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Multilevel topology optimization

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ABSTRACT
The paper proposes an extension of the Function-Behaviour-Structure (FBS) framework to multi-level design representation. The ontology based on function, behaviour and structure has been enriched with a new design entity, the topology, with the aim of connecting more levels of representation. According to this new paradigm, design activity is not focused exclusively on working principle, shape and material at macro level, but goes beyond, to greater levels of detail, designing for example how to dispose material in the inner structure of the product parts at microscopic level.
Structural optimizers are excellent tools to design the topology of a structure according to its function and behaviour, but they have been conceived for working only at mono-level.
This paper proposes a multi-step optimization process for improving the versatility of structural optimization tools allowing them working also in both macro and microscopic dimensional scales.

KEYWORDS
Structural optimization; topology; multilevel; FBS

1. Introduction

Nowadays, several factors are deeply influencing the way of producing things and thinking the design. An increasingly large network of people, share and disseminate knowledge about product design that are no longer the result of the design capacity of a single person but of a community. Furthermore, the recent technological developments are enabling everybody to be truly independent from industry in all project phases, enlarging the number of potential designers. As a consequence, new needs are coming out, both for structuring this huge "messy knowledge" and for stimulating already existent tools.

In this new scenario even most advanced tools like structural optimization tools showed deficiencies.

This paper deals with a new methodology based on "multi-level design" approach that foresees the development and integration of computer aided tools to support designers' work. Multilevel design is not intended here as the relationship between product design, service, system and society but as the design of products taking into account a plurality of design perspectives at different detail levels, from macro to micro. In the following, an overview of some research areas that involve a multilevel approach is introduced, including biomimetic, problem solving and material science, which constituted the basis for the definition of the new design methodology. In section 2 the ontological framework about multi-level design is introduced. Section 3 introduces the methodological basis on structural optimization and shortcomings, while the proposal for a new multistep optimization according to multilevel framework is presented in section 4. Finally, in section 5, conclusions and future developments are drawn.

1.1. Multilevel in biomimetic

Living beings offer an endless collection of examples of how the stunning mechanical characteristics of biological structures directly depend on the hierarchical organization of the organic material itself; such organization is defined at various dimensional scales, so that the properties at lower levels influence the behaviour of the structures at higher levels.

There are evidences of the fact that multilevel organization of organic structures is a competitive factor. For instance, it can be noticed that the "evolutionary trend" of living beings promotes the creation of more and more complex organisms, so that there has been an evolution from simple mono cellular bacteria to extremely complex animals, by the addition from time to time of more and more levels of complexity.

As an example, Raabe et al. [11] describe the arthropod's exoskeleton starting from the hierarchical organization of its structures, decomposing it in many structural

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layers, each one with many different functions and, consequently, different (mechanical) behaviours. According to this complex theoretical model, the local stiffness of the material of each structure may be calculated by considering the structural compliance of a twisted plywood pattern. Such compliance basically depends on the stiffness of its constituent chitin-protein honeycomb, which stands in another dimensional level of description.

Similarly, spider web is another remarkable example. It would be a mistake considers it only as a reticular structure; in fact, the only nature of its macroscopic design wouldn't explain its excellent mechanical (and not only mechanical) properties. Such features are the result of a very complex interaction between the organizations of the organic material at various dimensional scales [18].

Anyway, in biomimetic, the multilevel approach has been used mostly for analysis purposes. Many models have been created in order to understand and simulate the behaviour of the biological structures, but very few address the use of a multilevel approach in order to create a new paradigm for design. An effort has been presented in [13], trying to describe how natural structures overcome contradictions operating a different scale levels, and extend this concept as design strategy.

1.2. Multilevel in conceptual design and problem solving

On the other hand, most of the design studies merely describe the system at a single level of detail, creating an alternative abstract level of description based on functions, and getting down to a lower technical level of detail only for choosing the material [9]. One of the most representative design models is FBS. According to it, all systems can be described analysing at the same level of description how its structures collaborate together in order to create a certain behaviour that allows the system to fulfil a certain function [6].

Instead, more practical design approaches, such as TRIZ or psychological methods for problem solving [15] (e.g., lateral thinking), propose tools for changing point of view and facing the problem at different scales of representation. For example, one of the 11 TRIZ Separation principles (namely macro-micro) is dedicated to solutions working at Macro-Micro levels. It suggests how to overcome a physical contradiction trying to solve conflicting requirements at different levels of detail. For example, if it is required the design of a protective wall, very thick in order to be resistant, and very thin in order to be light, Macro-Micro separation suggests to think "a big layer at macro level, made of very thin layers at micro level, such as honeycomb, multilayers, porous materials, etc.".

