G^1 Hermite Interpolating with Discrete Log-aesthetic Curves and Surfaces

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Abstract. Log-aesthetic curve is a plane curve proposed as a curve with high quality curvature distribution. However, since the general expression is given in integral form, generation takes time, and there is a problem that drawing may not be possible for the given boundary condition depending on the shape parameter α. In this research, we implement the discretization of log-aesthetic plane curve and propose G^1 Hermite interpolation method based on the discretization to solve these problems. We also propose a method of generating discrete log-aesthetic surfaces and G^1 Hermite interpolation by extending the formulation of planar curves to surfaces.

Keywords: log-aesthetic curve, log-aesthetic surface, discretization , G^1 Hermite interpolation
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1 INTRODUCTION

Recently, aesthetic design which takes account of designability has become popular. In the aesthetic design, the creation of high quality curve and surface models is demanded. However, on current CAD systems, the operator must move control points by trial and error to obtain high-quality curves and surfaces. This incurs high costs and requires a great deal of expertise. Therefore, an efficient method to generate fair curves and surfaces is desirable to achieve high quality that will satisfy customers' aesthetic requirements.

"Aesthetic curves" were proposed by Harada et al. as curve whose logarithmic distribution diagram of curvature (LDDC) can be approximated by straight line. Miura et al. [3] derived analytical solution of the curves whose logarithmic curvature graph (LCG)- an analytical version of the LDDC is strictly given by a straight line and proposed these lines as general equations of aesthetic curves. For a given curve, we assume the arc length of the curve, the radius of curvature and slope of LCG are denoted by s and ρ and α, respectively. When $\alpha \neq 0$, one of the general equations of aesthetic curves is given by the following equation on 2D plane.

$$\rho^{\alpha} = cs + d \tag{1}$$

where, α, c and d are constants. In particular, α is the slope of LCG and a parameter for controlling the impression of the curve. Fig.1 illustrates log-aesthetic curves for various α values. Also, one segment of the log-aesthetic planer curve is uniquely determined by both the endpoints and tangent vectors there [2]. Hence, one can modify the log-aesthetic curve by changing these boundary conditions and α value. Since the log-aesthetic curve is defined by use of curvature as the above equation, its curvature distribution is smooth. In addition, it includes logarithmic (equiangular) spiral, clothoid, and circular involute as well as Nielsen's spiral.

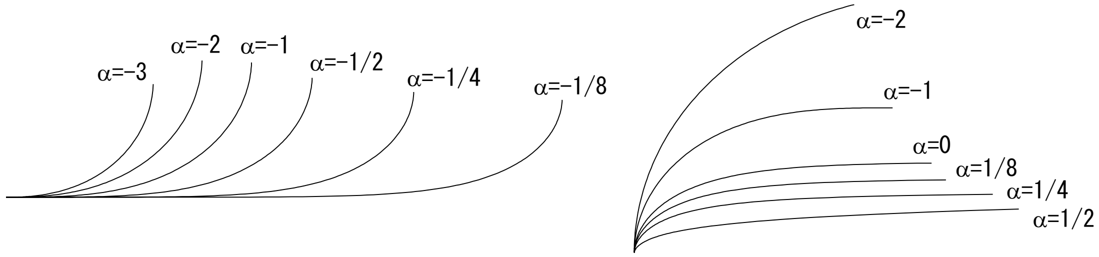


Figure 1: Log-aesthetic curves with various α's.

As a formulation of log-aesthetic surfaces, some surface formulas besides the minimum variation log-aesthetic surface have been proposed that generate free-form surfaces by sweeping the log-aesthetic curve [1, 6].Harada et al proposed the log-aesthetic curved surface [1]. It is defined as a sweeping surface using two profile curves, which are composed of log-aesthetic curves, and one guide line composed of a non-log-aesthetic curve. Saito et al. proposed the complete log-aesthetic surface [6]. It is defined as a pure sweeping surface with two log-aesthetic curves. This formulation also uses the log-aesthetic curve as the guide line and guarantees that all parametric curves are log-aesthetic. Suzuki et al proposed a new formulation of the minimum variation log-aesthetic surface(MVLAS) for scale-invariance and Parameterization-independence [8]. However, it takes time to generate these curves and surfaces.

