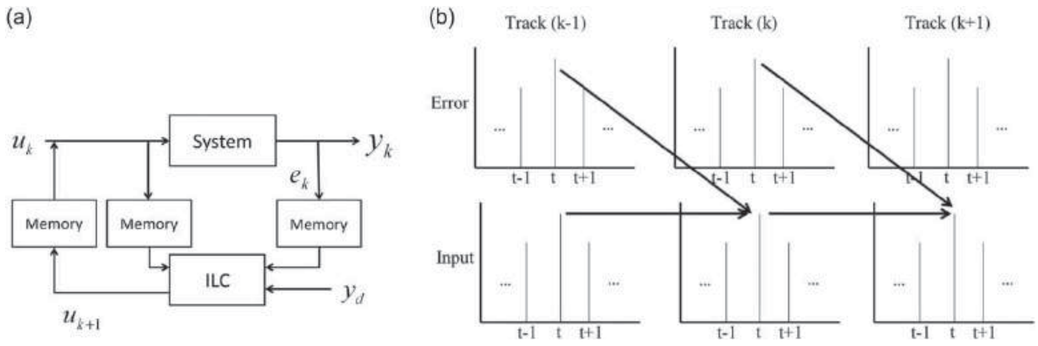

Fig. 4: Design and application of ILC algorithm: (a) Block diagram; (b) Compensation steps.

repetitively. The idea of ILC is to use error information from previous tasks to improve the performance of subsequent tasks by updating the control inputs from one trial to the next. ILC is usually applied to robot manipulators, machine tools, chemical batch processes, and so on. The development of an iterative learning control theorem has attracted the attention of researchers in the field of control. In the first instance, iterative learning control is easy to apply to repetitive tracking control and repetitive disturbance rejection [12]. It is an effective and practical solution to research into nonlinear systems. Because repetitive movement can be significantly improved by ILC, many applications rely on it to improve their motion. Unlike an online control architecture, ILC is an offline algorithm which uses data from the current process and records the error information into a memory buffer, which is then used as a correctional signal in subsequent iterations, thus reducing tracking errors as shown in Fig. 4(a)., below. The ILC algorithm has been subjected to several theoretical analyses to measure its stability, convergence, and robustness both in terms of time and frequency domains. The application of ILC to machine tools has been studied extensively.

In this study, the design concept of the ILC algorithm is employed to further reduce the volumetric errors that cause deviations from the planned machining contours described by the CNC part programs and to improve the volumetric accuracy despite time being limited. The ILC formula used for volumetric error compensation (Eqn. (3.1)) is shown below

$$u_{k+1}(t) = u_k(t) + \gamma \bullet e_k(t), \quad t = 1, \ldots, n \tag{3.1}$$

where $u_k(t)$ and $u_{k+1}(t)$ are the motion commands at the (k)th and (k + 1)th iteration tracks for the (t)th command point, respectively; $e_k(t)$ is the volumetric error at the (k)th iteration track for the (t)th command point; γ denotes the ILC gain that affects the rate of convergence during iterative learning processes, and n denotes the number of command points. Fig. 4(b) shows the compensation steps using ILC. Based on measured volumetric errors, the developed software compensation method first calculates the volumetric error $e_k(t)$ caused by the motion command $u_k(t)$ at the (k)th iteration track for the (t)th command point, and then generates the compensated motion command $u_{k+1}(t)$ by using the ILC formula given by Eqn. (3.1) for the (k + 1)th iteration track. All command points t, from 1 to n, are extracted from the NC codes in the part programs which form the desired machining contours. A CNC machine simulation program is further employed to calculate the entire set of motion commands $u_{k+1}(t)$. For each iteration period, the software compensation method calculates the compensation error, i.e. the difference between the compensated and non-compensated machine motions obtained using a CNC machine simulation program using compensated motion commands and original motion commands. The ILC formula is executed iteratively until the volumetric error of the compensated machine motion is minimized. Here, the formula for ILC inputs the motion command $u_k(t)$ and adds the volumetric error $e_k(t)$, multiplied by parameter γ, which has been tuned manually so that the ILC-based software compensation system can rapidly receive convergence motion commands without convergent oscillation after several iterations. The volumetric error obtained from all the motion commands is collected to form a volumetric error vector. The norm of the volumetric error vector $\|e\|$ is regarded as being the performance index for evaluating the learning performance of ILC.