Dynamics laws of evolution, always formulated by Altshuller in TRIZ theory, describe the Transition from macro to micro level: "The development of working organs proceeds towards a better exploitation of the resources at first on a macro and then on a micro level. The transition from macro to micro level is one of the main (if not the main) tendency of the development of modern technical systems that use energy fields in order to achieve better performance and control".

Even if some TRIZ tools (e.g., Multiscreen, some inventive principles, the concept of operative zone,) or other methodologies (e.g., lateral thinking) can help to move to a multilevel approach, until now, rare examples of the adoption of a real multilevel design have been investigated both in problem-solving [14] or for slightly different purposes, such as forecasting [8].

1.3. Multilevel in computational materials

One of the fields, which took most from the observation of the nature, is the study of materials, In the last years, many attempts have been done in order to characterize materials studying their own inner structures. Starting from nano-metric dimensional scales, different properties of the materials derive from different structures, and a central topic is the relation among the different structures, and how difference parameters influence one another [5]. Anyway, beside the most recent researches, such approach has been adopted in various forms since long time. An example is the study of different micro structural materials [4], which investigates the relation between the lattice topology and its mechanical characterization, underling the evidence that hierarchical design increases buckling strength.

To the authors' best knowledge, once again, all these researches have the limitation of realizing an analysis of materials in order to describe (and eventually simulate) the behaviour of the material (bottom up approach), but such considerations are not implemented in a methodology for supporting the conceptual design.

2. Multilevel definition

From the previous analysis, there is the evidence that multilevel comes out naturally while facing a certain number of scientific and technical issues. The use of multilevel approach, so heterogeneous and in different and far disciplines is reflected in a lack of a shared definition. Before describing the proposed methodology, it is convenient to introduce the main bricks of the future ontology for a multilevel framework, as reported in Table 1.

In the presented schema, the generic elements of the ontology are presented: the macro level, the micro level,

Table 1. Conceptual basis for a future multilevel ontology.

MACRO	At a first step, multilevel approach states that a structure may be schematized by a set of constitutive elements, and the interaction among them, under certain external conditions, defines the behaviour of the structure itself. The macro level is the level of the first main structure thought as one.
MICRO	The constitutive elements, thought as part of the structure, are elementary entities, which have their own (mechanical, but not only) properties, and should not be further divided (see point A in figure 1). At microscopic level, such elements may be represented as structures, which, in this new domain, are composed by a further set of elements (see point B in figure 1). In this way, the characterization of the elements at a macro level is the behavior of the same element thought as structure at micro level. The totality of the set of elements thought as separated entities represents the second level.
RELATION BETWEEN MACRO AND MICRO	Depending on the way the macro and micro levels are defined, the physical characterization of the elements at macro level depends on the behavior of the structures at micro level (this equivalence is indicated by the arrow connecting macro and micro level, at point C in figure 1). Such framework can be iterated, so that the lower level of the first iteration step represents the upper level for the further iteration step.
FUNCTION	Function is the list of the final goals the system has to fulfil at each level.
BEHAVIOUR	Each element of the system interacts with others for providing a function by creating a particular behaviour. It can be described looking at how structural topology trigs a physical phenomenon under a specific load condition.
STRUCTURE	Structure is the list of different elements that constitute the system, according to the Topology. It is crucial to identify all the geometrical links between elements composing the system.
TOPOLOGY	Topology is a factor that influences the global behaviour of a structure. In fact, the physical properties are directly influenced by the geometry with which the components of the structure are made, both at macro and micro level (see points D in figure 1). It means that the topology is as much important as the intrinsic mechanical properties of the basic constituents themselves.
	An example of the relevance of this fact is that different studies about auxetic materials have been developed. Counting on the contribution of the particular morphologies, these structures are able to globally behave like materials with negative Poisson coefficient. This is the classical example of how the distribution of the material in the space allows the material itself to obtain a certain property, in opposition of its homogeneous form, so to actually fulfil a specific function.

the basic relation between them, and the possibility of iterating the scheme itself.

For example, from a certain point of view, FEM analysis schema belongs to a bi-level framework; in fact, in this kind of analysis, the continuum spatial domain is decomposed in a certain number of discrete elements, so that we have a passage from a macroscopic level to a lower scale domain.

Extending this concept, in order to model a system, it is necessary to provide a description of the elements at all the scale levels. The dimensional levels will be conventionally called macro, micro, meso, etc. levels. For each level there must be a core functional decomposition into Structure, Behaviour, Function with a particular attention to Topology. figure 1 shows a schematic resume of the relation between the conceptual entities above described.

2.1. Example of the methodological framework

As an example of the proposed methodological framework the spider's orb web is analysed using the described multilevel approach (figure 2).

