Efficient Shape-editing Method for Curves and Surfaces based on the Quantification of Aesthetic Properties

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ABSTRACT

In this paper, we propose a method of shape editing based on the quantification of aesthetic properties for more efficient and effective industrial design. The aesthetic measure, proposed in this paper, consists of two different characteristics; namely, the Class A condition and the minimum energy approach. Based on this idea, we defined two aesthetic measures; the curve aesthetic value and the surface aesthetic value. The former is used to evaluate the aesthetic property of a given curve, and the latter is used for a given surface, respectively. These values were tested on some examples, and their usefulness was shown.

Keywords: aesthetic property, class A, energy minimization, shape editing.

1. INTRODUCTION

In this paper, we propose a method of shape editing based on the quantification of aesthetic value for industrial design, in which high-quality and aesthetically pleasing shapes are preferred. In conventional CAD tools, it is not easy to create or modify shapes by simultaneously considering shape quality and designer's aesthetic intent. We defined aesthetic measures to evaluate curves and surfaces so that designers and CAD engineers can cooperate on the basis of an objective aesthetic quantity of the shape. This method should increase the efficiency of cumbersome shape modification processes.

The evaluation and modification of surfaces have been attempted by a variety of methods; however, each method has some shortcomings. The evaluation using monotonic curvature conditions, also known as the condition of typical curves [17] or Class A curves [5], suggests parameter domains for high-quality shapes; therefore there are likely to be a number of design solutions, and designers have to choose one solution from them. For the surface evaluation using highlight lines, a designer needs to visually check each surface for any abnormality; therefore, it is difficult to evaluate surfaces quantitatively. Lastly, for the evaluation using geometrical energy, the energy can only be used as a design solution when the energy

reaches the optimum value; thus, it cannot be used as a design tool owing to its lack of operability of shapes. Consequently, to perform an objective evaluation of curves and surfaces, we propose a new method in which a shape evaluation criterion based on the monotonic curvature condition and the strain energy minimization condition is used. This paper describes the fundamental idea and some examples are given to show the effectiveness of the proposed method.

2. RELATED WORK

The aim of this paper is to support procedures of style design by defining the "aesthetics" of curves and surfaces, for which we establish an objective evaluation method. Conventionally, curves and surfaces have been evaluated by their curvature profile, highlight lines, and bending energy. Regarding curvature, monotonic curvature has been considered attractive in the context of industrial design. The importance of monotonic curvature has been pointed out by many researchers including Jones [14], Farin and Sapidis [6], and Birkhoff [1]. Miura, and Yoshida and Saito have studied expressions generating aesthetic curves [18,19], where the logarithmic curvature distribution

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diagram defined by Harada et al. [9] was used. Yoshi-moto and Harada also discussed the association of curve with sensitivity evaluation [20]. From a different viewpoint, a method of controlling or evaluating surfaces by changing the reflection lines was proposed by Klass [16] and Kaufman and Klass [15]. Reflection lines indicate the highlight lines conventionally used in the automotive industry. Also, Chen et al. proposed a method that allows the highlight lines on a NURBS surface to be changed directly [4]. Wang et al. proposed a designer-supporting method that meets the requirements of monotonic curvature variation by performing shape control that can take account of the geometric constraint that expands the highlight lines [21]. Furthermore, Higashi et al. proposed a method of generating curves and surfaces by using evolutions [9]. As for the study on the relation of shape modeling and implicit aspect of design, there was an important project named FIORES II that was an European project for character preservation and modeling in aesthetic and engineering design, in which Giannini and Monti [7] presented a method to relate shape and emotional character for developing helpful CAD tool.

There is an idea that a shape in nature is beautiful because its elastic energy takes a minimum value. Regarding the generation of surfaces by energy minimization, Hosaka and co-workers defined the total squared curvature as a curve evaluation function, which was used to smooth curves automatically [11]. Then, Hosaka and co-workers applied this idea to a practical model [12]. The minimization of surface energy has been studied by Bloor and Wilson [2]. Brakke considered the optimum shape of triangulated meshes on a surface to edit the surface interactively [3]. Regarding the minimization of the strain energy of a curve, a method of searching for the minimum value for a curve passing through two arbitrary points in a given direction was presented [10]. Inoue et al. [13] extended this and realized surface optimization via the strain energy by considering a surface made of parametric curves.