4. RESULTS OF EXPERIMENTS

The developed software compensation method was implemented with a LEADWELL MCV-OP CNC milling machine and an ITRI-0M/0 T PC-based CNC controller. The travel distances were 405 mm for the X axis and 270 mm for the Y and Z axes. An OPTODYNE MCV-500 LDDM instrument was used to measure the volumetric errors. This instrument uses a sequential diagonal measurement method that differs from the traditional method in that each axis is moved separately and the

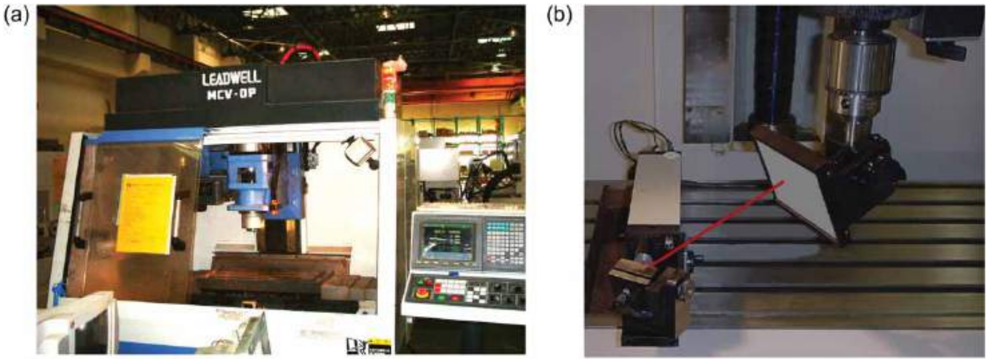

Fig. 5: Experimental setup: (a) LEADWELL MCV-OP CNC machine; (b) OPTODYNE MCV-500 LDDM instrument.

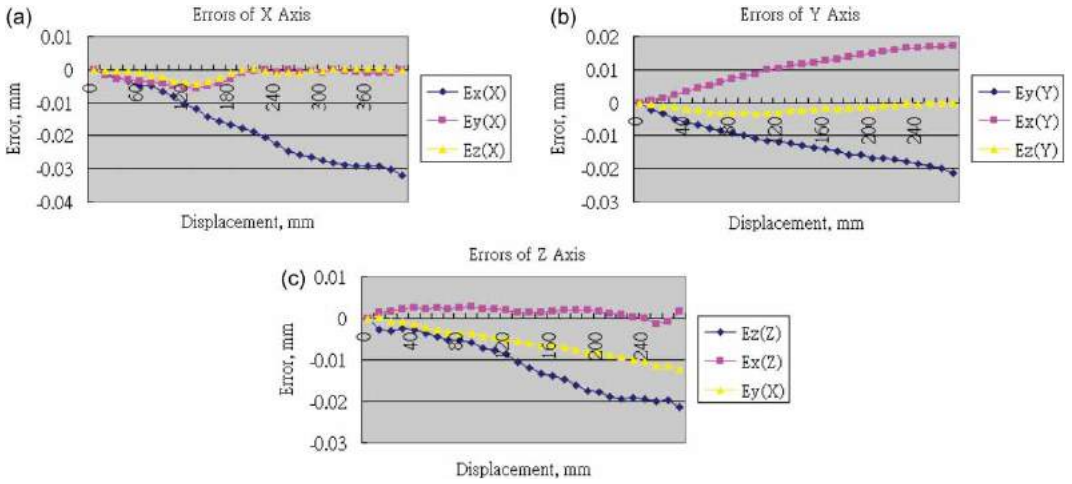

Fig. 6: Measured volumetric error along the (a) X-, (b) Y-, and (c) Z-axes.