At a Macroscopic Level, the orb web is composed by two kinds of threads: the frame silk, which is disposed in radial direction from the centre of the structure to the edge of the structure itself, and the viscid spiral.

The web changes its behaviour, according to the external conditions and the several functions that must be fulfilled in a certain moment. The first behaviour is related to the "structural optimization" which is realized by the spider using a strategy similar to the application of the Clerk Maxwell's lemma stating that the maximum force is in correspondence of an elongation of 1.25 (εmax $= 0.25$), which, is the maximum deformation of the dragline silk actually.

The second behaviour is typical of the part of the web composed by viscid silk: the main characteristic of such threads is that they are able to dissipate a huge amount of energy, realizing a huge deformation.

The frame silk must fulfil functions such as realizing a web which is able to preserve structural integrity, using less material as possible, ideally using threads that may work as safety lines (which absorbs the force of the falling spider), and, obviously, realize an efficient trap.

Threads are the structures at the Micro Level, and their role is to provide elements (at the Macro Level) characterized by different yield stresses, strength, and maximum elongations. There are two kinds of threads, and their different behaviours are determined by the number and the physical properties of the elements they are made of, and the way these elements are disposed. Frame silk is composed by a high number (4 or 5) of strands, which have high initial Young modulus, high strength, and relatively low elongation. Viscid silk is composed of 2 strands with a lower Young modulus, a lower strength, and higher elongation. Both silks consist of elements of the same kind with a parallel disposition.

The described structures for the silk have their own behaviour, which, in both cases, corresponds to the mechanical response of the thread. Due to the parallel disposition of the strands, and for the said topology, the effect is just to influence the cross section of the thread, preserving the stress-strain diagram, and, on the other