The evaluation and editing of surfaces have been attempted by a variety of methods. However, each method has some shortcomings. The evaluation using the monotonic curvature condition is a condition evaluation that gives a parameter region; therefore, there are likely to be a number of design solutions, and each solution cannot be evaluated objectively in comparison with the others. For the evaluation using highlight lines, a designer needs to visually check each surface for any abnormality; therefore, it is difficult to evaluate the surface quantitatively. Regarding the evaluation using surface energy, this approach can only be used as a design solution when the energy reaches the optimum value; thus, it cannot be used as an aesthetic measure. Consequently, in this paper, evaluation by aesthetic measures based on the monotonic curvature condition and the strain energy minimization condition is proposed as a means of performing an objective evaluation of shapes.

3. PROPOSED METHOD

3.1. Class A Condition

A mathematical condition for a curve with monotonic curvature called the Class A condition was proposed by Farin [5], which is based on the *typical curve* proposed by Mineur et al. [17]. In [5], a Bézier curve was applied to Class A curve generation. A Bézier curve is a 2D or 3D parametric curve whose points $\boldsymbol{x}(t)$ are defined by

$$\boldsymbol{x}(t) = \sum_{i=0}^{n} \boldsymbol{b}_i B_i^n(t), \tag{1}$$

where $\boldsymbol{b}_i$ are control points, and $B_i^n(t)$ are Bernstein polynomials. The derivative or hodograph of a Bézier curve is given by

$$\dot{\boldsymbol{x}}(t) = n \sum_{i=0}^{n-1} \Delta \boldsymbol{b}_i B_i^n(t) \tag{2}$$

where $\Delta \boldsymbol{b}_i = \boldsymbol{b}_{i+1} - \boldsymbol{b}_i$.

In [5], a class of special 3D Bézier curves is treated; namely, those having hodographs with control vectors:

$$\Delta \boldsymbol{b}_i = \boldsymbol{v}_i = M^i \boldsymbol{v}; \quad i = 0, \ldots, n-1 \tag{3}$$

where M is a 3×3 matrix and $\boldsymbol{v}$ is an arbitrary unit vector. This concept is illustrated in Fig. 1. Farin defined the Class A condition by using the vector $\boldsymbol{v}$ and the matrix M. Geometrically, the definition states that the line segment $\boldsymbol{v}$ and $M\boldsymbol{v}$ does not intersect the sphere with radius $\boldsymbol{v} = 1$. This condition is equivalent to the following equations;

$$v^T[M^T + M - 2I]v \geq 0 \quad \text{and} \quad v^T M^T M v \geq v^T v \tag{4}$$
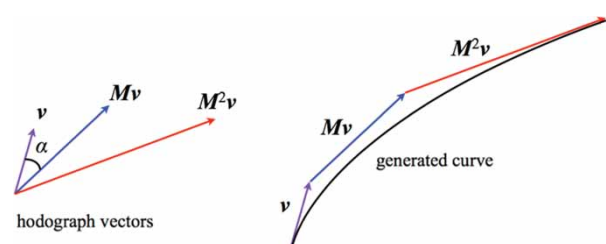

Fig. 1: The concept of Class A condition [5].

In addition to this, a further restriction on the singular value σ_i ($i = 1, 2, 3$) of the matrix M is required. Namely, it is $\sigma_3^2 \geq \sigma_1$ in three-dimensional case. When the matrix M consists of a rotation angle α and a scale s, the following relations

$$s > 1/\cos\alpha, \qquad s < \cos\alpha \tag{5}$$

are correspond to the above conditions. In two-dimensional case, the resulting curve is identical to the *typical curves* presented by Mineur et al. [17]. In

the rest of this paper, we use this condition; namely, the matrix M takes the form of

$$M = \begin{bmatrix} s\cos\alpha & -s\sin\alpha \\ s\sin\alpha & s\cos\alpha \end{bmatrix} \quad (6)$$

Note that the curves tested in this paper are space curves but they do not have torsion; thus, the curves will be treated as if plane curves. As expressed by Eqn. (4), a Class A region exists in the α–s plane, which is shown in Fig. 2. The lines in this figure are called boundary curves of Class A in this paper.