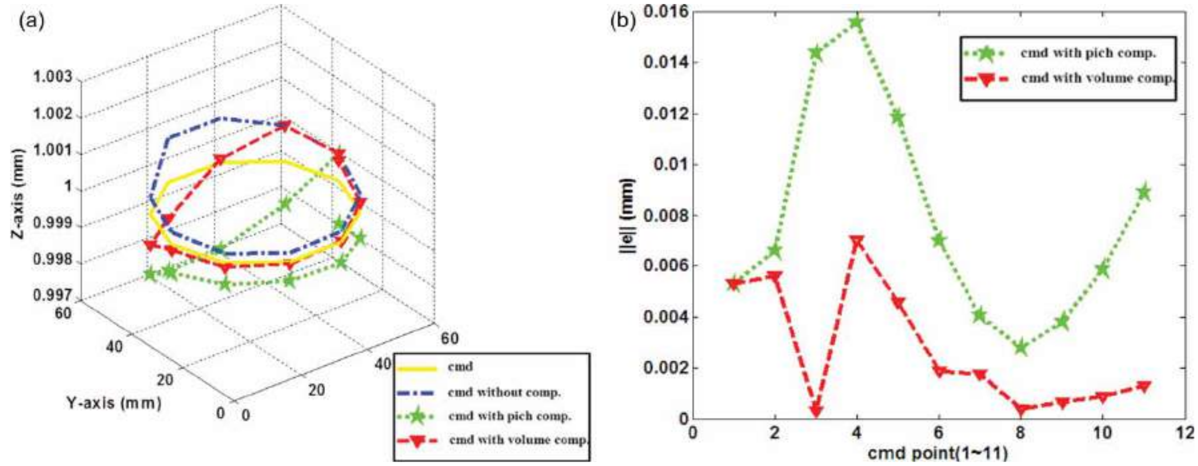

Fig. 7: Results of experiments using developed software compensation method with 10-part motion command points: (a) command contour (yellow); contour with pitch compensation (green); contour with volumetric compensation (red); (b) volumetric error.

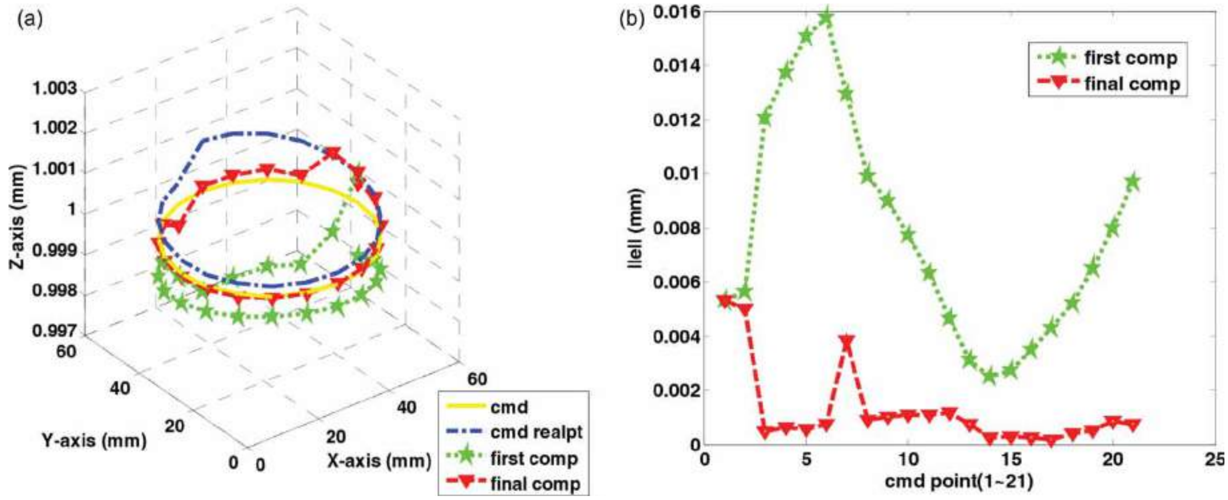

Fig. 8: Results of experiments using developed software compensation method with 20-part motion command points: (a) command contour (yellow); contour with pitch compensation (green); contour with volumetric compensation (red); (b) volumetric error.