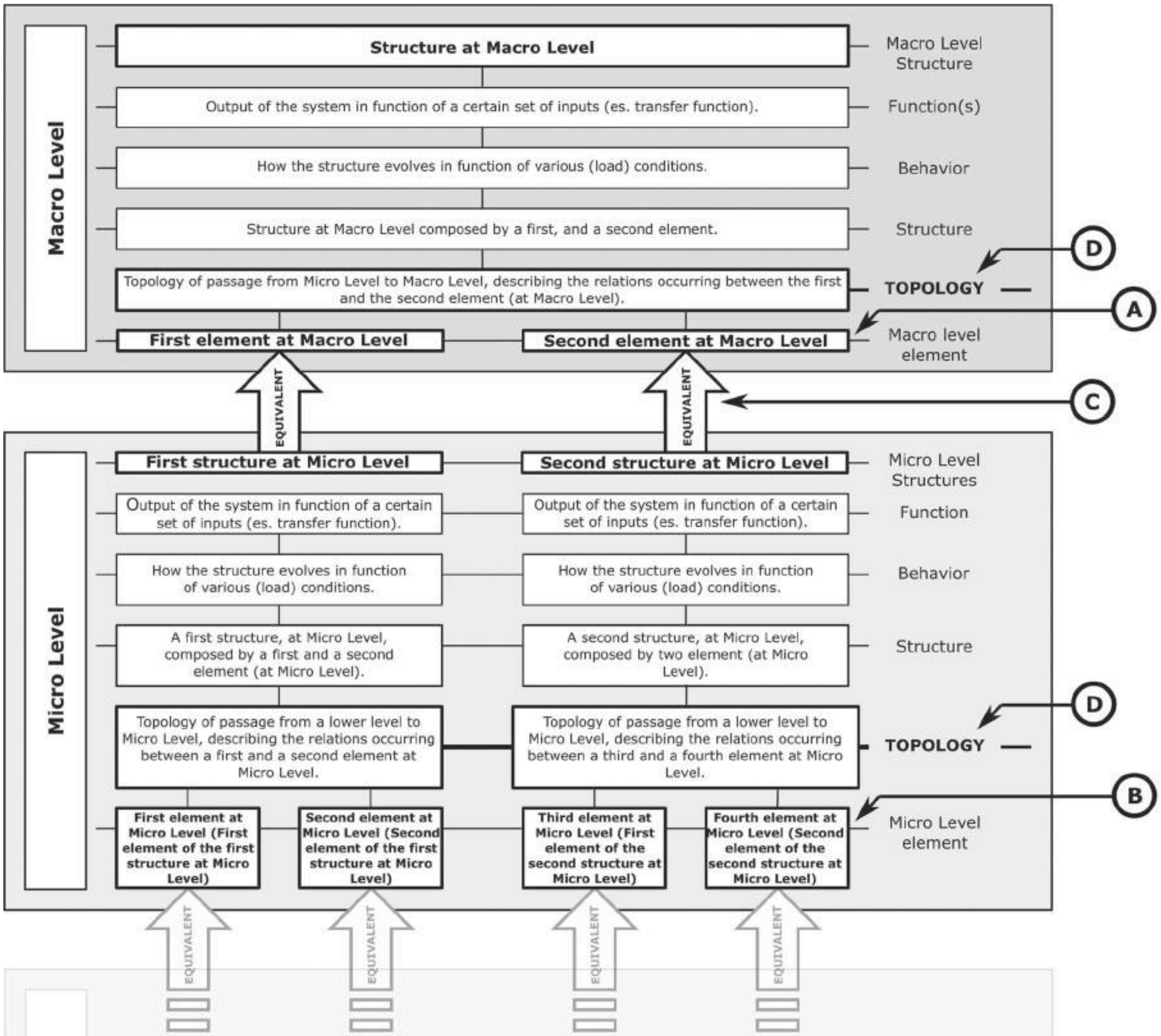

Figure 1. Hierarchical organization of a system model based on FBS multilevel decomposition framework.

hand, influencing the maximum load, and the energy dissipated, that, in both structures, is more or less equal.

The viscid silk is able to absorb a certain amount of energy realizing a high elongation, and frame silk is able to absorb the same amount of energy but being less deformed.

At this level, the described structures have the function of fulfilling the requirement described at Macro Level. The mechanical responses of the frame and viscid threads, which are structures at the Micro Level, obviously correspond to the characteristics of the components at the macro level. In particular, for the viscid silk, the higher admissible force is in correspondence of a very high elongation, while for frame silk, the higher admissible force is in correspondence of an elongation compatible around 1,25.

3. Structural optimization tools

The scope of the present work is to investigate the possibility of integrating a new optimization level in structural optimization software according to multilevel design. Therefore, the proposed design methodology foresees the use of structural optimization tools adopting a multilevel approach.

Under the name of "optimization of structures" there is a number of different approaches to solve the problem of identifying the best design for a structure. We take into account only topological optimization that is the research of the ideal distribution of material in a certain region of space in order to fulfil a number of specific goals, usually regarding the stress configuration and compliance of the structure itself. The word "topological" refers to the idea