3.2. Strain Energy

Then, the strain energy of the parametric curve $(x(t), y(t))$ is expressed as follows according to Inoue et al. [13].

$$E = \int_0^1 \frac{(x''(t)y'(t) - x'(t)y''(t))^2}{\{(x'(t))^2 + (y'(t))^2\}^{5/2}}\, dt \quad (7)$$

The minimum strain energy curve is considered to be the most natural shape under the given conditions. Therefore, it is possible to search for the minimum
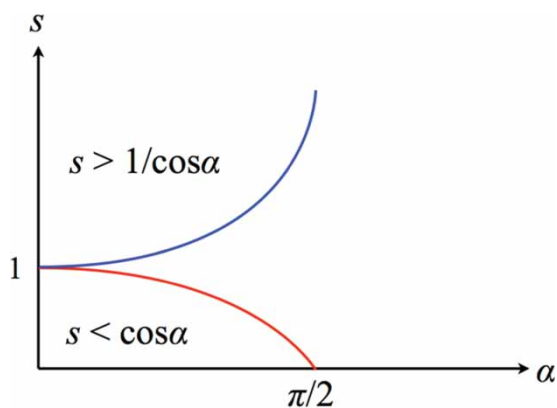

Fig. 2: The schematic illustration of the αs plane.

energy shape by changing parameters using an optimization method. By combining the Class A condition and this, we developed a new evaluation method.

In our research, when searching for the minimum energy curve, it is found that imposing sufficient conditions on the curve is required. In other words, it is necessary to designate the starting control point, the terminal control point region, the rotation angle range and the scale range, and to impose the Class A condition. These initial conditions function as a representation of the intention of the designer. Searching without setting these conditions would lead to undesirable shapes, such as a very long curve or straight line, being derived as a solution. Fig. 3 shows a case where only the starting point and Class A condition are imposed. The original and calculated curves are shown on the left and the α–s plane is shown on the right. Without enough conditions, although its position in the α–s plane was within the Class A region, the calculated curve, shown in blue, was distorted greatly. On the other hand, Fig. 4 shows a case where appropriate conditions are imposed. The calculated curve became a better curve with respect to the energy.

3.3. Curve Aesthetic Value

Now, we define the evaluation value of a curve by the curve aesthetic value (CAV). The CAV is determined by a position and distance L in the α–s plane of the given Class A region. Here, the position is determined by the values of α and s of the given curve in the α–s plane, and the distance L is the distance between the position and the origin, which is given by the minimum energy point. Tab. 1 shows the classification of the curve and Fig. 5 shows a schematic illustration of the classification given in Tab. 1.

The numbers from 1 to 5 in Fig. 5 are equivalent to those in Tab. 1. In this figure, the Class A region, which is expressed by Eq.(5), is colored green, and the origin of the measure, the minimum strain energy point, is denoted as number 1. The black points are
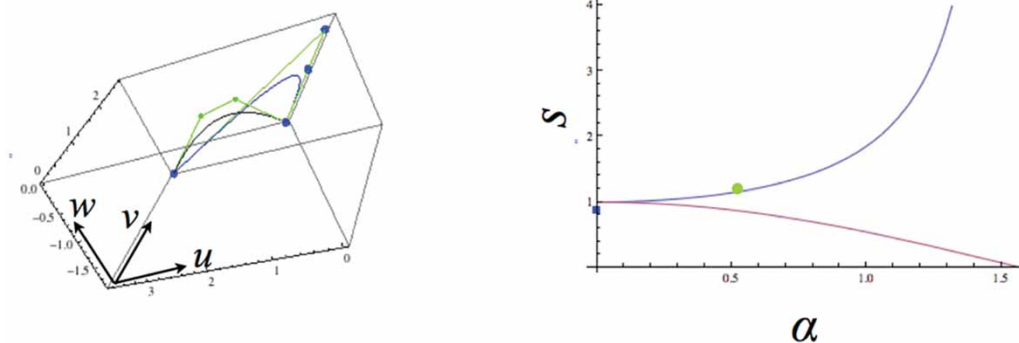