In this research, in order to solve the problem, focusing on discrete curves that can be expected for high speed in generation, we propose discretization of log-aesthetic plane curves based on point sequence interpolation by discrete clothoid curves, and G^1 Hermite interpolating method that generates curves from end points and the tangential direction there. In addition, we extend the method used in the curves to the surfaces.

2 RELATED WORK

In plane case, Schneider et al proposed a algorithm to construct an interpolating closed discrete clothoid spline (DCS) purely based on its characteristic differential equation[5]. In the algorithm, firstly, initial polygon are specified and interpolation points are inserted between the polygon so as to interpolate between initial point sequences so that curvature change becomes monotonous. Furthermore, they extended the algorithm of planar clothoid splines to closed surfaces of arbitrary topology[5].

3 EULER-LAGRANGE EQUATION

The Euler-Lagrange equation in the variational problem is a partial differential equation that characterizes a functional, In this research, we transform the discrete curve and surface by this Euler-Lagrange equation. From the variational principle, the log-aesthetic curve satisfies $\rho^{\alpha} = cs + d$, so it is reformulated as a curve that minimizes the energy between two points in the space with arc length s on the horizontal axis and $\sigma = \rho^\alpha$ on the vertical axis[4]. Then, the functional of the log-aesthetic curve is given by Eq.2[7].

$$K_{LAC} = \int \sigma_s^2 ds \tag{2}$$

where σ_s is the derivative of σ with respect to s.
Therefore, the Euler-Lagrange equation is expressed as Eq.3.

$$\frac{d}{ds}\left(\frac{\partial \sigma_s^2}{\partial \sigma_s}\right) = \frac{d}{ds}(2\sigma_s) = 2\sigma_{ss} = 0 \tag{3}$$

Moreover,

$$\frac{d}{ds}(\kappa^{-\alpha}) = -\alpha\kappa^{-(1+\alpha)}\kappa_s \tag{4}$$

where κ is curvature.

$$\begin{aligned} \frac{d^2}{ds^2}(\kappa^{-\alpha}) &= \frac{d}{ds}(\alpha\kappa^{-(1+\alpha)}\kappa_s) \\ &= \alpha(1+\alpha)\kappa^{-(2+\alpha)}\kappa_s^2 - \alpha\kappa^{-(1+\alpha)}\kappa_{ss} \end{aligned} \tag{5}$$

Hence, we obtain the following expression.

$$\begin{aligned} (1+\alpha)\kappa^{-(2+\alpha)}\kappa_s^2 - \kappa^{-(1+\alpha)}\kappa_{ss} = 0 \\ (1+\alpha)\kappa_s^2 - \kappa\kappa_{ss} = 0 \end{aligned} \tag{6}$$

When $e_i = 1, 2(min, max)$ as a unit principal direction vector and $\sigma = (\rho^i)^{\alpha_i}, \sigma_i^i = \frac{d\sigma^i}{de_i}$, the functional K_{LAS} of MVLAS is defined as Eq.7 by extending the functional (Eq.2) of the log-aesthetic curve to surfaces with respect to the principal curvature [8].

$$\begin{aligned} K_{LAS} &= \int \sum_{i=1}^{2} (\sigma_i^i)^2 dA \\ &= \int \sum_{i=1}^{2} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 dA \end{aligned} \tag{7}$$

On the other hand, the unit principal direction vector e_i is given by Eq.8 when the eigen vector that corresponding to e_i is (ξ_i, η_i).

$$\hat{e}_i = \frac{\partial S}{\partial u}\xi_i + \frac{\partial S}{\partial v}\eta_i \tag{8}$$

From Eq.8, the principal direction differential $\frac{d\kappa_i}{de_i}$ of the principal curvature becomes as shown in Eq.9, paying attention to the fact that the principal direction is a unit vector.

$$\frac{d\kappa^i}{de_i} = \frac{1}{\sqrt{E}}\frac{d\kappa^i}{du}\xi_i + \frac{1}{\sqrt{G}}\frac{d\kappa^i}{dv}\eta_i \tag{9}$$

