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New Simplified Inverse Kinematics Method for 5-Axis Machine Tools

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ABSTRACT

A novel method to physically understand the inverse kinematics of a 5-axis machine tool is presented based on visual observation and manually jogging the machine to the handshake position where the tool and a real material Cutter Location (CL) vector merge. This creates a better physical understanding and can also be used to verify the mathematical formula in the exact computations. The Denavit Hartenberg (DH) method is applied to the 5-axis machine and analyzed. Finally, two new simplified methods are developed that minimize the number of frames and simplifies the computations with minimal number of parameters.

Keywords: 5-axis, inverse kinematics, post processor, CL Data, DH method.
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1 INTRODUCTION

The 5-axis machine is viewed as two cooperating robots, one carrying the tool and one carrying the workpiece. The motion of the two robots must be such that the two robots “handshake”. The “handshake” is defined as coincidence of the tool vector and tooltip coordinate with the CL vector and CL point coordinate. Many studies classify 5-axis milling machines into three basic categories [1]. By embedding a coordinate frame, using homogenous transformations matrix (HTM) and D-H convention, we can describe the relative position and orientation between these coordinate frames. D-H [2] proposed a method which reduces the number of basic HTM transformations. D-H parameters to procure the direct kinematics are discussed by [3]. D-H method applied on the 5-axis machine tool is presented by [4]. [5] Implemented the D-H convention to describe the geometry of the milling machine. The normal D-H notation uses only four parameters [6]. A modified D-H notation using five parameters has been discussed by [7]. The machine kinematics of four-half axis systems was presented using the modified D-H notation [8]. [9] modified the D-H notation to enhance the flexibility of the location of basic joint types, links, and cutting devices. A geometric postprocessing method that does not require the forward and inverse kinematics equations has been discussed by [10]. [11] presents the forward kinematics and closed form solution. A cutter kinematic chain using D-H conventions is proposed by [12]. Author [13] classified the possible kinematics structure of five-axis machine tools.

In this paper, we propose a simplified inverse kinematics method to physically understand the inverse kinematics of a 5-axis machine tool, based on visual observation and manually jogging the machine to the handshake position where the tool and a real material CL vector merge. The first method is based on orienting the required coordinate systems in the same direction as the machine tool axes and minimizing the numbers of required coordinate frame to compute the inverse kinematics. The second method simplifies further the computations of the machine rotations based on the geometrical approach. Tab. 1 gives a review of the existing kinematic models. The paper is organized as follows: An overview is given in this section. New conceptual interpretation is proposed in the next section, and Analysis is presented in the section after. Then, simplified inverse kinematics methods based on minimizing numbers of required coordinate frame to compute the inverse kinematics and computations of the machine rotations on the geometrical approach are outlined. A conclusion is drawn at the end.

Kinematic Model developed by Authors Ref [ ]	**Type of 5-axis machine Tool used**	**No of Frames applied for transformation**	**No of Frames applied for transformation using D-H**	**No of Frames applied for transformation without using**	**Number of Transformation Matrix**	**Total number of Parameters**	**Approach for Post-processing**	Outcome of Kinematic Model
[1]	Mikron UCP 600 (XYZAC)	6	-	-	5	20	Inverse Kinematics	A general postprocessor has been implemented using the D-H method and used for position milling.
[8]	Huron KX8 (XYZAC)	6	-	-	5	20	Forward Kinematics	An optimal post-processing module has been tested for validation on many different complex surfaces.
[6]	5-Axis CNC Mill (XYZAC)	-	6	-	5	25	Inverse Kinematics	The generalized kinematics model is implemented for hybrid parallel serial 5-axis postprocessor.
Proposed method by authors	Maho600E (XYZAB)	-	-	4	3	5	Inverse Kinematics	A novel method to physically understand, to minimize the number of required frames and to compute the machine rotations.

Table 1: A list of the existing kinematic models using D-H convention.

2 NEW CONCEPTUAL INTERPRETATION OF INVERSE KINEMATICS OF MACHINE TOOL

From Fig. 1, the machine tool can be seen as two cooperating robots. One robot is carrying the work piece and one robot is carrying the tool. The motions must be such that the positions of the tool and work piece are according to the CL file. It is well known that this inverse problem is difficult and has many possible solutions for the same cutter location. It is important to realize that the post processor should select one of these solutions based on certain constraint that include the travel limits, collision avoidance and tool path length minimization.