Fig. 3: Original and calculated curves (left) and energies in the αs plane (right).
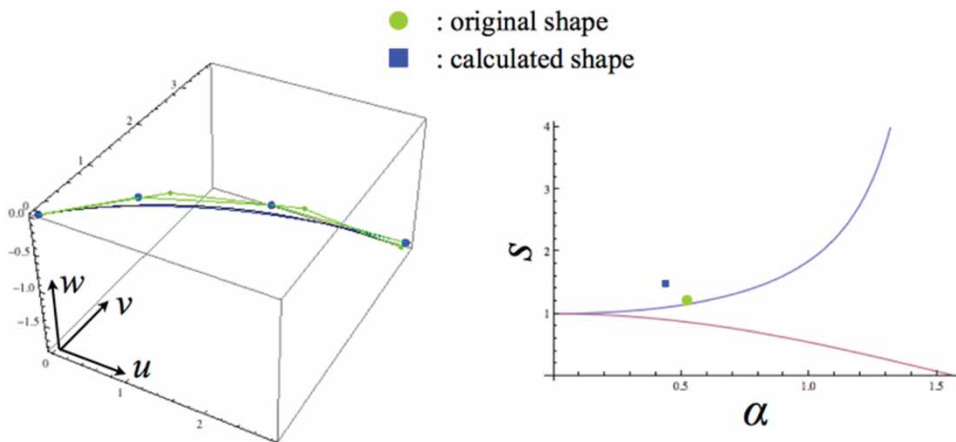
Fig. 4: Original and calculated curves and the αs plane (all conditions are imposed).

Curve type	Description
1	M is in Class A region and the strain energy is minimum.
2	M is in Class A region.
3	M exists out of Class A region.
4	M exists but is not one matrix.
5	Control point matrix M does not exist.

Tab. 1: Classification of the curves.

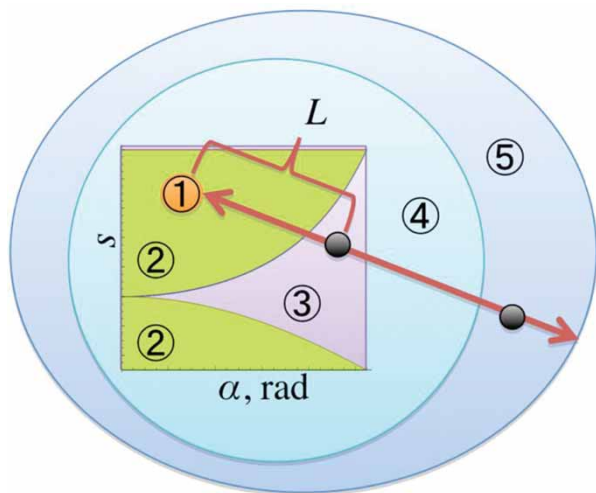| Curve type	CAV
1	0
2	L
3	$|(0, 1), (\alpha_{max}, 1/\cos\alpha_{max})| \times (1 + L)$
4	$|(0, 1), (\alpha_{max}, 1/\cos\alpha_{max})|^2 \times |(0, 2), (\alpha_{max}, 0)|^2$
5	$|(0, 1), (\alpha_{max}, 1/\cos\alpha_{max})|^3 \times |(0, 2), (\alpha_{max}, 0)|^3$

Tab. 2: How to calculate the CAV.

Fig. 5: Relationship between the α–s plane and classification of the curves.

examples of arbitrary given curves. Once a curve is given, the parameters α and s are calculated from it, and then the position is determined, as shown in this figure, and the distance L can be calculated.

When a given curve exists near the origin, it is evaluated as a high-quality and natural curve, which will be categorized as type 2. Of course, whether a curve is beautiful is dependent on individual sense;

therefore, this aesthetic measure should only be used as a geometrically objective measure. We consider that this measure will be useful as a communication tool between designers and engineers, as a shape-analyzing tool for constructing design databases and as a shape-editing tool in CAD systems.

