Tolerance Mathematical Modeling and Analysis Method Based On Control Points of Geometric Elements

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ABSTRACT

Tolerance is one of the important factors that cannot be ignored in modern manufacturing process, it not only affects the machining accuracy and production quality of parts, but also has a vital significance in process route, testing, manufacturing costs, assembly of final products, etc. In this paper, the tolerance mathematical modeling and analysis method based on control points of geometric elements are proposed, in which the geometric tolerance is indicated according to the position variation of control points. The first step is to define the inherent direction of geometric elements. The inherent direction of point is the same with the one of the benchmark points, and the inherent direction of linear is the linear itself while the inherent direction of plane is the one of the normal directions. The second step is to establish the tolerances coordinate system according to the inherent direction of geometric elements, then defining and classifying the degrees of freedom of geometric elements, taking the direction of DOF as the change direction of control points, using the position parameter domain of geometrical elements as the tolerance zone. The third step is to simulate the dimensional tolerance, position tolerance and shape tolerance according to the variation of geometric elements parameters. Finally this method is applied in actual tolerance analysis of parts, and the calculation results show that the mathematical modeling and analysis methods are in accordance with all relevant tolerance principles and regulations, which have a vital significance in the studying of whole tolerance field.

Keywords: Tolerance analysis, mathematical modeling, control points of geometrical elements.

1. INTRODUCTION

For mechanical designing, manufacturing and assembling, tolerance analysis is to determine the variation of geometric parameters parts within prescribed scope for the goal of compatibility, coordination and generality[1-3]. Scholars at home and abroad have acquired great achievements in tolerance mathematical modeling and representation, tolerance analysis and synthesis, computer-aided tolerance design, concurrent design, function tolerance research, etc [4,5,7].

In terms of tolerance modeling and analysis, the tolerance mathematical modeling on plane of polyhedron has been conducted by Roy, which based on the analysis of the factors DOF, including the dimensional tolerance, shape tolerance, orientation tolerance and location tolerance [20]; Wang established a DOF analysis modeling based on tolerance domain-deviation space mapping and studied the tolerance by using this model in comprehensive research and analysis for geometric features [23]; Chiabert put forward CAT modeling based on the theory of the new generation Geometrical Product Specification and verification, and successfully applied the position tolerance analysis and processing in CATIA software [6].

Wu developed a tolerance mathematical modeling based on control points of geometric elements variation, indicating size and geometric tolerance with the position variation of control points of geometric elements [1]. Bourdet introduced small displacement vector of the cluster (SDT) in the field of tolerance [3]. SDT is the vector composed of six component motion showing tiny displacement of rigid body, which is used to describe the shape, location, direction and size deviation of geometric elements. Davidson put forward a tolerance graph model compatible with ASME standards. T_Map is a hypothetical point set space with shape of convex polyhedron, its shape and size reflect types and possible changes of target object, so that there exists one-to-one correspondence between the Tolerance changes of target and various points in the T_Map [8].

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However, the tolerance mathematical modeling and analysis method described above somewhat have disadvantages. For instance, the traditional method of dimension chain and tolerance zone graphic method only can calculate the extreme value of tolerance in one direction, while other directions of tolerance cannot be analyzed[9–13,15]. The variation surface modeling based on geometric operation is complicated, and the kinematics modeling has some problems, such as complex manual modeling method, inconsistence with the tolerance standard, etc[14,16,17]. The accuracy of results depends on the modeling and its process is unable to automate. The parameter constraint cannot include all types of tolerance based on this analysis method. Although the mathematical definition of ASME can solve the uncertainty of traditional tolerance definition, it still cannot be directly applied to the computer[18,19,21,22]. The SDT parameter cannot reflect the interaction between tolerances, and there is no corresponding relationship between SDT parameters and tolerance. The T-map modeling process is complex and has difficulty in the visualization and practical application. The above various problems of mathematical model have restricted the integration research and development of tolerance[23–26].

To solve above problems, this paper puts forward the tolerance mathematical modeling and analysis method based on control points of geometric elements. It can analyze various corresponding tolerance of geometric elements, which be put in combination with entity CAD modeling easily. It not only can indicate the tolerance which is convenient for analyzing and conversion, but also be applied to the computer aided tolerance designing. It will have extensive application in the field of tolerance analysis.

2. THE MATHEMATICAL MODEL OF CONTROL POINTS OF GEOMETRIC ELEMENTS

2.1. The Degrees Of Freedom of Control Points of Geometric Elements

Traditional rigid body elements have six degrees of freedom, namely translational DOF along the X, Y, Z axis and rotation DOF along the X, Y, Z axis, but the geometric elements are different from the rigid body element. The number of DOF is less than six because of the uncertainty of geometric elements tolerance.