2.1 Visual Inverse Kinematics Experiment

An experiment that the authors recommend every 5-axis machine operator, programmer and student is described below. A single real physical CL vector is placed on the machine as shown in Fig. 1 with the machine in the chosen reference position (all machine axes on zero). The machine is than moved in manual jog mode until the CL vector and the tool vector coincide as shown in Fig. 2. The position of the machine shown on the CNC control will then be one of the solutions of the inverse kinematics for the physical CL vector. The accuracy will be poor (+/- 1mm; +/- 1°) as this is done through visual observation but more than accurate enough to later on verify the equations.

2.1.1 Machine Tool Reference Position

At the start of the CNC program the machine is in a precise defined reference. It is assumed here that the display of the machine shows the coordinates for all slides position x_{ref}, y_{ref}, z_{ref}, A_{ref} and B_{ref}. (Best all on zero or X=Y=Z=A=B = 0). A typical situation like this is shown in figure 1. The centerline of the tool intersects with the apparent intersection of the A and B axis. All this centerlines are mutually perpendicular.

2.1.2 CL Vector

The tool and a typical CL vector are also shown in Fig. 1. Notice that the CL vector here is a real physical arrow put on a sample work piece .The authors have often used a piece of foam (polystyrene) as work piece with a pencil pierced inside as CL vector for a fast demonstration. The required accuracy for this "experiment" is +/- 1mm and +/- 1° as this is sufficient for the purpose of checking and verifying the postprocessor equations after the experiment.

2.1.3 Workpiece Coordinate System

Before the work piece is fixed to the machine it is important to physically draw the work piece coordinate system on the work piece and measure the location and components of the CL vector in this coordinate system. Note that the workpiece coordinate system does not have the same orientation as the machine axis. Y_1 is opposite Y_4; X_1 is opposite to Z_4 and Z_1 is opposite Z_4.

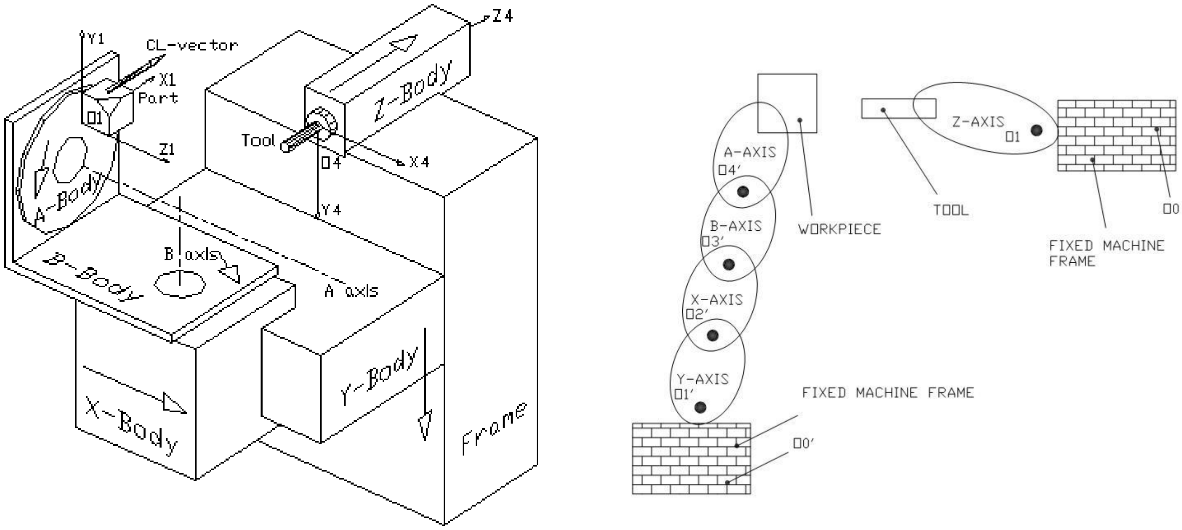

Figure 1: Workpiece Coordinate System O_1 and Machine Coordinate System O_4 in the Reference Position and corresponding Kinematic Link Diagram.

2.1.4 Machine Tool Coordinate System

The above work piece with the physical CL vector is then fixed to the machine. The tool length is also measured relative to the spindle nose. The next step is to physically identify a point as machine origin. This origin should be fixed to the floor or machine frame. Often it is not clear where this should be. This point is selected on the spindle reference point O_4. There many possible choices but here an easy and tangible choice is the best. Later it will be shown that there are choices for the machine coordinate system that are better for simplifying the equations.

2.1.5 Jog Mode Inverse Kinematics

2.1.5.1 Rotations

The machine tool is started in manual jog mode once the tool and work piece are fixed on the machine and the machine is in the reference position. The first step in the manual jog mode is to rotate the rotary axes until the CL vector is in the same direction as the tool axis. O_2 is a reference frame with the origin on the A centerline and in the same orientation as the machine tool axes it is not needed during the experiment but it is convenient here to illustrate different A rotation solutions. The O_3 reference has the same purpose but with the origin on the B centerline. These frames O_2 and O_3 are not needed in the experiment.