How to calculate the value of the CAV is listed in Tab. 2. In the case of curve type 1 and 2, the way of evaluation is obvious; namely, we use the Euclidean distance L from the energy minimum point in the α–s plane. When the curve is judged as type 3, the given curve has single (α, s) set that is out of Class A region. To make the difference to the type 2 clear, 1 is added to make the value greater than 1; then, a weighting value is multiplied. In this paper, as an example of the weight, a length of the diagonal line is applied. This diagonal line begins with $(0, 1)$ and ends at a point on the Class A boundary curve. A user is required to specify the upper limit of alpha (α_{max}), which gives the terminal point of this diagonal line. In the case of type 4 and 5, it is impossible to evaluate the curve because it does not have a single (α, s) set. Therefore, we just use two large values that are determined by a user as a maximum value of angle α. This means that the given curve is out of the α–s plane and cannot be built by single matrix M; this curve is not assumed to be beautiful in our method.

The procedure of this calculation is summarized as follows.

(1) A curve is given by a designer.
(2) The curve is analyzed. If the curve is given by designating the parameters α and s, these values are directly used to construct the matrix M. When the curve is given by free style, the curve will be approximated by a Bézier curve of suitable degree. Then, the control points are obtained and used for analysis.
(3) The matrix M or the control points are investigated to know which curve type is assigned.
(4) The CAV value is calculated as the expressions given in Tab. 2.

3.4. Surface Aesthetic Value

Next, we define the evaluation value of a single surface patch as the surface aesthetic value (SAV). The SAV is a sum of CAVs; namely, we assume that a surface consists of eight network curves, as shown in Fig. 6, and the CAVs are obtained from these curves.
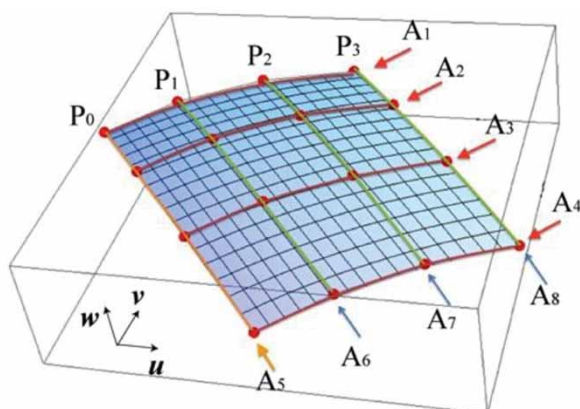

Fig. 6: Definition of network curves.

In *uvw* space, when four Bézier curves (A1– A4) in the direction of *u* and one Bézier curve (A5) in the direction *v* are specified, a Bézier surface is determined. Since the remaining curves (A6 – A8) are automatically determined, the calculation targets are eight network Bézier curves (A1 – A8). The SAV is the sum of each CAV; thus, the SAV is given by the following equation:

$$SAV = \sum_{i=1}^{8} CAV_i \qquad (8)$$

4. EXAMPLES OF APPLICATION

4.1. Example of Curve Evaluation

To begin with, an example of a curve edit is presented. As a possible boundary condition, the following is used; namely, the starting point is (0, 0, 0) and the tangent vector is (1, 2, 0). Then, we assumed that the given curve has $\pi/4$ of rotation angle and 1.05 of scale value, which does not meet the Class A condition, therefore, the curve is in type 3. Then, a curve of same starting point and tangent vector with variables of α and s is prepared. The strain energy of this curve is constructed; namely, a function of strain energy with parameter α and s is made. In order to produce a meaningful result, we added another variable, which is a scalar β multiplied to the tangent vector. As a result, there are three variables in the strain energy function. The following is the explicit form of the strain energy function of this example using Eq. (7);

$$f(t, \alpha, \beta, s) = \int_0^1 \frac{0.596285 \, s^2 \, \beta^2 \, \sin^2 \alpha}{(\beta^2(1 - 2t + (1 + s^2)t^2) - 2s(t-1)t \, \cos \alpha)^2)^{1.5}} \, dt \qquad (9)$$