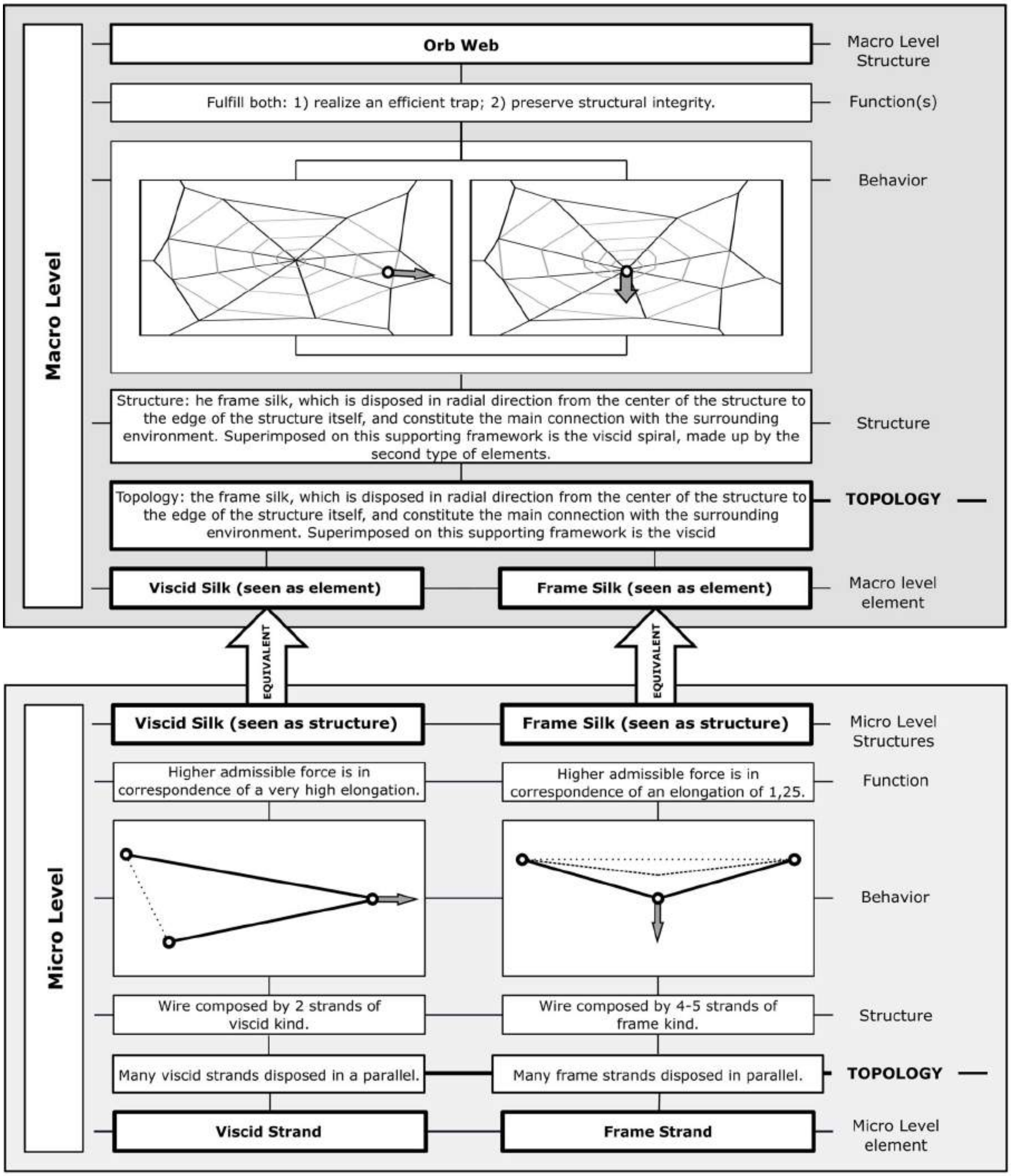

Figure 2. FBS multilevel decomposition framework applied to the orb web example.

that a certain procedure, in order to identify the "ideal layout" for the structure, changes the topological class of the initial design space, generally adding or subtracting a certain quantity of material. According to multilevel design, topological optimization can be rethought adding a new optimization at a deeper level of detail.

Usually, in FEM, the "continuity" of mechanical (thermal, fluidic, etc.) parameters inside the elements is ensured by the use of the shape functions. Using the multilevel approach, the elements themselves can be thought as structures, and their mechanical (thermal, fluidic, etc . . .) behaviour may be set in a more accurately way. This offers the possibility to operate a topology optimization at many different dimensional levels, so that the way the elements at "lower" dimensional scales are "built" affects the global performance of the main design.

In literature, many techniques of structural optimization have been developed, depending on the formulation of the problem, the domain (continuum or discrete structures), the optimization algorithm (gradient based or non-gradient based methods), and number of the objectives (single or multi objective). Anyway, a very first discriminating factor in topological optimization is the choice between macro-structural or micro-structural strategy [3]. Such initial setting is not secondary, because it may deeply affect the final result.

The main methodologies used in topological optimization are three (even if there are many other variants):

- **SIMP** (Simplified Isotropic Material with Penalization) is the most implemented methodology to create a topology based on the structural analysis [1],[20]. The fundamental principle behind its use requires a density design variable dependent material constitutive law that penalizes intermediate density material in combination with an active volume constraint. According to it, the load capacity of the structure is progressively taxed more for the intermediate densities than solid and void densities. The synergy between the penalization of intermediate densities and the resource constraint leads to solid–void structural design.
- **BESO** (Bi-directional Evolutionary Structural Optimization) method has acquired great success to solve topology optimization problems in different areas of structural engineering but also to design microstructures for materials [12]. It aims at simultaneously adding or removing elements from the finite element model of the structure. All of the BESO schemes that have been introduced so far apply the idea of ground structure, in which its elements cover the whole design domain including solid and void regions. The BESO turns these elements on and off, but keeps the record of their geometrical information through the whole optimization procedure [10], and introduces a new methodology for solving engineering problems related to the design of materials.
  In both SIMP and BESO algorithms, a filter may be applied after the sensitivity analysis in order to produce a well-defined topology for the resulting structure.
- **Level set method** is employed for tracking the motion of the structural boundaries under a speed function and in the presence of potential topological changes. An explicit jump immersed interface method is used for computing the solution of the elliptic problem in complex geometries without using meshes. The approach is also an evolutionary one. The principal idea is to remove material in regions of low stress and to add material in regions of high stress. A removal rate is established representing a percentage of the maximal initial stress below which material may be eliminated, and above which material should be added. The removal rate determines the closed stress contours along which new holes are cut and also the velocity of the boundary motion [16],[19].

Looking at the literature on structural optimization for microscopic applications, there is the evidence that it is necessary to use composite materials (i.e., microstructure) to improve the mechanical characteristics of the optimized structure. However, even if the optimizers usually generate well-defined geometry at macroscopic level [1],[3], a microstructural approach should be taken into account.

Algorithms implemented in FEM software are based on constitutive equations (e.g., for linear analysis of structures they are the equations of elasticity), which are not valid at every dimensional level, because, at lower levels, for example, the hypothesis of isotropic elastic continuous material fails. This means that, moving from a level to a lower one, the mathematical model changes. This could be a limit of our iterative method, unless the new physical model is considered.