The CL vector should point from the tool tip towards the inside of the tool. The accuracy to align tool and CL vector by only rotary axes motion can be done by visual inspection. The accuracy of +/- 1 degree is sufficient. The resulting rotation angles of the machine rotary axes can be read from the machine tool control display. These values are one of the possible inverse kinematics solutions for i, j, k -> A,B. After this first experiment it is easy to identify the other possible solutions to transform i, j, k -> A,B. A_1 and A_2 rotate the CL vector towards the tool (positive z). A_3 and A_4 rotate the CL vector away from the tool (negative z). The next rotation B rotates the CL vector in the same direction as the tool.

2.1.5.2 Translation

Once the CL vector is aligned with the tool axis it is possible to make the CL point x, y, z coincide with the tooltip by moving the machine in x, y and z in jog mode (Fig. 2). Again only visual accuracy is sufficient. The resulting values of A, B, x, y, z displayed on the machine tool control are one of the solutions to the inverse kinematics with an accuracy of 1 degree and 1 mm. By repeating, all feasible inverse kinematics solutions can be obtained by experiment. This set of solutions that can be used to verify the correctness of the inverse kinematics transformation equations. The Fig. 3 shows the possible inverse kinematics solutions for the rotation. The solutions for the A rotation are 4. The CL vector can be rotated in the plane perpendicular to the B axis (horizontal) by rotations A_1 and A_2 as show in Fig. 3.

The CL vector can be rotated in the opposite direction of the tool vector also in two ways A_3 and A_4. It is clear that at the end of the above experiment we have 4 possible A solutions and two possible B solution B_1 and B_3, as B_2 and B_4 are larger than π. There are two corresponding X, Y and Z motions one set for A_1 and A_2 and one for A_3 and A_4.

2.2 General Format of the inverse kinematics transformation equation of a multi-axis machine tool

Two sets of equations are necessary. The first set are the equations that transform the CL data unit vector components i, j, k into the corresponding machine tool rotations. This first set of equations is independent of the translations as was demonstrated by above experiment (for 5-axis machines with orthogonal rotation axes). The second set is equations to compute the machine translations. To physically understand these equations, it is good to use the “handshake” concept. The first handshake is between the CL vector i,j,k and the tool vector (i=0,j=0,k=1 for Fig. 1). The CL vector direction must coincide with the tool vector T direction after the rotations on the machine tool. This can be written as

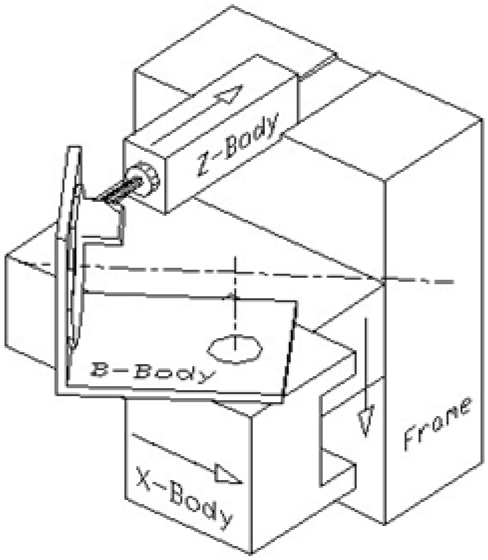

Figure 2: Handshake Position after moving in A,B, and X,Y,Z.

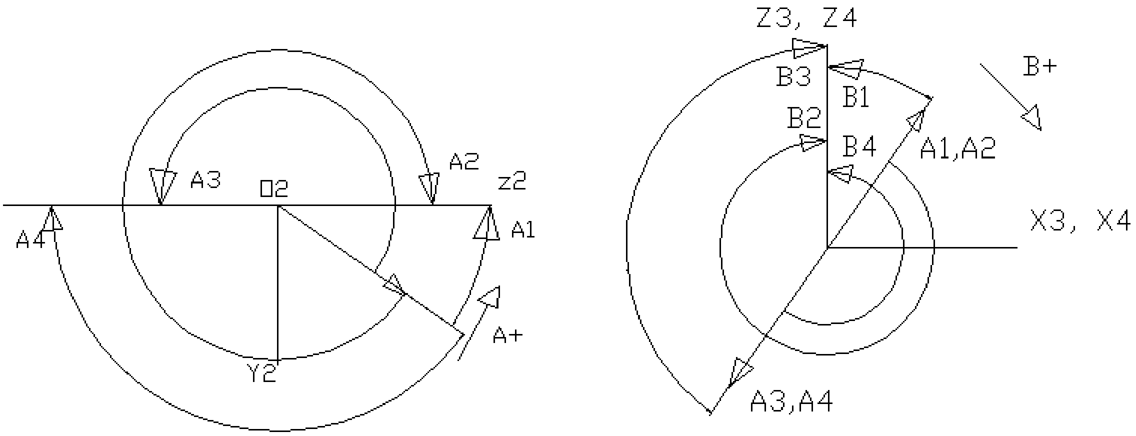

Figure 3: A and B rotations to rotate the CL vector in horizontal and vertical plane.