Moreover, $\xi_1 = \eta_2$ are 1, $\xi_2 = \eta_1$ are 0, when put $E = G = 1$ and consider curvature line coordinates s, t. Hence Eq.7 will be following equation.

$$\begin{aligned} K_{LAS} &= \int\int \sum_{i=1}^{2} \frac{1}{g_{ii}} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 \sqrt{EG - F^2} dsdt \\ &= \int\int \sum_{i=1}^{2} (\alpha_i (\kappa^i)^{-(1+\alpha_i)} \kappa_i^i)^2 dsdt \\ &= \int\int \sum_{i=1}^{2} (\sigma_i^i)^2 dsdt \end{aligned} \tag{10}$$

where, $F = 0$ because the principal directions are orthogonal to each other. From Eq.10, we obtain the following Euler-Lagrange equation.

$$\sum_{i=1}^{2} \sigma_{ii}^i = 0 \tag{11}$$

$$\sum_{i=1}^{2} \left((1 + \alpha_i)(\kappa_i^i)^2 - \kappa^i \kappa_{ii}^i\right) = 0 \tag{12}$$

Schneider et al defined Discrete Clothoid Spline as a curve where the discrete curvature satisfies the following equation[5].

$$\Delta\kappa_i = \kappa_{i-1} - 2\kappa_i + \kappa_{i+1} = 0 \tag{13}$$

κ_i can be updated by following equation.

$$\kappa_i = \frac{\kappa_{i-1} + \kappa_{i+1}}{2} \tag{14}$$

Moreover, Schneider et al extended this theory of the curves to surfaces, considering a one-ring at the vertex, and updated the inner vertex of the triangular mesh by Eq.15.

$$\kappa_i = \frac{1}{6} \sum_{l=1}^{6} H_{i,l} \tag{15}$$

Likewise, the curvature of the discrete log-aesthetic surface is updated the vertex of the Quadrilateral mesh from the following expression.

$$\sum_{j=1}^{2} \sigma_i^j = \frac{1}{8} \sum_{j=1}^{2} \sum_{l=1}^{8} \sigma_{i,l}^j \tag{16}$$

Hence, when $\alpha_1 = \alpha_2 = -1$,

$$\sum_{j=1}^{2} \sigma_i^j = \sum_{j=1}^{2} \rho_j^{-1} = \kappa_1 + \kappa_2 = 2H \tag{17}$$

since Eq.16 is twice the average curvature, the result of the optimization agrees with the case of Schneider.

4 PROCESSING PROCEDURE

We show the flow chart of our method in Fig.2 and explain its procedures in detail as below.

1. In curve case, input the start point, the end point, the tangent vector at there, α, and number of subdivision. In surface case, input the surface that to determine the boundary conditions, α, and number of subdivision.
2. While keeping the boundary conditions to satisfy G^1 continuity, put the initial points.
3. In order to maintain G^1 continuity, optimize the position of the target vertex by minimizing the objective function while fixing two points from the start point and end point respectively. Implement the optimization until the convergence condition is satisfied.
4. Repeat processing for the specified number of subdivision, and output the curve (or mesh) when finished.

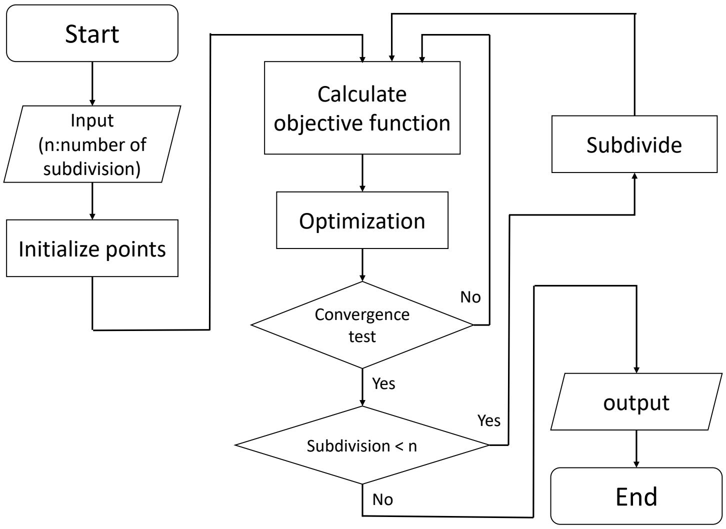


Figure 2: Processing procedure.