Despite the evidence that the optimization of a structure means even an optimization at lower dimensional scales, at the moment, all the main commercial software providing topological optimization are based on variants of the SIMP density based method, and mostly aim at producing well defined results, where (isotropic) material and void are well separated at macroscopic level.

Another limitation is the so-called Pareto approach to multiple task optimizations: for the nature of the algorithms itself, after a certain level of optimization of the structure, it is impossible to optimize a certain goal without worsening another objective.

Multilevel approach can implement a strategy differently from the Pareto one to fulfil multiple objectives optimization. In fact, it could be possible to assign the optimization of different parameters to different levels of details. For example, if the compliance optimization is obtained defining the macro-topology of the structure, an independent optimization of the frequency response may be obtained working at lower level of structural material.

4. Proposal for a multistep topological optimization process

As it had been already stated, it is well known that the better performances for structural purposes are obtained realizing objects, which, at a microscopic level, are constituted by elements (or cells), which are not isotropic, so that the singular elementary unit may globally have, along the three principal directions, different values of Young and Poisson moduli [1],[3]. Furthermore, the orientation of the single constitutive elements may lie along the principal direction for stress and strain, which have been obtained by the macroscopic analysis.

According to the multilevel philosophy for the topological optimization, the main idea is to involve different optimization strategies at different dimensional levels in order to take into account the evolutionary trend of the systems, which imposes the reordering of material at different hierarchical levels (Topology). Figure 3 portrays the architecture of the proposed solution. The multilevel approach will be implemented in an algorithm, which will provide a first structural optimization over a certain work space; in a second step, the "survived" elements of the resulting structure are considered as structural cells, and their design will be defined in a second optimization step.

In other words, the purpose of this research work is to develop a method in order not only to characterize the direction and the mechanical characteristic of the constitute microstructures, but also their topology optimization. Once the mechanical parameters for an elementary cell have been defined, it is possible to apply various strategies defining the structure of the cells themselves. In some homogenization methods, the main topology of the elementary structures is already defined using square cells or more complex microstructures [2],[17].

The idea is to perform a further topological optimization (at micro level) to define the best microstructure topology of the constitutive elements (figure 3).

In principle, the procedure can be extended to lower dimensional levels even if this requires further considerations.

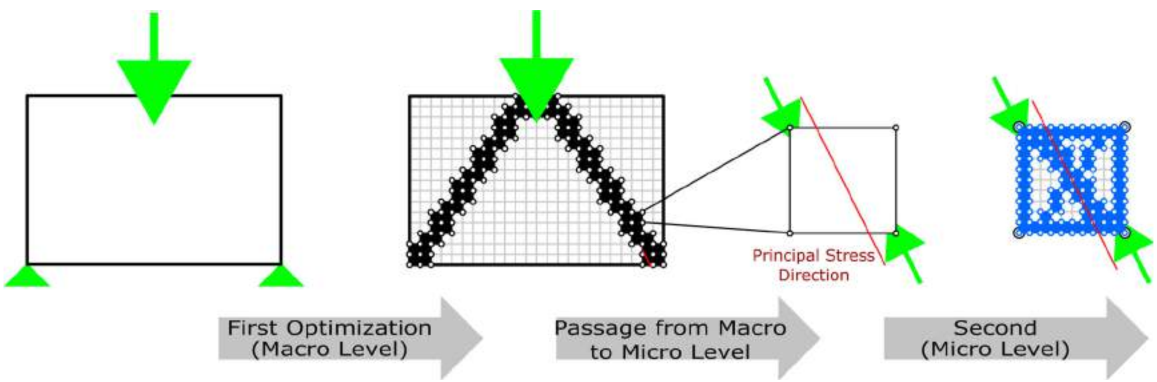

Figure 3. Two-level Optimization.