$$T = [Brot[Arot[CL]]]. \tag{1}$$

This will give three equations to compute the value of the two machine rotations. The rotations are often written in matrix form. In most multi-axis machine tool, the axes of rotations are parallel to the translations of the machine. There are however machine where the rotation axes are not parallel to the translations [13]. The formula for the rotation around and axis with an arbitrary orientation exist and could be used [14]. It is however not recommended to use this approach as it is very easy to make errors in signs and references. The best approach is to decompose all the rotations in rotations around axes parallel to the current coordinate system. The benefit of this approach is that it is not required to use formula from books as it can be done with simple easy rules that can be checked for correctness in a straightforward way (see section 2.2.2). After the solution for the machine tool rotations are found it is possible to compute the required machine translations. The translations and rotations that move the CL vector and the tool vector are applied. The motions must be such that the CL coordinate x,y,z coincides or "handshake" with the tool tip (tool reference point). For the machine in Fig. 1 the handshake equations are

$$[Ztrans\ [T]] = [Xtrans\ [Ytrans\ [[Brot\ [Arot[\ CL]]]]] \quad (2)$$

This will give three equations to compute the required machine translations. The kinematic link diagram (Fig. 1) shows the corresponding motions and sequence of motions to be applied to the CL vector and Tool vector

2.1.1 Choice of coordinate systems

It is clear that when a body is rotated that the coordinate system used to compute the coordinates must be fixed to another reference body relative to which the rotation is performed. The ISO standard recommends the use of a right handed orthogonal Cartesian coordinate system. However on many machines this positive reference is not always adhered to. The best is to use the reference that coincides with the concerned specific machine tool. An easy rule to get the correct equations for rotations around the x, y and z axis of a Cartesian is given below.

2.2.2 Rotation of angle C around the z-axis

In this case it is clear that only the x and y coordinate will change The point X,Y,Z will be transformed in the point X(C), Y(C), Z(C). The formula is always of the following form

$$X(C) = X \operatorname{Cos}[C] + Y \operatorname{Sin}[C];\ Y(C) = Y \operatorname{Cos}[C] - X \operatorname{Sin}[C];\ \ Z(C) = Z \quad (3)$$

The coordinate corresponding to the rotation axis before and after rotation does not change. The two other coordinates are multiplied by the cosine and sine of the angle of rotation. The new coordinate after rotation is always equal to the same corresponding coordinate before multiplied by the cosine and the other coordinate multiplied by the sine of the angle with a plus or minus sign. The required sign for the sine term can easily be found by imagining a small positive rotation and observe what will happen to the corresponding coordinate. In case the corresponding coordinate decreases the sine term must have the minus sign. The other sine term for the second equation must in this case be positive. The same as above applies for rotations around the y-axis and the x-axis.

3 ANALYSIS OF 5-AXIS MACHINE TOOL KINEMATICS

3.1 Denavit Hartenberg Method

The DH method [2] [14] is a very general method for the kinematic analysis of a robot or mechanism. It can be observed the tool is moved by the z-axis and the workpiece is move by Y and X linear motions and A and B rotations (Fig. 1).

3.1.1 DH Links and Joints

A link is defined as a rigid solid body. Looking at Fig. 1 we see that there are the following links: Frame, Y-body, X-body, B-body, A-body in the workpiece kinematic chain. The workpiece can also be considered as a separate body. The Frame, Z-axis and tool are the links in the tool kinematic chain. Following the DH method we start numbering the links from the frame onwards. The frame is numbered 0 the next links in sequence 1, 2… n. Fig. 4 shows the numbered links in the two kinematic chains. The links are connected by joints. The first joint from the frame is numbered 1 and the next in sequence shown in the Fig. 4.