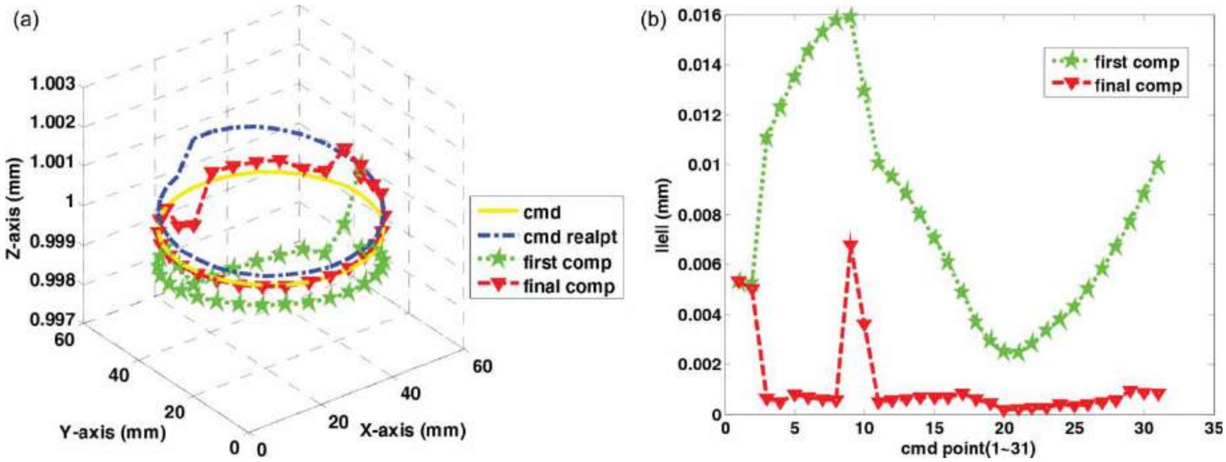

Fig. 9: Results of experiments using developed software compensation method with 30-part motion command points: (a) command contour (yellow); contour with pitch compensation (green); contour with volumetric compensation (red); (b) volumetric error.

positioning error is collected after each single movement of the X, Y, and then the Z axes [18,19]. For this reason, three times more data is collected, while the positioning error due to each single axis movement can be separated. Fig. 5 shows the experimental setup used in this study, and Fig. 6 shows the measured volumetric errors for the CNC machine.

To test the developed software compensation method, a circular contour with a diameter of 50 mm on the X-Y plane of the machine volume was divided into 10, 20, and 30 parts to generate motion command points; Fig. 7, Fig. 8, and Fig. 9 show the corresponding results using a CNC machine simulation program. When the circular contour was divided into 10 parts, the norm for the volumetric error was 8.858 μm, as determined from the motion results to which no compensation was applied, and 5.104 μm for the motion results to which only pitch error compensation is applied, as is usually the case with conventional CNC machine tools. However, the norm for the volumetric error is reduced to 3.571 μm by applying the proposed software compensation method with ILC; the rate of reduction reaches 59.69% and 30.04%, respectively, relative to previous approaches. A similar trend is observed regarding the reduction in the volumetric error that occurs for a circular contour divided into 20 and 30 parts. For the circular contour divided into 30 parts, the norm of the volumetric error is 8.965 μm for the motion results to which no compensation is applied, which is similar to the