In the tolerance analyzing, the new DOF needs to be established to build the relationship between constraint and benchmark so that it can distinguish the sense difference of DOFs. Firstly the inherent direction of geometric elements should be defined. The inherent direction of point is the direction of benchmark points, the inherent direction of linear is the linear itself, and the inherent direction of plane is the one of normal directions. The DOFs of control points of geometric elements are shown as Fig. 1(a-c):

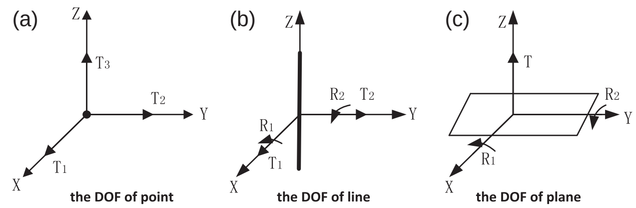

Fig. 1: The DOFs of control points of geometric elements.

Among the DOFs of control points of geometric elements shown in Fig. 1, the factors include three translational DOFs T_1, T_2 and T_3. The linear elements include two translational DOFs T_1, T_2 and two rotational DOFs R_1, R_2. The planes elements include the translational DOF T along the normal and two rotational DOFs R_1, R_2.

2.2. Tolerance Coordinate System for Geometric Elements

Before establishing the tolerance mathematical modeling, the coordinate system based on the DOF is set up to represent the position and the change direction, which is consistent with relative ideal location. The origin of tolerance coordinate system is in coincidence with the center of geometric elements which based on nominal position. It means that the origins of coordinates geometry elements of point, linear and plane are the point itself, the halfway point of the straight linear and the center of the plane bounding box. Moreover, the X axis direction of tolerance coordinate system is the direction that benchmark constraints goal points to the degrees of freedom, the Z axis is the geometric elements inherent direction.

The directions of tolerance coordinate axis are the inherent direction of geometric elements. The benchmark constraints goal points which can be used to fit the changing direction and range of geometric elements. The change direction is axis direction and the change range is the tolerance value in the tolerance coordinate system.

But there exists a special situation when analyzing the shape tolerance. It is not necessary build the tolerance coordinate because the shape tolerance controls the actual changes of geometric surface relative to the fitting elements. For example, the changes of straight linear extraction only exists in the vertical plane. Therefore, it does not need to establish the coordinate system when expressing the shape error, which may be determined by the types of tolerance and fitting elements.

The geometric elements of control points can be divided into bearing and shape points, among them the azimuth control points resolve the position of fitting elements and the shape control points controls the extraction of object elements. The azimuth control point is the point itself, the azimuth control points of linear elements are the two endpoints on both ends of the straight linear and the azimuth control of plane are three random points in this plane. The change direction of azimuth control point is the translational DOF direction of geometric elements, but the coordinates of azimuth control point can only change in this change direction, the coordinate values outside change direction remain the same.

The change direction of azimuth control point is the direction of translational DOF. The change direction of azimuth control point for linear is the translational DOF direction and vertical translational DOF direction. The change direction of azimuth control point for plane is the normal translational DOF direction. According to the variation of geometrical element DOF , all kinds of tolerance can be showed. The azimuth control point of change Geometric elements must be consistent with the meaning of tolerance, the biggest changes which exist in the quantity control points is the tolerance value. The azimuth control point of change Geometric elements must be consistent with the meaning of tolerance, the biggest changes which exist in the quantity control points is the tolerance value.

The space shape of geometric elements can be summarized as follows:

(1) The space shape of point variation in the rectangular coordinate system, cylindrical coordinate system and spherical coordinate system are cube, cylinder and sphere respectively;
(2) The space shape of linear variation may be cube and cylinder;
(3) The space shape of plane variation is flat cube.

The relative position between the azimuth control points can also present position variation of geometric elements. The surrounding area divided by the maximum relative position is the tolerance zone. The shape points are composed of geometric elements points, in which the shape control point is the point itself, and the shape control points of linear and plane are located in the geometric elements and boundary. The direction of shape control point variation are specific direction or normal direction.

The variation space of shape geometry elements control points constitutes the tolerance domain, such as be in accordance with the requirements of tolerance. The shape of linear control point variation direction has orthogonal direction and circumferential direction, and the variation space of shape control points are cube and cylinder respectively. The variation direction of shape control points for plane elements is along with the normal surface, so the variation space of plane is flat cube. The shape control points changes in the location is relative to the ideal geometry location, so the shape tolerance field must depend on the ideal location of geometric elements.