3.1.2 DH Coordinate System Assignment

To each joint we will assign a right handed Cartesian coordinate system in the following way: The z-axis is aligned with the joint axis. For a linear motion we align the z-axis with the direction of motion. For a rotational motion we align the z-axis along the centerline of the rotation. The positive direction can be chosen arbitrary. The origin of the z-axis can be chosen anywhere in the case of a linear motion. In the case of a rotational degree of freedom the origin should be on the centerline of the rotation. The Fig. 4 shows a possible assignment of the joint z-axis. The z-axis of the frame coordinate system is oriented along the first joint axis. As the 5-axis machine consists of two kinematic chains starting with joint axis along different orientations we define two coordinate system fixed to the frame, O_0 for the tool side, O_0' for the workpiece side. The x-axis of the coordinate systems is oriented pointing to the next z-axis and perpendicular to it. The x axis of the workpiece coordinate system (x_5') should be perpendicular to the z_4' axis or the centerline of the A axis. The y-axis direction for each coordinate system is found by the right hand rule.

3.1.3 DH Parameters for Joint Axes and Transformation Matrices $^{i-1}A_i$

The position of each coordinate system relative to each other is now characterized by 4 parameters.
a_i: distance between z_{i-1} and z_i (this value is always positive because the way the x-axis is defined in the DH method)
d_i: Coordinate in oi of the intersection of the x_{i-1} axis with the z_i axis
α_i: The angle over which the z_{i-1} axis must be rotated around x_i to make it coincide with z_i .
θ_i: The angle over which the x_{i-1} axis must be rotated around z_i to make it coincide with x_i .
The Fig. 4 shows the resulting coordinate systems and the Fig. 5 presents the values of the parameters for the first joint.
The joint parameter that will change during the machine tool motions for the case of Fig. 1 are
Workpiece side
$d_{1'}$ -> Y translation; $d_{2'}$ -> X translation; $\theta_{3'}$ -> B rotation; $\theta_{4'}$ -> A rotation
Tool side
d_1 -> Z translation
The Tab. 2 and Tab. 3 summarize the parameters in the DH matrices.

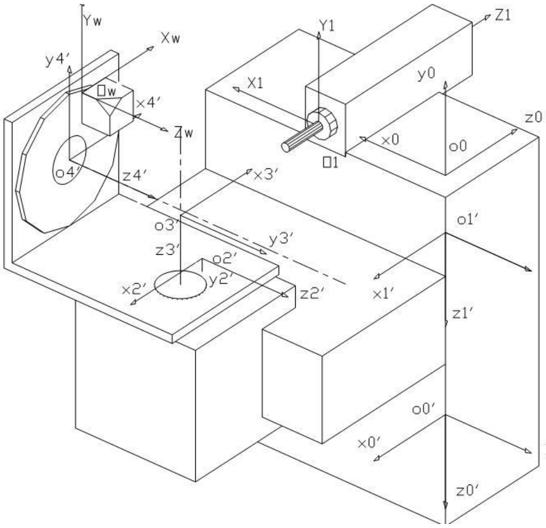

Figure 4: DH coordinate systems.

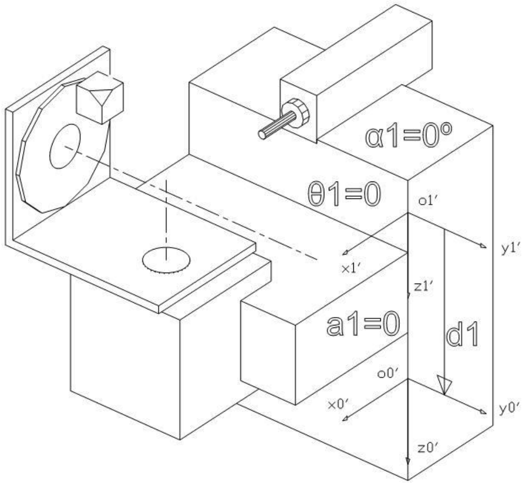

Figure 5: DH Joint Parameters Joint.

X, Y, Z. A and B are the translations and rotations of the machine relative to the reference position. There are two more constant transformations that in general are not having the same parameters as the DH parameters in Tab. 2-3. The matrix ${}^{4'}A_{5'}$ Transforms the workpiece coordinate system to the coordinate system O_4'. The matrix ${}^{O'}A_O$ transforms the frame coordinate system used for the tool kinematic chain O_0 to the frame coordinate system for the workpiece kinematic chain O_0'. In the above Tab. 2-3, we have set some of the constant parts of the parameter a_i and d_i to zero. Taking this into account we have the coordinate systems 0, 0′, 1′, 2′, 3′ coincide at the point O_3' on the B-axis centerline.

i	a_i	d_i	α_i	θ_i
1′	$a_1' = 0$	$d_1' + Y$	$\alpha_1' = 0°$	$\theta_1' = 0°$
2′	$a_2' = 0$	$d_2' + X$	$\alpha_2' = 90°$	$\theta_2' = 0°$
3′	$a_3' = 0$	$d_3' = 0$	$\alpha_3' = 90°$	$\theta_3' = 180° + B$
4′	$a_4' = e$	$d_4' = 0$	$\alpha_4' = 90°$	$\theta_4' = 180° + A$

Table 2: Parameters for workpiece kinematic chain.

i	a_i	d_i	α_i	θ_i
1	0	$d_1 + Z$	$\alpha_1 = 0°$	$\theta_1 = 0°$

Table 3: Parameters for tool kinematic chain.