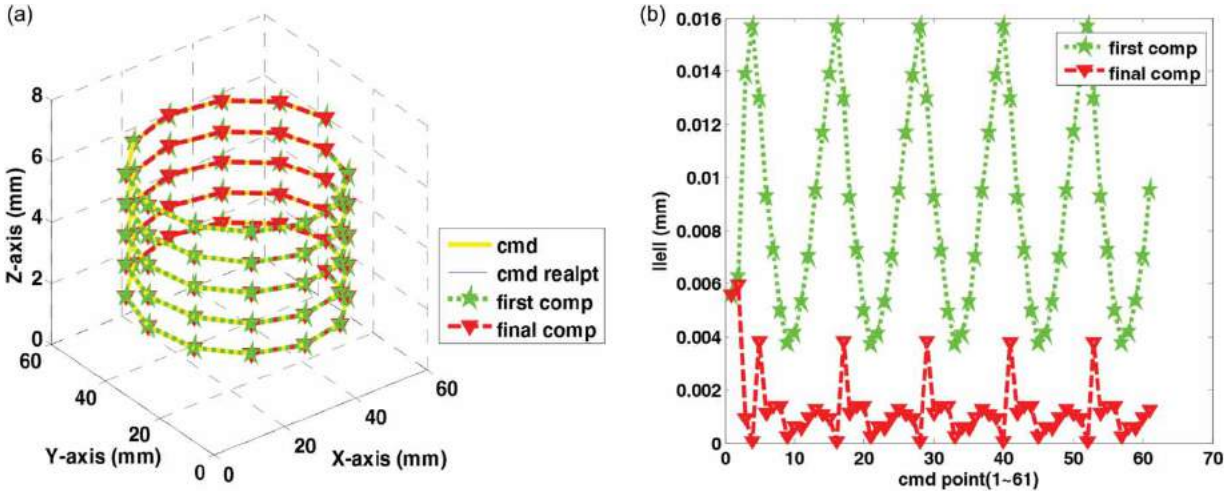

Fig. 10: Results of experiments using developed software compensation method for spiral contour with 60-part motion command points: (a) command contour (yellow); contour with pitch compensation (green); contour with volumetric compensation (red); (b) volumetric error.

Contour type	Circular contour		Spiral contour	
Number of divided parts	10	20	30	60
Norm of volumetric error $\|e\|$ (μm)				
Without compensation (VE)	8.858	8.777	8.965	8.890
Pitch error compensation (PE)	5.104	4.385	4.629	4.352
Proposed ILC compensation (ILC)	3.571	1.919	1.973	1.103
Rate of reduction (%)				
VE vs. ILC	59.69%	78.14%	77.99%	87.59%
PE vs. ILC	30.04%	56.24%	57.38%	74.66%

Tab. 1: Summary of experimental results.

value obtained for the circular contour divided into 10 parts; however, the norm of the volumetric error is significantly reduced to 1.973 μm as a result of applying the proposed approach. The rate of reduction reaches 77.99%, relative to the results obtained without using any compensation, and 57.38% relative to the results obtained using only pitch error compensation. Therefore, for the motion results to which no compensation is applied, the number of divisions does not affect the improvement of the volumetric accuracy; however, for the proposed software compensation method, the number of divisions significantly affects the rate of improvement. Thus, for a planned contour described by a CNC part program, it is recommended that the number of divisions be increased as far as possible to further improve the volumetric accuracy.

To test the application of the developed software compensation method to spatial contours, a spiral contour was divided into 60 parts to generate motion command points. Fig. 10. shows the results of experiments using a CNC machine simulation program. Obviously, the proposed software compensation algorithm significantly reduces the norm of the volumetric error. The rate of reduction reaches 87.59% relative to the results obtained when no compensation is applied, and 74.66% relative to the results obtained when only pitch error compensation is applied. Tab. 1. lists all the data values; thus, the results of experiments validate the developed software compensation method for volumetric error compensation for CNC machines and they also demonstrate the feasibility of this method for significantly improving the spatial motion accuracy of CNC machines.