The relationship between the absolute range and relative changes can represent the relationship between the position and direction tolerance, so there is a corresponding relationship between the interaction of different tolerance types and variation in quantity, such as the absolute changes and relative changes of two straight linear azimuth control points along a coordinate direction can represent location tolerance and direction tolerance.

3. TOLERANCE MATHEMATICAL MODELING BASED ON CONTROL POINTS OF GEOMETRIC ELEMENTS

The geometric elements have a variety of tolerance types and its position variation can be marked in the rectangular tolerance coordinate system, shown in Fig. 2.

3.1. Tolerance Mathematical Modeling Of Point Elements

According to the error definition, the only tolerance type of point elements is position error. When the point changes from P_0 to P_i , the tolerance can be expressed as:

$$L_{SL} \le \sigma = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \le L_{SU} \tag{1}$$

The L_{SU} and L_{SL} are upper and lower deviation respectively. The constraint conditions are :

$$\begin{cases} L_{SLX} \le x_i - x_0 \le L_{SUX} \\ L_{SLY} \le y_i - y_0 \le L_{SUY} \\ L_{SLZ} \le z_i - z_0 \le L_{SUZ} \\ max(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2 \end{cases} \tag{2}$$

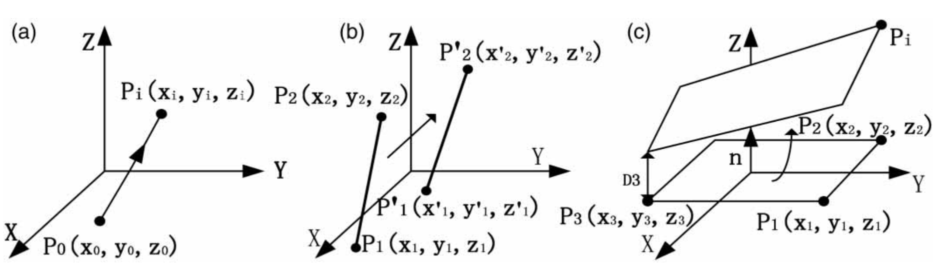

Fig. 2: The position variation of control points in rectangular coordinate system parameters.

The tolerance domain of point has a variety of forms such as straight line, round, rectangular and cylindrical. In the right angle tolerance coordinate system, three parameters of control points(X, Y, Z) can be used to simulate business field, which correspond to length, width and height of cube shape tolerance domain which is shown in Fig. 3 (a). When the tolerance domain of point is spherical, similarly can also be said with the three parameters (Δx, Δy, Δz), as shown in Fig. 3 (b): θ stands for the point position variation on the plane XOY, α is the position change angle.

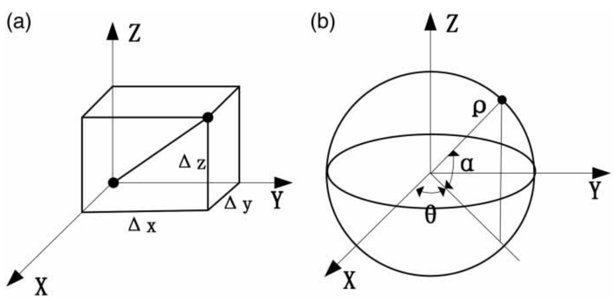

Fig. 3: The position variation parameters of control points in the tolerance coordinate system.

When the control points vary in the spherical coordinate system, the location parameters can be said:

$$\begin{cases} \Delta x = \rho \cos\alpha \cos\theta \\ \Delta y = \rho \cos\alpha \sin\theta \\ \Delta z = \rho \sin\alpha \end{cases} \tag{3}$$

When the control points vary in the cylindrical coordinate system and the cylindrical radius is R, the position parameters can be showed:

$$\begin{cases} \Delta x^2 + \Delta y^2 \leq R^2 \\ L_{SLZ} \leq \Delta z \leq L_{SUZ} \end{cases} \tag{4}$$

3.2. Tolerance Mathematical Modeling of Linear Elements

In the right angle tolerance coordinate system, the linear elements can use two endpoints P_1P_2 as the tolerance control points. The range of position parameter is tolerance value and the dimensional value is D, the position parameters can be showed:

$$D_{SL} \leq \sigma = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} \leq D_{SU} \tag{5}$$