The coordinate system 4′ is offset by the value e in the direction of x_3' and lies on the centerline of the A rotation. This eccentricity is a systematic error and must be included in the transforms. We could also have moved the workpiece coordinate system but it was not done because of the common use in practice of a workpiece coordinate system and a tool length controlled by the user. The surface of the machine tool table π and the spindle nose surface ψ are reference surfaces and it is convenient to assign a coordinate system plane to coincide with it. There is however no theoretical objection to move the workpiece coordinates system to O_4' and the tool coordinate system to the point O_3'.

The matrix ${}^{O'}A_O$ =

0	0	-1	0
-1	0	0	0
0	-1	0	0
0	0	0	1

The matrix ${}^{4'}A_{5'}$ =

1	0	0	${}^{O4'}X_{O5'}$
0	1	0	${}^{O4'}Y_{O5'}$
0	0	1	${}^{O4'}Z_{O5'}$
0	0	0	1

The parameters (e, ${}^{O4'}X_{O5'}$, ${}^{O4'}Y_{O5'}$, ${}^{O4'}Z_{O5'}$) that are now left must be determined very accurately. Some of the parameters will not change if the reference position is not changed such as the values e and d_1. The values in the matrix ${}^{4'}A_{5'}$ will change every time when the origin of the workpiece is changed. The orientation of the workpiece coordinate system was selected in the same direction as the coordinate system O_4'. If this is changed the matrix ${}^{5'}A_{4'}$ will also include constant rotations and will include more parameters then above case. The coordinate of the tooltip z_1 will also change when another tool or tool length is used. The other tooltip coordinates x_1 and y_1 will be zero. The tool

vector will be having the constant components $i_1=j_1=0$ and $k_1 = 1$. This is because the tool axis is always along the Z-axis of the machine tool for the machine in Fig. 1.
The solution of the above inverse kinematics problem is the most important part of a 5-axis machine tool postprocessor.
The coordinates from the system i-1 can be transformed in the coordinates of the system i by following transformations:
The i-1 coordinate system is translated over the distance di along the z_{i-1} axis.
The i-1 coordinate system is then rotated over the angle θi around the z_{i-1} axis.
The coordinate system is now translated over the distance a_i along x_i .
Next the coordinate system is rotated α_i around x_i
The result of the above sequence of transformations is often represented as ${}^{i-1}A_i$

$${}^{i-1}AT_i = T(d_i , z_{i-1})\, T(\theta_i ,z_{i-1})\, T(a_i ,x_i)\, T(\alpha_i ,x_i) \quad (4)$$

To transform the coordinates from a point in the coordinate system i to i-1 we multiply by the homogenous matrix ${}^{i-1}A_i$.

3.1.4 DH Method Complications for 5-Axis Machines

The above application of the DH method is very systematic and lead to the correct solution but it is tedious and error prone. For example the DH parameter a_i is always positive because the rule for the DH method is that the xi axis should be perpendicular to the z_{i+1} axis and z_i axis pointing towards z_{i+1}.. This could lead easily to an error in the case of the 5-axis machine (Fig. 1) where the eccentricity between the A and B axes centerlines will always be positive if the DH rule is used. However the direction of the x_2 axis will change depending on the relative position of these two centerlines. As this eccentricity is function of the specific machine instance it will lead to different DH coordinate systems.

4 SIMPLIFIED INVERSE KINEMATICS METHODS

The first method is based on orienting the required coordinate systems in the same direction as the machine tool axes and minimizing the numbers of required coordinate frame to compute the inverse kinematics. The second method simplifies further the computations of the machine rotations based on the geometrical approach.