Note that the parameter t is vanished by numerical integration. When searching for the minimum value of the strain energy, we used two constraint condition; that is, the Class A condition and terminal point correspondence. The terminal point correspondence means that a Euclidian distance between each terminal control point, before and after the modification, should be within a small threshold value; namely, 0.01 was used in this example. We used the function FindMinimun of Mathematica to search for the solution, the results were $\alpha = 0.65$, $s = 1.44$, and $\beta = 0.64$. The original and calculated α and s are plotted in Fig. 7. The curves of the original and the minimum energy are shown in Fig. 8. As shown in this figure, each end point is almost coinciding, and the initial tangent direction is preserved. The original curve can be modified to be Class A and minimum energy with preserving initial design intention.
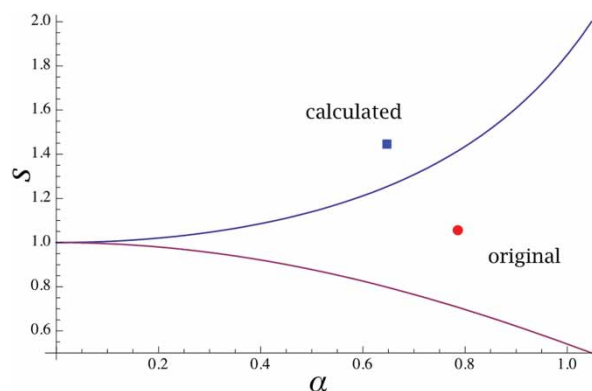

Fig. 7: The α-s plane of the example. The original curve is given as $(\pi/4, 1.05)$, and the calculated minimum energy point is (0.65, 1.44).
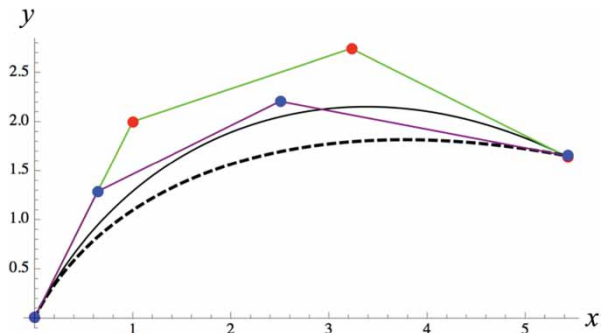
Fig. 8: The original curve is shown as solid black line, and the modified curve is shown as dotted line. The direction of the curve-beginning tangent vector is preserved, and the terminal points are almost coinciding.

4.2. Example of Surface Evaluation

Figures 9 and 10 show surfaces used to perform next experiment. The surfaces are shown in the left part and the curvature profiles are shown in the right part.

These surfaces have the following characteristics.

1. Surface (a): all network curves are Class A.

2. Surface (b): six network curves have control point matrices but they are not Class A, and the other two network curves do not have single control point matrix.

In these cases, the aesthetic measure under the given conditions becomes as follows.

$$0 \ (M_{all} = M_{E_{\min}}) \ \rightarrow \ 12 \ (M_{all} = M_{Class \ A})$$
$$\rightarrow \ 38 \ (M_{all} \neq M_{Class \ A})$$
$$\rightarrow \ 110 \ (M_{all} \neq M_{sym.}) \ \rightarrow \ 407 \ (M_\phi) \tag{10}$$