Where:

$$\begin{cases} \Delta x = \dfrac{(x_1' + x_2') - (x_1 + x_2)}{2} \\ \Delta y = \dfrac{(y_1' + y_2') - (y_1 + y_2)}{2} \\ \Delta z = \dfrac{(z_1' + z_2') - (z_1 + z_2)}{2} \end{cases} \tag{6}$$

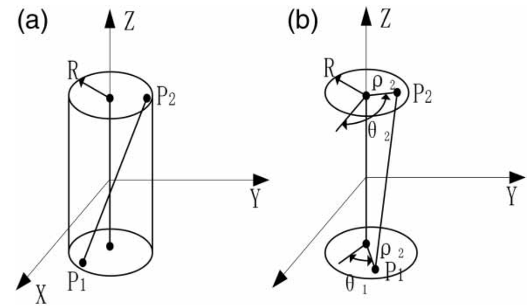

Fig. 4: The linear location variation in tolerance position system.

When the tolerance field is cylinder and the tolerance for domain radius is R, as shown in Fig. 4 (a) :

The position tolerance can be expressed as follows:

$$R_{SL}^2 \leq \sigma = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \leq R_{SU}^2 \tag{7}$$

The direction error expressed by vector angle as follows:

$$\theta_{SL} \leq \alpha = \arccos \frac{|aa' + bb' + cc'|}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}} \leq \theta_{SU} \tag{8}$$

Where the vector S (a, b, c) and S '(a, b, c) are the direction vectors before and after the line change respectively, the θ is the tolerance value.

When the line varies in cylindrical coordinate system, as shown in Fig. 4 (b), the azimuth control points coordinates ρ_1 and ρ_2 vary in the range from 0 to R, and θ_1 and θ_2 vary in the range from0 to π, the tolerance field can be expressed as:

$$\begin{cases} (\rho_1 \cos\theta_1 - \rho_2 \cos\theta_2)^2 \leq \Delta x^2 \\ (\rho_1 \sin\theta_1 - \rho_2 \sin\theta_2)^2 \leq \Delta y^2 \end{cases} \tag{9}$$

3.3. Tolerance Mathematical Modeling of Plane Elements

Since the plane elements has three DOFs, the control parameters of plane elements are three control points which exist in the nominal plane coordinate. It must fit the plane tolerance domain according to the variation scope of control parameters, setting the primitive equation plane S as:

$$Ax + By + Cz + D = 0 \tag{10}$$

Where n (A, B, C) is the plane normal vector, the distance L from the variation plane to the original one can be represented as:

$$L_{min} \le \sigma = \frac{|Ax_i + By_i + Cz_i|}{\sqrt{A^2 + B^2 + C^2}} \le L_{max}(i = 1, 2, 3) \quad (11)$$

When σ gets the max value, the plane S' including P_i (X_i, Y_i, Z_i) parallels plane S and the equation is:

$$Ax + By + Cz - (Ax_i + By_i + Cz_i) = 0 \quad (12)$$

The area between the two parallel surfaces is the plane tolerance zone. The change parameters of control points can be got by finite element analysis and the size tolerance value can be simulated in the business field.

4. CASE ANALYSIS

A simplified diagram for flexible support is shown in Fig. 5 and the tailstock bottom is constraint benchmark for support hole. The normal direction in the bottom is constraint for DOF direction of stent hole axis , namely the X axis direction in the tolerance coordinate system for the centerline of the bracket hole, and the center linear of the top pinhole is Z axis direction while the origin of coordinate system is the middle of the axis.

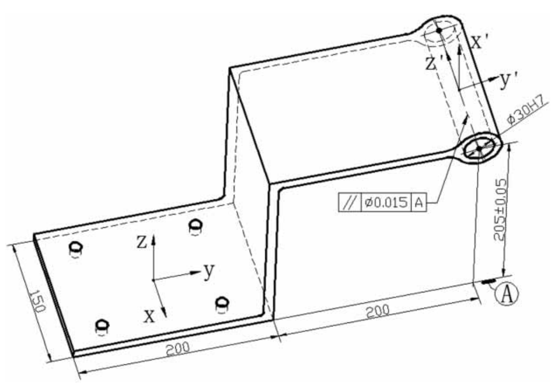

Fig. 5: Flexible stent simplified diagram.

According to the tolerance Settings, the variation parameter for two top axial endpoints of the support have size tolerance and the tolerance value is 0.015 mm. The axial endpoint must be within the parallelism tolerance zone, at the same time the position constraint under the axis on both ends of the transverse size is 0.05 mm.