4.1 Machine Axes Oriented Handshake (MAO-Handshake) Method

We propose now to orientate the joint axes of the two rotary axes along the centerlines of these rotations following the rule: The coordinate axis along the centerline of the rotations is having the same name as the machine tool axis that is parallel with it. The origin of these axes is chosen at the endpoints of the normal between the two centerlines. The other axes are oriented in the same direction as the machine tool coordinate system In general this rule will result in two orthogonal Cartesian coordinate systems with the origins on the rotation axis centerline. For the case of the machine in Fig. 1, we obtain the Cartesian coordinate systems O_2 on the A-axis centerline and O_3 on the B-axis as shown in the Fig. 6. The workpiece coordinate system can have any orientation but we make the coordinate axes planes parallel to the machine axes planes. In the origin of the workpiece coordinate system, we introduce a coordinate system O_1. The origin O_1 coincides with the workpiece coordinate system origin, however the O_1 axes are oriented along the machine tool coordinate system. At this stage we have 4 coordinate systems O_1, O_2, O_3 and O_4. The last coordinate system O_4 is the machine coordinate system O_4 that we can fix anywhere in space but fixed to the machine frame. A good choice is to locate the origin at point O_3 when the machine is in the reference position (another choice is on the reference surface of the spindle and on the centerline of the spindle when the machine is in the reference position) When the machine slides move all the coordinate systems will move except the machine coordinate system O_4.

With above coordinate systems is possible to derive the inverse kinematic equations of the machine. Example, the case of the machine in the Fig. 1, Based on the kinematic link diagram, the set of axes that moves the workpiece and the tool can be determined. We compute the position of the CL Point and CL vector after the rotations A and B and the translations X and Y. We also compute the position of the tooltip coordinate and the tool vector after the translation Z. After this the tool tip, tool vector, CL point and CL vector must coincide. This will give us 6 equations that allow us to compute the required slide motions. The three relations between the tool vector and CL vector are independent of the linear motions. Solving this equation first will give us all the solutions for the machine tool rotations.

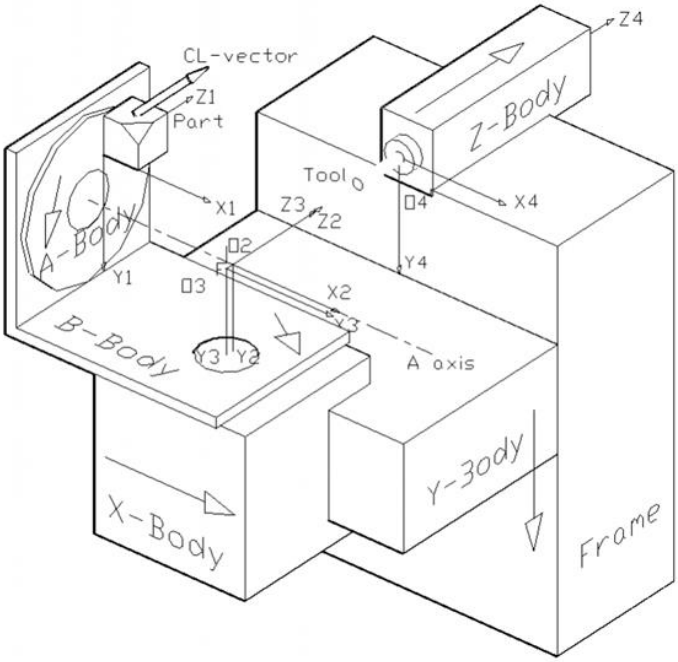

Figure 6: Coordinate Systems O_1, O_2, O_3 and O_4 Oriented as Machine Axes Directions.

Rotation A around X_1 axis of the CL vector

$i_1(A) = i_1;\; j_1(A) = j_1 \,Cos[A] - k_1 \,Sin[A]\;; \quad k_1(A) = k_1 \,Cos[A] + j_1 \,Sin[A]$ (5)

Coordinate transform O_1 to O_2 (as this is a translation it has no effect on the vector) So

$i_2 = i_1(A)\;;\; j_2 = j_1(A)\;;\; k_2 = k_1(A)$ (6)

Rotations B around Y2 of the CL vector

$i_2(B) = i_2 \,Cos[B] + k_2 \,Sin[B]\;;\; j_2(B) = j_2\;;\; k_2(B) = k_2 \,Cos[B] - i_2 \,Sin[B]$ (7)

As the tool kinematic links do not include rotation in this case the tool vector is constant.

$i_T = 0;\; j_T = 0\;;\; k_T = 1$ (8)

As the tool vector and CL vector must coincide after A and B rotation we obtain

$i_T = 0 = i_2(B) = i_2 \,Cos[B] + k_2 \,Sin[B]; \quad j_T = 0 = j_2(B) = j_2;$

$k_T = 1 = k_2(B) = k_2 \,Cos[B] - i_2 \,Sin[B]$ (9)

After substituting

$i_1 \,Cos[B] + (k_1 \,Cos[A] + j_1 \,Sin[A]) \,Sin[B] = 0$

$j_1 \,Cos[A] - k_1 \,Sin[A] = 0$ (10)

$(k_1 \,Cos[A] + j_1 \,Sin[A])\; Cos[B] - i_1 \,Sin[B] = 1$

We get by solving above three equations for A and B

$A = ArcTan[j_1/k_1] + k\,\pi\;;\; B = -\,ArcSin[i_1] + 2k\pi$ (11)

4.2 Machine Axes Oriented Geometric Handshake (MAO-GEO Handshake) Method

The above results can also be obtained in a more straightforward geometric way.