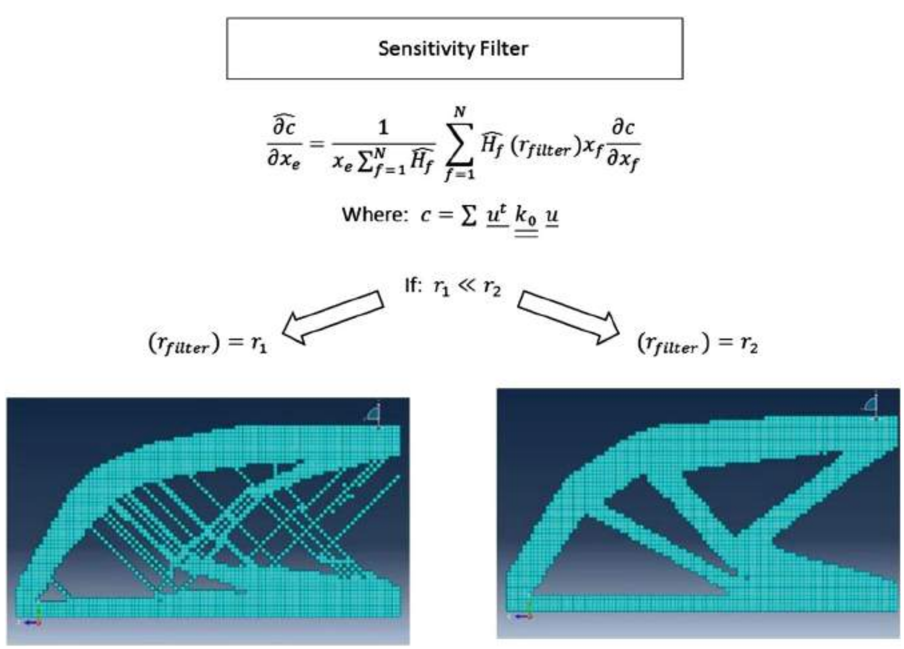

Figure 4. Influence of the radius of the sensitivity filter on the topology of the resulting structure.

4.1. First level optimization (macro level)

It is well known that there is a multitude of feasible methods in order to realize a structural optimization.

The first step (at macro level) is the topological optimization of the structure using a hard kill method, for example the BESO, as discussed in [21]. Anyway, BESO is not the only feasible algorithm.