5. CONCLUSIONS

The machining qualities of CNC machines are highly dependent on the volumetric accuracy of the machine volume. This study focused on the development of a software compensation method that modifies the NC codes in part programs by applying machine coordinate simulation and volumetric error computation, and by referring to previously measured volumetric errors; moreover, to precisely calculate the motion commands, the concept of iterative learning control is further applied to the developed software compensation method to significantly reduce the volumetric errors that are newly generated when the applied machine tool sequentially moves according to the modified motion commands. To test the developed software compensation method, several experiments and simulations were performed on a LEADWELL MCV-OP CNC milling machine, using an OPTODYNE MCV-500 LDDM instrument to measure the volumetric errors in the machine volume of the CNC machine. Here, the volumetric error obtained from all the axial motion commands is collected to form a volumetric error vector and the norm of the volumetric error vector is regarded as being the performance index for evaluating the execution performance of ILC. The experimental results indicate that the developed software compensation method can significantly reduce the volumetric errors incurred by axial motion of the CNC machine. For the tested circular contour, the volumetric error was reduced by 77.99% relative to the results of experiment without any compensation and by 57.38% relative to the results of experiment using pitch error compensation only. For the tested spiral contour, the volumetric error was reduced by 87.59% relative to the results of experiment without any compensation and by 74.66% relative to the results of experiment with pitch error compensation only. Thus, the results of experiments validate the feasibility of the developed software method for compensating for volumetric errors in CNC machines. Moreover, the developed software compensation method can be integrated into existing CAD/CAM systems to generate CNC part programs for high-precision machining.

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REFERENCES

[1] Aguado, S.; Samper, D.; Santolaria, J.; Aguilar, J. J.: Identification strategy of error parameter in volumetric error compensation of machine tool based on laser tracker measurements, International Journal of Machine Tools and Manufacture, 53(1), 2012, 160–169. http://dx.doi.org/10.1016/j.ijmachtools.2011.11.004

[2] Barakat, N. A.; Elbestawi, M. A.; Spence, A. D.: Kinematic and geometric error compensation of a coordinate measuring machine, International Journal of Machine Tools and Manufacture, 40(6), 2000, 833–850. http://dx.doi.org/10.1016/S0890-6955(99)00098-X

[3] Chapman, M. A. V.: Limitations of laser diagonal measurements, Precision Engineering, 27(4), 2003, 401–406. http://dx.doi.org/10.1016/S0141-6359(03)00041-2

[4] Chen, G.-S.; Mei, X.-S.; Li, H.-L.: Geometric error modeling and compensation for large-scale grinding machine tools with multi-axes, International Journal of Advanced Manufacturing Technology, 69(9-12), 2013, 2583–2592. http://dx.doi.org/10.1007/s00170-013-5203-7

[5] Chen, J.-S.; Yuan, J.-X.; Ni, J.; Wu, S.-M.: Real-time compensation of time-variant volumetric error on a machining center, Proceedings of Winter Annual Meeting of the American Society of Mechanical Engineers, 55, 1991, 241–253.

[6] Eskandari, S.; Arezoo, B.; Abdullah, A.: Positional, geometrical, and thermal errors compensation by tool path modification using three methods of regression, neural networks, and fuzzy logic, International Journal of Advanced Manufacturing Technology, 65(9-12), 2013, 1635–1649. http://dx.doi.org/10.1007/s00170-012-4285-y

[7] Fan, J.-W.; Guan, J.-L.; Wang, W.-C.; Luo, Q.; Zhang, X.-L.; Wang, L.-Y.: A universal modeling method for enhancement the volumetric accuracy of CNC machine tools, Journal of Materials Processing Technology, 129(1-3), 2002, 624–628. http://dx.doi.org/10.1016/S0924-0136(02)00669-6

[8] Fletcher, S.; Postlethwaite, S. R.; Ford, D. G.: Volumetric compensation through the machine controller, Proceedings of the Fifth International Conference on Laser Metrology, Machine Tool, CMM and Robot Performance, 2001, 321–330.