As the rotations in the case of the machine tool (Fig. 1) are orthogonal and aligned with the machine coordinate axis we can obtain the same solutions as above by:
Rotate the tool vector in the horizontal plane X_3/Z_3. If we look in the vertical plane Y_3/Z_3 it can be observe that is can be done in 4 different ways with A smaller than 2π as shown (Fig. 3) Solutions A_1 and A_2 align the CL vector in the Z_3 direction. Solutions A_3 and A_4 align the CL vector with pointing in the opposite direction of the Z_3 axis. Once the CL vector is rotated in the horizontal plane by one of the above solution we can rotate over B to align the CL vector with the tool vector. The solutions are shown in Fig.3.

The rotation B_1 corresponds to the solutions A_1, A_2 and B_2 corresponds to the solutions A_3, A_4. This geometric interpretation is very straight forward and useful. In some case it is however not possible to use this geometric way to find the possible solutions for the inverse kinematics rotations. In the case of the machine tool with non-orthogonal axes it is not possible to use the geometric solution. The three equations for the rotation angles must be solved analytically.
Once the solutions for the rotations have been found it is possible to find the corresponding linear slide motions. In this case we need also to take the translations between the coordinate systems into account.
In the case of the machine in Fig. 1 we obtain:
Coordinate transform from O_1 to O_2
$x_2 = x_1 + x_{o1o2}$; $y_2 = y_1 + y_{o1o2}$; $z_2 = z_1 + z_{o1o2}$ (12)
Rotation A around X_1 axis of the CL Point
$x_2(A) = x_2$; $y_2(A) = y_2 \,Cos[A] - z_2 Sin[A]$; $z_2(A) = z_2 \,Cos[A] + y_2 Sin[A]$ (13)
Coordinate transform O_2 to O_3 (as this is a translation it has no effect on the vector)
$x_3 = x_2(A) + x_{o2o3}$; $y_3 = y_2(A) + y_{o2o3}$; $z_3 = z_2(A) + z_{o2o3}$; (14)
Rotations B around Y_3 of the CL vector
$x_3(B) = x_3 \,Cos[B] + z_3 Sin[B]$; $y_3(B) = y_3$; $z_3(B) = z_3 Cos[B] - x_3 Sin[B]$ (15)
The CL point has now been rotated over the angles A and B.
The translations Y and X also move the CL point. The resulting CL point position is:
$x_{4w}(Y,X,B,A) = x_3(B) + X$; $y_{4w}(Y,X,B,A) = y_3(B) + Y$; $z_{4w}(Y,X,B,A) = z_3(B) + z_{03O4}$ (16)
The tooltip will translate by Z
$x_{4T}(Z) = x_{4T}ref$; $y_{4T}(Z) = y_{4Tref}$; $Z_{4T}(Z) = z_{4Tref} + Z$ (17)
With our choice of the Machine tool coordinate system we have
$x_{4Tref} = y_{4Tref} = 0$
z_{4Tref} = Tooltip z coordinate (tool length) in O_4 (18)
The values of the machine translations X, Y and Z are found by solving the following equations:
$x_{4w} = x_{4T}$; $y_{4w} = y_{4T}$; $z_{4w} = z_{4T}$ (19)

5 CONCLUSIONS

A novel method to physically understand the inverse kinematics of a 5-axis machine tool was presented based on visual observation and manually jogging the machine to the handshake position where the tool and a real material CL vector merge. The 5-axis machine is viewed as two cooperating robots, one carrying the tool and one carrying the workpiece. Method is as precise as all other methods. However, the initial validation of the inverse kinematics equation is straight forward and simple, because values and signs of the machine coordinates are calibrated based on the visual experiment. The manual experimental method not only creates a better understanding but it is also used to verify the exact computation in the discussed methods above. After introducing the DH coordinate systems and minimizing the number of non-zero parameter the inverse kinematics solution was outlined and the analyzed. Finally, we proposed two new methods (MAO & MAO-GEO Handshake) that make the inverse kinematics as simple as possible improving the understanding and reducing to complexity. The first method is based on orienting the required coordinate systems in the same direction as the machine tool axes and minimizing the numbers of required coordinate frame to compute the inverse kinematics. The second method simplifies further the computations of the machine rotations based on the geometrical approach.

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