5 G^1 HERMITE INTERPOLATING WITH DISCRETE LOG-AESTHETIC CURVES

Schneider et al define an initial point sequence as $P = \{P_1, ..., P_n\}$, and defined a point sequence $Q = \{Q_1, ..., Q_m\}$ as a discrete clothoid curve when the point sequence Q including P satisfies Eq.18, Eq.19 [5].

$$\|Q_{i-1} - Q_i\| = \|Q_i - Q_{i+1}\|, \quad Q_i \notin P \tag{18}$$

$$\Delta^2 \kappa_i = \kappa_{i-1} - 2\kappa_i + \kappa_{i+1} = 0, \quad Q_i \notin P \tag{19}$$

Discrete curvature κ_i at Q_i is expressed by Eq.20.

$$\kappa_i = 2\frac{\det(Q_i - Q_{i-1}, Q_{i+1} - Q_i)}{\|Q_i - Q_{i-1}\| \; \|Q_{i+1} - Q_i\| \; \|Q_{i+1} - Q_{i-1}\|} \tag{20}$$

In this research, in order to generate aesthetic curves we applied shape parameter α to the Eq.19. Hence, the Eq.19 will be the following equation.

$$\Delta^2 \kappa_i^{-\alpha} = \kappa_{i-1}^{-\alpha} - 2\kappa_i^{-\alpha} + \kappa_{i+1}^{-\alpha} = 0, \quad Q_i \notin P \tag{21}$$

Curve shape is updated by iterative calculation so as to satisfy the definition of the above equations. Start processing from the initial shape Q_0 of the 0th step, update the point Q_i^{k+1}(Fig.3) that is newly placed in the $k+1$th step $Q^k \rightarrow Q^{k+1}$ using Eq.22.

$$Q_i^{k+1} = \frac{1}{2}\left(Q_{i+1}^k + Q_{i-1}^k\right) + tR\left(\frac{\pi}{2}\right)\left(Q_{i+1}^k - Q_{i-1}^k\right) \tag{22}$$

where $R(\theta)$ is rotation matrix and t is parameter to determine amount of movement.

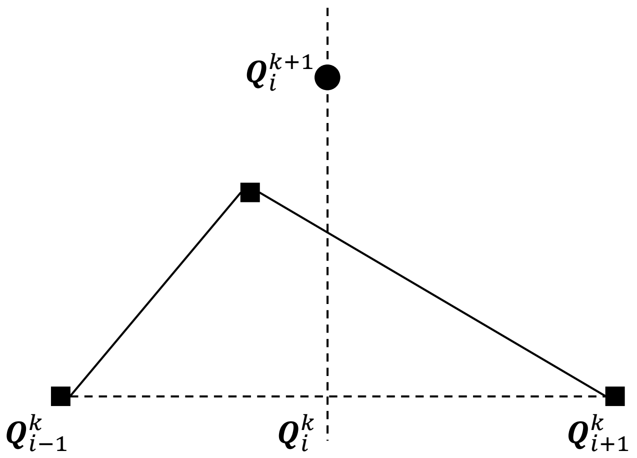


Figure 3: Updating vertex.