In this regard, $M_{all} = M_{E_{\min}}$ means that the all network curves are minimum in strain energy, and $M_{all} = M_{Class \ A}$ means that the entire control point matrices M are symmetric and Class A, and M_φ means that no network curves have a control point matrix M. For example, in surface (a) all the curves are Class A; therefore, their CAVs are between 0 and 12. The SAVs of these surfaces are summarized in Tab. 3. It is almost impossible to distinguish these surfaces by visual observation; however, their SAVs are clearly different, which means the proposed quantity is helpful to investigate surfaces objectively. Thus, the SAV is able to show the difference in the characteristics of
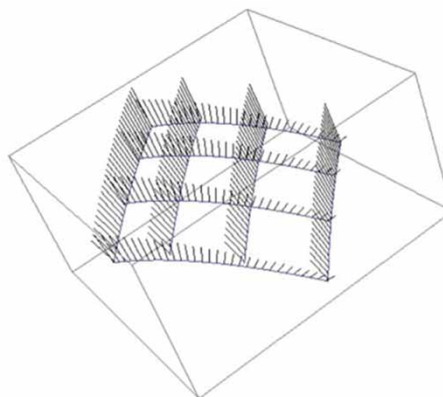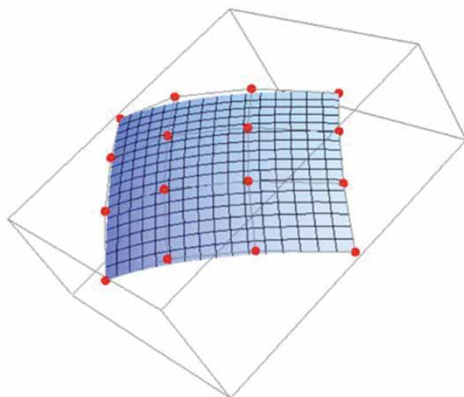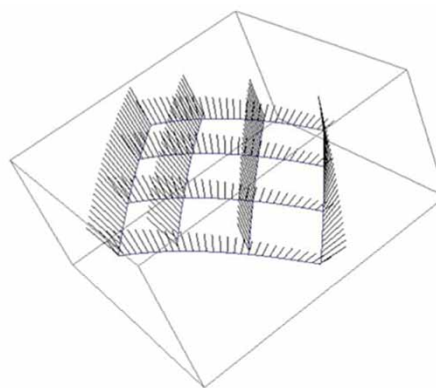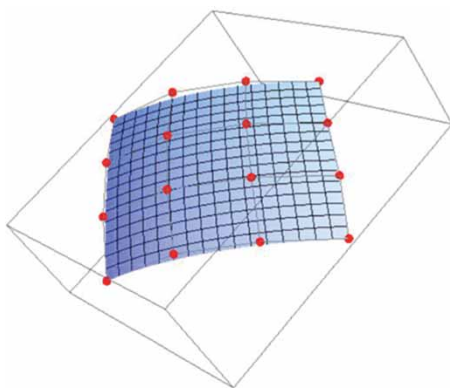

Fig. 9: Surface (a).

Fig. 10: Surface (b).

	SAV
Surface (a)	1.9
Surface (b)	130.5

Tab. 3: The SAVs of surface (a) and surface (b).

surfaces. Moreover, in the case of some low-aesthetic-value curves, even if their number is small, the SAV becomes high. This property would be helpful to check the existence of poor network curves.

4.3. Example of Surface Editing

The SAV is useful for not only evaluation but also editing surfaces. Here we consider surface (c), which was manually rendered by trial and error, in which no network curve has a control point matrix. In other words, its quality is very low.

Definitions of the modification are as follows. In this case, the original value of SAV was 290, which could be reduced to 0 by editing. Surface editing was carried out by parameter change. First, the curve conditions were obtained, which produces each CAV, then, the minimum energy points are searched. By this way, we had the optimal values of α and s for all

curves. Finally, all the network curves were changed to the most beautiful shape; it is not only Class A but also minimum energy shape. Fig. 11 shows original surface (c) that is before editing and Fig. 12 shows the result of editing.

Surfaces are shown in the left part and curvature profiles are shown in the right part. In addition, Fig. 13 shows zebra maps of the surface before and after editing, respectively. The surface modeling software Rhinoceros 5.0 was used to generate these zebra maps. The zebra map before the editing has some twisted patterns; however, the edited one has well-formed zebra patters. This editing was carried out without changing the given initial condition, which can be interpreted as designers' intention or design regulations. The simple change of aesthetic parameters enabled this relatively difficult shape-editing operation. Thereby, the usefulness of the proposed method for editing surfaces was proved.