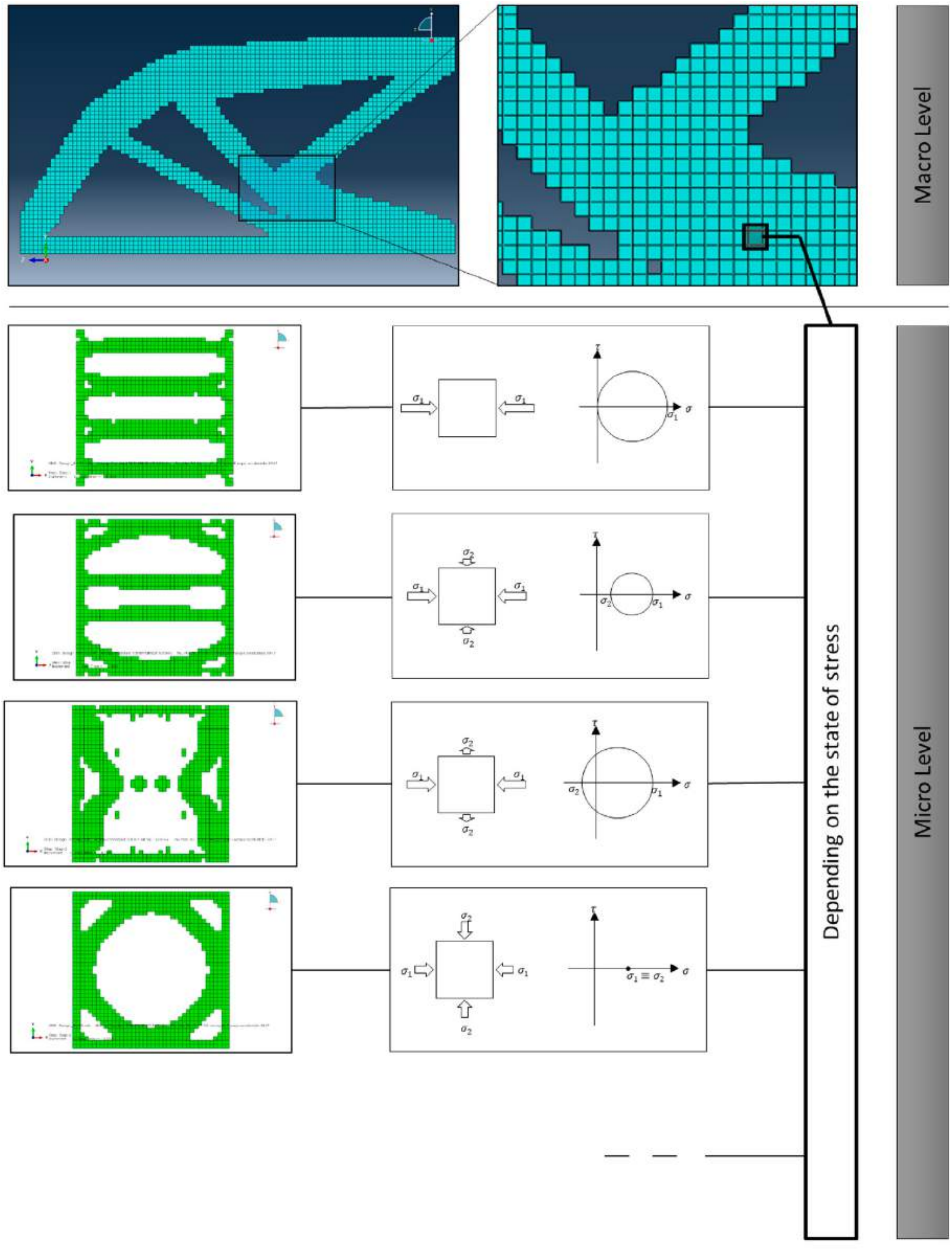

Figure 5. Examples of feasible topologies at micro-level for the elements of the macro structure.

Since we are considering a “global” optimization at macroscopic level, all methods, which define macro-zones of material-void, can be used; for example, the Level Set method could be an alternative as well.

Anyway, BESO method is suitable to produce high quality designs [7], in particular if coupled with a mesh independency filter (sensitivity filter) with a relatively low number of iterations. In fact, the main features of such algorithm are the production of high quality topology solutions, an excellent computational efficiency, simplicity in understanding and implementing.

The main strategy of BESO is to add or suppress the elements in the structure, depending on the value the objective function (for instance the compliance) assumes in the correspondence of the elements. The function that allows the algorithm to decide where there must be material or not, is the sensitivity function. It is typical of the “gradient based methods”, and indicates if the addition or subtraction of material promotes the optimization of the objective function.

Anyway, such “raw” sensitivity is normally modified for many reasons. One of these reasons is due to the “mathematical” tendency of the optimization methods to redesign the initial continuous workspace in a solid provided of “checkboards” zones. Usually, a filter is applied to avoid the creation of such microstructures, in favour of more “smooth” topologies. Briefly, this is obtained by calculating the sensitivity not as property of a single element, but as average between elements standing at a certain distance from the given element, as shown in figure 4. Depending on the radius of the filter, the “microstructural” nature of the result will be more or less enhanced.

After a first distribution of material, the second step consists in applying the SIMP (Solid Isotropic Microstructure with Penalization) method.

4.2. Second level optimization (micro level)

In the first step a BESO algorithm is adopted to manage the creation/elimination of the elements, coupled with the FEA analysis. Moreover, it has been applied the low-pass filter to reach a less fragmented structure (a mean sensitivity with a high filter radius), even if our purpose is to realise a microstructural solid. This means that the second configuration depicted in figure 4 is more suitable for our purposes.

At this point, a second level of optimization can be applied to the “survived” elements. The main idea is to realize the best sub-structure to fulfil the “requirement” depending on the state of tension. For each solid element, which is the output of the first step, an ideal topology is identified according to the stress configuration, or, in other words, to the direction of the principle tensions and their moduli.

Micro-structured cells will be realized, but, differently from the other existing algorithms, such schema is implemented after a first step of optimization in accordance with the ontological framework presented in the section 2.

From macro to micro level scale, the mechanical (but not only) characteristics of the structure are set, element by element, to fulfil the objective function. Such “continuous” variation of the element characteristics is realized by the structure of the element itself at the lower dimensional scale.

For this purpose, an object-oriented language is ideal in order to organize the different levels of information. Stresses, strains, deformations, and so on, may be arranged in different data structures, which ideally belong to different dimensional levels. Moreover, as further development, it may be possible to create custom “conditions” at the different levels.

The described procedure is visualized in figure 5. An example of optimized geometry for a beam is presented, where there is a first raw optimization, and the defined main macrostructure. The microstructure of the constitutive elements is identified considering the stress configuration for all the “survived” ones. As an example, some possible microstructural topologies are presented together with the corresponding stress configuration.

5. Conclusions

This paper analyses the multilevel concept and its use in different research fields. On this basis, we extended this concept to topological optimization with the aim to develop a new design methodology integrating both optimization and multilevel approaches. The underlying idea is to drive the designer to a more aware and efficient design activity and a choice of materials more suitable to her/his needs, also thanks to the possibility to realize products with complex structures using new production techniques, such as additive manufacturing.

A new ontological schema for describing multilevel design is proposed, according to the already existent mono level FBS framework and integrating new entities like topology.

In order to optimize the topology according to the multilevel perspective, we introduced a new methodology consisting in the use of a multi-step topological optimization.

At present, the proposed framework has been investigated using as a biomimetic example (i.e., the spider web orb) and a simple case study (i.e., a beam). The implementation of the methodology is still under development with the main goal to provide the designer with a next generation optimisation tool.

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