According to the tolerance mathematical modeling, the control points can be expressed as P_1 (x_1', y_1', z_1') and P_2 (x_2', y_2', z_2'). The radius within the axis changes is 30 mm, in which the base hole level is 7 precision system, the upper deviation is 0.021 mm, the lower deviation is 0. Then the mathematical modeling can be represented as:

$$\sigma = \sqrt{x_i'^2 + y_i'^2} \le 30.021 \quad (13)$$

The lower end is the benchmark for the axis and the tolerance size is 0.05 mm. The equation for lower end plane is:

$$10x' + 2050 = 0 \quad (14)$$

The distance L from One of the two endpoints in the axis to the ground can be represented as:

$$204.95 \le \frac{|10x_i' + 2050|}{\sqrt{10^2}} \le 205.05 \quad (15)$$

The parallel degree controlled by the variation parameters for two top axial endpoints is 0.015 mm. The radius within the control points included in the axis shape is 0.015 mm. According to this kind of regulation, the mathematical modeling can be represented as:

$$\sigma = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2} \le 0.015 \quad (16)$$

The deformation deviation of flexible support can be stimulated by using the computer finite element, as shown in Fig. 6:

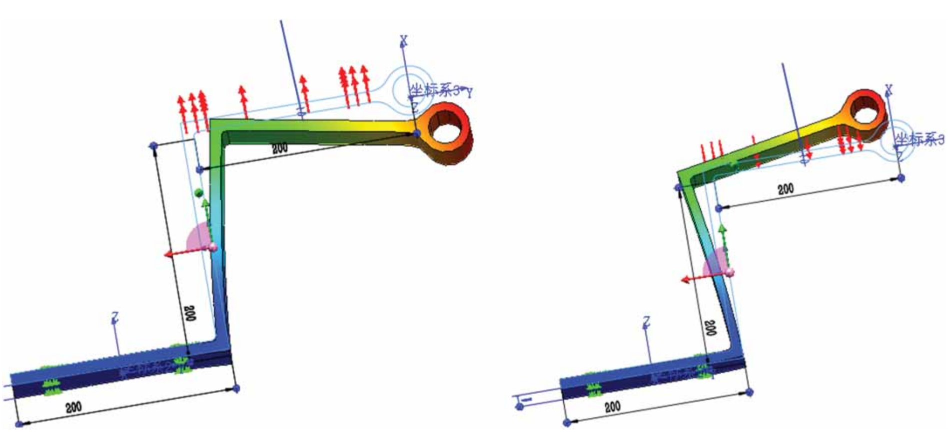

Fig. 6: The top of the flexible support drawing tolerance changes.

5. THE EXTENDED APPLICATION OF CONTROL POINTS OF GEOMETRIC ELEMENTS IN PSO ALGORITHM

5.1. The Definition and Origin of PSO

Particle swarm optimization algorithm (PSO) is a kind of evolutionary computation technology, and its solution (called particles) is related to the speed. Similar to the genetic algorithm, PSO is a optimization tool based on the iteration. It has a advantage of easy implementation and less parameters needed to be adjusted compared with genetic algorithm.

The PSO algorithm has been widely used in function optimization, neural network training, fuzzy system control, and other applications of genetic algorithm. First of all, a group of random particles (random solutions) are initialized in PSO to find the optimal solution through iteration. In each iteration, the particles update themselves by tracking two extreme value. The first optimal solution is found by the particles themselves , known as the individual extremum, the others are found by the entire population called the global extremum.

5.2. The Application of Algorithm in the Mathematical Modeling

According to the the speed and position of particles interaction, The PSO can be used to analyze the geometric tolerance. First of all, the mathematical modeling under the condition of minimum zone should be established, then the objective function is given to find the optimal solution.

The application of control points of geometric elements in tolerance mathematic modeling has been described before, and it can also be extended into PSO algorithm to analyze the tolerance. For example, the discrete measured points elements are be used to calculate the shape error.

According to the minimum zone method, assessing shape error is a optimization problem in essence based on the PSO algorithm. The definition of shape error can be described in the extremum method:

$$\begin{gathered}\min[\max F(U,V)-\min F(U,V)]\\ \min F(U,V)\\ \max F(U,V)\end{gathered} \tag{17}$$

Where F (U, V) is the objective function of shape error calculation and U is shape function of ideal geometry elements, X is the actual position function of geometric elements as follows:

$$V=[f(x),f(y),f(z)] \tag{18}$$

The shape error is the change of single actual measured elements relative to the ideal ones. According to the definition for form error, its evaluation can only be carried out under the minimum condition. The tolerance zone shape can be judged according to the specific design requirements, calculating the tolerance objective function expression of each project.