If a designer tries to modify the surface without any support, it will take much time to obtain a satisfactory shape because manipulation of many control points is far from the way of designer who prefers more natural approach. In other words, it would be a non-intuitive and non-productive activity. On the other hand, the proposed method provides more objective and relatively analytical design methodology. Although the designers are not mathematicians,
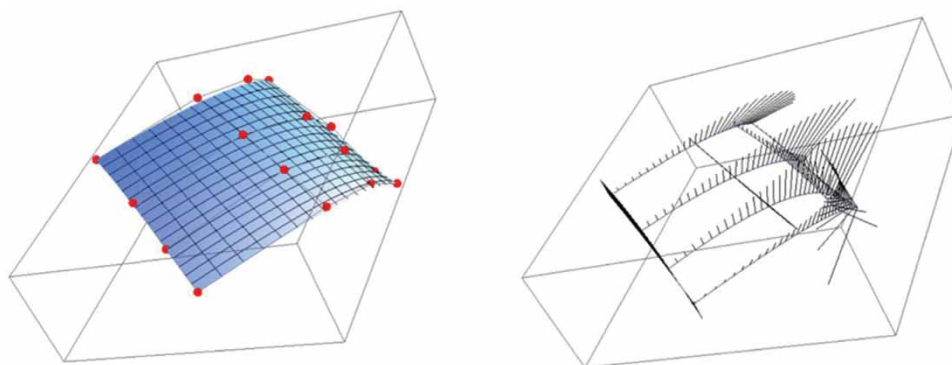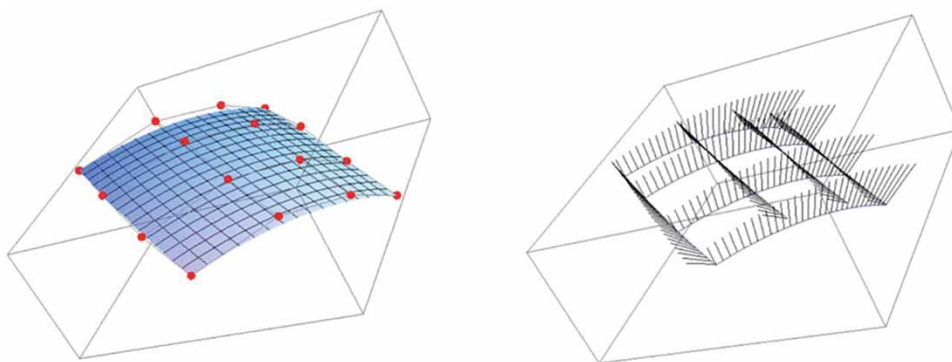

Fig. 11: Original surface (c).
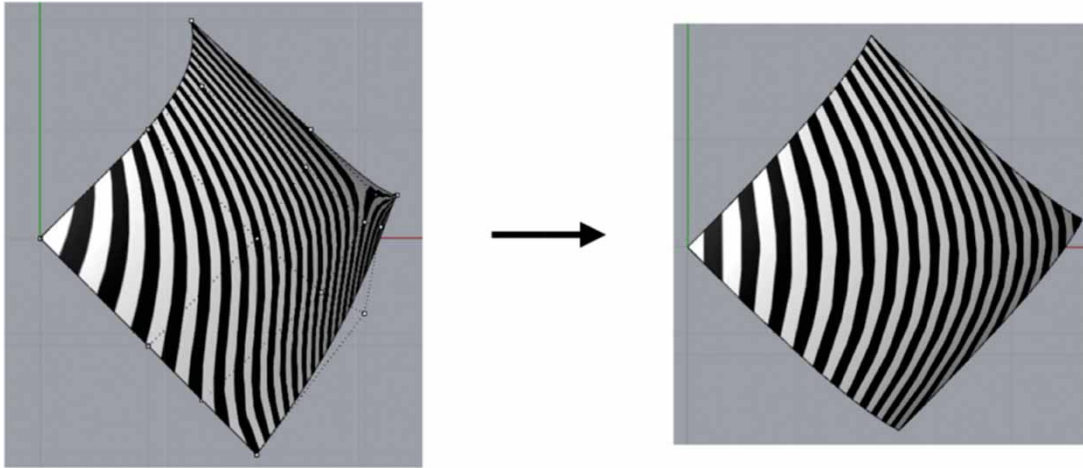

Fig. 12: Edited surface (c).

Fig. 13:   Zebra maps of the surface before and after the modification.

they can use this method easily; namely, owing to its objectivity, the proposed method can be systemized and combined with CAD systems; the mathematical aspect can be hidden.

5. CONCLUSION

In this paper, we propose a new method of editing curves and surfaces based on the evaluation of their aesthetic properties. We defined geometrically objective aesthetic measures for a curve and surface using the monotonic curve condition, known as the Class A condition, and the strain energy minimization. Through some examples, we demonstrated the advantageousness of the proposed method. A surface can be edited to improve its quality simply by changing aesthetic parameters. Conventional surface editing requires the manipulation of control points, which is cumbersome task that can be simplified by the proposed approach.

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