[9] Fujimori, T.; Taniguchi, K.; Ellis, C.; Aoyama, T.; Yamazaki, K.: A study on error compensation on high precision machine tool system using a 2D laser holographic scale system, Journal of Advanced Mechanical Design, Systems and Manufacturing, 6(6), 2012, 999–1014. http://dx.doi.org/10.1299/jamdsm.6.999

[10] Jung, J.-H.; Choi, J.-P.; Lee, S.-J.: Machining accuracy enhancement by compensating for volumetric errors of a machine tool and on-machine measurement, Journal of Materials Processing Technology, 174(1-3), 2006, 56–66. http://dx.doi.org/10.1016/j.jmatprotec.2004.12.014

[11] Khan, A. W.; Chen, W.: A methodology for systematic geometric error compensation in five-axis machine tools, International Journal of Advanced Manufacturing Technology, 53(5-8), 2011, 615–628. http://dx.doi.org/10.1007/s00170-010-2848-3

[12] Kim, D.-I.; Kim, S.: On iterative learning control algorithm for industrial robots and CNC machine tools, Proceedings of the 19th International Conference on Industrial Electronics, Control and Instrumentation, 1, 1993, 601–606.

[13] Kiridena, V. S. B.; Ferreira, P. M.: Computational approaches to compensating quasistatic errors of three-axis machining centers, International Journal of Machine Tools and Manufacture, 34(1), 1994, 127–145. http://dx.doi.org/10.1016/0890-6955(94)90044-2

[14] Liu, R.; Lv, M.; Zhang, G.: Modeling of volumetric error for CNC gear hobbing machine based on multibody system theory, Proceedings of the 3rd International Conference on Measuring Technology and Mechatronics Automation, 2, 2011, 623–626.

[15] Ni, J.; Wu, S.-M.: On-line measurement technique for machine volumetric error compensation, Journal of engineering for industry, 115(1), 1993, 85–92. http://dx.doi.org/10.1115/1.2901643

[16] Shen, H.; Fu, J.; He, Y.; Yao, X.: On-line asynchronous compensation methods for static/quasi-static error implemented on CNC machine tools, International Journal of Machine Tools and Manufacture, 60, 2012, 14–26. http://dx.doi.org/10.1016/j.ijmachtools.2012.04.003

[17] Vahebi Nojedeh, M.; Habibi, M.; Arezoo, B.: Tool path accuracy enhancement through geometrical error compensation, International Journal of Machine Tools and Manufacture, 51(6), 2011, 439–449. http://dx.doi.org/10.1016/j.ijmachtools.2011.02.005

[18] Wang, C.: Laser vector measurement technique for the determination and compensation of volumetric positioning errors. Part I: Basic theory, Review of Scientific Instruments, 71(10), 2000, 3933–3937. http://dx.doi.org/10.1063/1.1290504

[19] Wang, C.: Current issues on 3D volumetric positioning accuracy: Measurement, compensation, and definition, Proceedings of the Seventh International Symposium on Instrumentation

and Control Technology: Measurement Theory and Systems and Aeronautical Equipment, 2008, 7128.

[20] Wang, J.; Guo, J.: Research on volumetric error compensation for NC machine tool based on laser tracker measurement, Science China Technological Sciences, 55(11), 2012, 3000–3009. http://dx.doi.org/10.1007/s11431-012-4959-6

[21] Wang, J.; Guo, J.; Deng, Y.; Li, H.: Method of volumetric error compensation for 3-axis NC machine tool, Advanced Materials Research, 472–475, 2012, 2371–2376. http://dx.doi.org/10.4028/www.scientific.net/AMR.472-475.2371

[22] Wang, K.; Sheng, X.; Kang, R.: Volumetric error modelling, measurement, and compensation for an integrated measurement-processing machine tool, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 224(11), 2010, 2477–2486. http://dx.doi.org/10.1243/09544062JMES2200

[23] Zhu, W.; Wang, Z.; Yamazaki, K.: Machine tool component error extraction and error compensation by incorporating statistical analysis, International Journal of Machine Tools and Manufacture, 50(9), 2010, 798–806. http://dx.doi.org/10.1016/j.ijmachtools.2010.05.004