Describing the minimum tolerance zone according to control points of geometrical elements. Its shape is the same as the ideal elements to determine the minimum position and direction. The tolerance area corresponding to the size of the minimum area is the shape error.

5.3. The Evaluation of Flatness Error

According to the mathematical definition of the tolerance, the flatness error refers to the distance between two parallel planes for all points of measured contour as shown in Fig. 7.

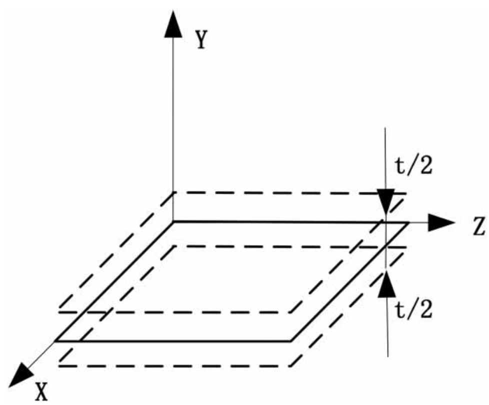

Fig. 7: The evaluation of flatness error.

Analyzing the Flatness tolerance according to the minimum zone method, the key is to find two parallel planes including the key point with the minimum distance. It also can be transformed into a plane, then measure the distance from each key point P_i (X_i, Y_i, Z_i) (i = 1, 2, ... , n) on the measured contour to the plane. If the difference between the maximum and minimum values for all distances is the minimum value, the flatness tolerance value is the distance difference, setting the plane equation as:

$$z=ax+by+c \tag{19}$$

The distance from the point P_i (X_i, Y_i, Z_i) to the plane is:

$$d_i=\frac{|ax_i+by_i+c-z_i|}{\sqrt{a^2+b^2+1}} \tag{20}$$

The objective function of flatness error is:

$$F(a,b,c)=\min[\max(d_i)-\min(d_i)] \tag{21}$$

5.4. The Evaluation of Straightness Error

5.4.1. The Evaluation of Plane Straightness Error

According to the mathematical definition of tolerance, the plane straightness error is the minimum distance between two parallel straight lines including all the points, as shown in Fig. 8.

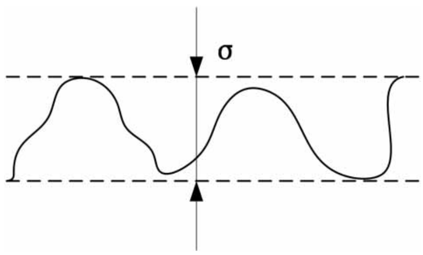

Fig. 8: The evaluation of plane straightness error.

Analyzing the plane straightness error according to the minimum zone method, the key is how to find the two parallel lines including measured lines with the minimum distance. Likewise, it can also be converted into a straight line, gaining the minimum difference between the minimum and maximum distance from each measurement point to the straight line. The distance difference value is the plane straightness error. The equation of line is:

$$\mathrm{y} = ax + b \tag{22}$$

The distance from the key point $\mathrm{P_i}$ ($\mathrm{X_i}$, $\mathrm{Y_i}$) (i = 1, 2, ..., n) to the line is:

$$\mathrm{d_i} = \frac{|y_i - ax_i - b|}{\sqrt{a^2 + 1}} \tag{23}$$

The objective function of plane straightness error is:

$$\mathrm{F}(\mathrm{a}, b) = \min[\max(d_i) - \min(d_i)] \tag{24}$$

5.4.2. The Evaluation of Spatial Straightness Error

Spatial straightness is defined as the minimum diameter of the cylinder including the measured contour. According to the definition of form error, space straightness tolerance zone is a cylinder, containing all key points with the minimum diameter. The diameter of the cylinder is the straightness error value.

In the rectangular coordinate system shown in Fig. 9, the Z axis direction is the length of the direction of ideal linear assumption. The control points for coordinate values are $\mathrm{P_i}$ ($\mathrm{X_i}$, $\mathrm{Y_i}$, $\mathrm{Z_i}$) (i = 1, 2, ..., n), and the space equation of straight line is:

$$\begin{cases} x = a_1 + b_1 z \\ y = a_2 + b_2 z \end{cases} \tag{25}$$

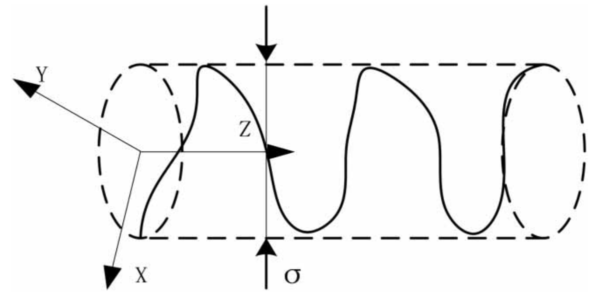

Fig. 9: The evaluation of spatial straightness error.

Where:

$$\begin{aligned} a_1 &= \frac{\sum(x_i z_i)\sum z_i - \sum z_i^2 \sum x_i}{(\sum z_i)^2 - n\sum z_i^2} \\ a_2 &= \frac{\sum(y_i z_i)\sum z_i - \sum z_i^2 \sum y_i}{(\sum z_i)^2 - n\sum z_i^2} \\ b_1 &= \frac{\sum x_i \sum z_i - n\sum(x_i z_i)}{(\sum z_i)^2 - n\sum z_i^2} \\ b_2 &= \frac{\sum y_i \sum z_i - n\sum(y_i z_i)}{(\sum z_i)^2 - n\sum z_i^2} \end{aligned} \tag{26}$$

The distance from key points $\mathrm{P_i}$ ($\mathrm{X_i}$, $\mathrm{Y_i}$, $\mathrm{Z_i}$) to the line is :

$$\mathrm{d_i} = \sqrt{[x_i - (a_1 + b_1 z_i)]^2 + [y_i - (a_2 + b_2 z_i)]^2} \tag{27}$$

The objective function of spatial straightness error is:

$$\mathrm{F}(a_1, a_2, b_1, b_2) = 2\min[\max(d_i)] \tag{28}$$

5.5. The Evaluation of Roundness Error

According to the mathematical tolerance definition, in order to meet the minimum zone condition, the roundness error is the minimum radial distance between two adjacent concentric circles. It makes the actual boundary of workpiece measured the section contained in the annular region between the two concentric circles, taking the radius difference as tolerance value R shown in Fig. 10.

In rectangular coordinate system shown in Figure 10, the coordinate values at each measurement section on sampling is $\mathrm{P_i}$ ($\mathrm{X_i}$, $\mathrm{Y_i}$) (i = 1, 2, ..., n). The center coordinate is (a, b) and the distance from measurement point to circle center is:

$$\mathrm{R_i} = \sqrt{(\mathrm{X_i} - a)^2 + (y_i - b)^2} \tag{29}$$

The objective function of roundness error is:

$$\mathrm{F}(\mathrm{a}, b) = \min[\max(R_i) - \min(R_i)] \tag{30}$$

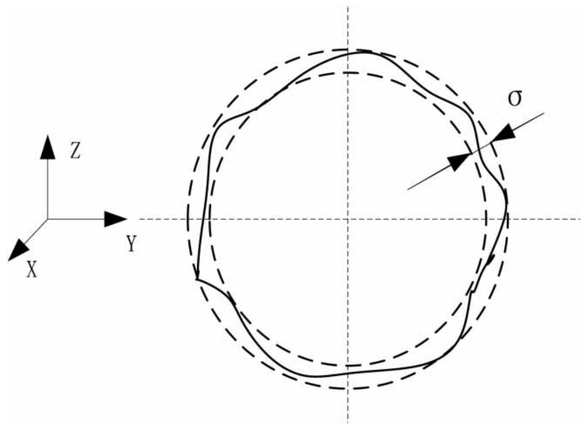

Fig. 10: The evaluation of roundness error.

5.6. The Evaluation of Cylindricity Error

Cylindricity error is the tolerance area between the two coaxial cylinders, taking the radius difference as the tolerance σ. It can limit the change value range from actual cylinder to the ideal cylinder.

In the space rectangular coordinate system shown in Fig. 11, the z axis direction is the length direction of cylindrical surface. The coordinate values of sample points on the transverse cross section of measurement are P_i (X_i, Y_i, Z_i) (i = 1, 2, ..., n). Comparison between actual and ideal cylinder surface is required in analyzing on the cylindricity error. Assuming that the axis of the cylinder is ideal for L, the direction of L is determined by two parameters l and m. The value of L is determined by two parameters a and b, then the axis of ideal cylinder can be represented as:

$$\frac{x - a}{l} = \frac{y - b}{m} = z \tag{31}$$

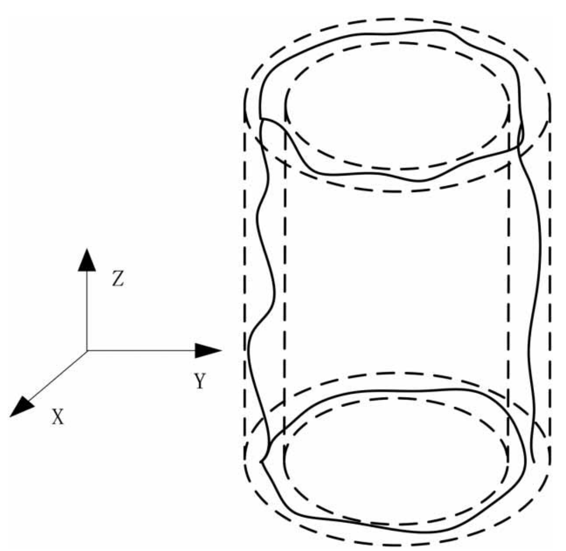

Fig. 11: The evaluation of cylindricity error.

The distance d_i from the key points on the Cylinder to the axis is:

$$d_i = \sqrt{[x_i - (lz_i + a)]^2 + [y_i - (mz_i + b)]^2} \tag{32}$$

According to the minimum zone method analysis , the key to find cylindricity error tolerance is to seeking two ideal coaxial cylinder of being in practical cylinder practical and having the minimum radius.

The objective function of cylindricity tolerance can be defined as:

$$F(a, b, l, m) = \min[\max(d_i) - \min(d_i)] \tag{33}$$

5.7. The Evaluation of Sphericity Error

According to the minimum tolerance area , the key to analyze sphericity error is to find two concentric spheres with the minimum radius difference including the actual measured contour. There are many measured contour concentric spheres, but only a couple of them is the smallest. As shown in Fig. 12, the analysis of sphericity error is to seek two concentric spheres with the minimum radius.

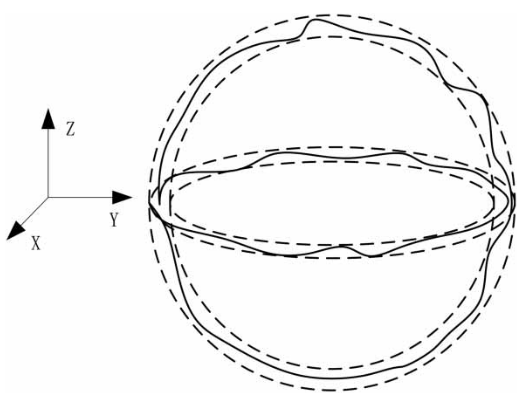

Fig. 12: The evaluation of sphericity error.

Set the center of sphere coordinates is(a, b, c)and the spherical measurement point is P_i (X_i, Y_i, Z_i) (i = 1, 2, ..., n), R_i as the distance from for measurement points to center as follows:

$$R_i = \sqrt{(x_i - a)^2 + (y_i - b)^2 + (z_i - c)^2} \tag{34}$$

The sphericity tolerance can be defined as:

$$F = R_{max} - R_{min} \tag{35}$$

Where:

$$\begin{aligned} R_{max} &= \max\sqrt{(x_i - a)^2 + (y_i - b)^2 + (z_i - c)^2} \\ R_{min} &= \min\sqrt{(x_i - a)^2 + (y_i - b)^2 + (z_i - c)^2} \end{aligned} \tag{36}$$

The objective function of sphericity tolerance can be defined as:

$$F(a, b, c) = \min[R_{max} - R_{min}] \tag{37}$$

6. CONCLUSIONS

In this paper, the tolerance mathematical modeling based on geometric elements control is presented. The coordinate parameter domain of control point is tolerance zone, the absolute position and relative position relations of control points in the tolerance coordinate system can be used to express size tolerance, orientation tolerance and location tolerance.

The parameters relationship of mathematical modeling tolerance based on control points is direct and concise, which can be easily deposit in data structure of solid models in the CAD data , applicable for various tolerance analysis methods. The nominal distance between control points is the nominal size of geometric elements and the relationship between the tolerance and accuracy can be easily established according to the variation of control points and the ratio of nominal distance.

At the same time, the geometrical elements of control points can be combined with the PSO algorithm to establish the tolerance mathematical modeling. For example, the form error can be calculated according to the measured element discrete points. It also calculates the coordinate parameters by using computer aided tolerance design, calculating the variation and the corresponding tolerance type. Compared to the traditional tolerance analysis, this method has advantageous for the data storage structure, simple calculation, accurate precision.

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