5.1 Determination of Initial Shape

G^1 Hermite interpolation generates curves that satisfy their boundary conditions, specifying both endpoints of the curve and the tangent direction there. The start and end points of the curve P_s, P_e and the unit tangent vector there T_s, T_e are input. Then, as shown in Fig.4, initial points P_1, P_2, P_3 are defined such that the distance between each point is l.

$$P_1 = P_s + lT_s \tag{23}$$

$$P_3 = P_e - lT_e \tag{24}$$

$$P_2 = P_1 + l\frac{P_3 - P_1}{\|P_3 - P_1\|} = P_3 - l\frac{P_3 - P_1}{\|P_3 - P_1\|} \tag{25}$$

where l is Eq.26, $dP = P_e - P_s, sT = T_s + T_e$, and θ is angle between dP and sT.

$$l = \frac{\|sT\| \cos\theta - \sqrt{4 - \|sT\|^2 \sin^2\theta}}{\|sT\|^2 - 4} \|dP\| \tag{26}$$

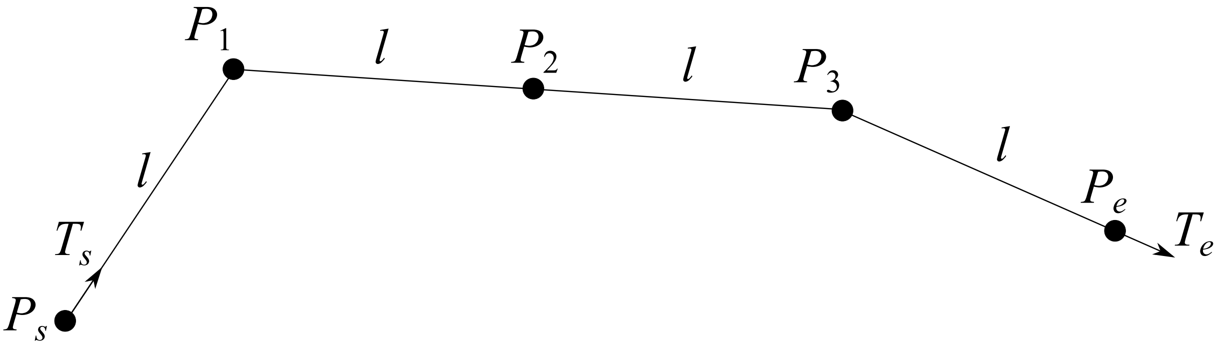


Figure 4: Initial shape of curves.

5.2 Optimization

First, interpolate the initial polygon P by the point sequence Q_i^0. The newly arranged Q_i^{k+1} is a vertical bisector of Q_{i-1}^k and Q_{i+1}^k, and is expressed as Eq.22. t is calculated by following equation.

$$t = \frac{-\tilde{\kappa}_i^{k+1} \left\|Q_i^k - Q_{i-1}^k\right\| \left\|Q_{i+1}^k - Q_i^k\right\|}{2\left\|Q_{i+1}^k - Q_{i-1}^k\right\|} \tag{27}$$

Calculate the discrete curvature $\kappa_s^{-\alpha}, \kappa_e^{-\alpha}$ at start point and end point respectively. Moreover, calculate the arc length l_c, and the slope a_κ of the $l_c - \kappa^{-\alpha}$ diagram is determined as follows.

$$a_\kappa = \frac{\kappa_e^{-\alpha} - \kappa_s^{-\alpha}}{l_c} \tag{28}$$

In order to maintain G^1 continuity, two points at both end points are fixed and updated. The curvature κ after movement is

$$\kappa = \left(\kappa_s^{-\alpha} + s a_\kappa\right)^{-\frac{1}{\alpha}} \tag{29}$$

where s is the arc length from start point to Q_i^j

6 G^1 HERMITE INTERPOLATING WITH DISCRETE LOG-AESTHETIC SURFACES

6.1 Subdivision Method in Discrete Log-aesthetic Surface Generation

In this research, discrete Log-aesthetic surface is expressed by square grid mesh. In order to define the surface uniquely, we use G^1 continuity at the boundary, and input the four boundary lines(NURBS) and tangent vectors that direct to the inside of the surface at the boundary lines. Also, as in the case of curves, the surface is subdivided and optimization of inner vertices is performed at each division step. Here, when the mesh grid consists of $m \times m$ vertices, the interior vertices refer to the $(m-4) \times (m-4)$ vertices from which the boundary and the vertex inside one of the boundaries have been removed. Fig.5 shows the subdivision of the discrete log-aesthetic surface. Here, n is the number of divisions.

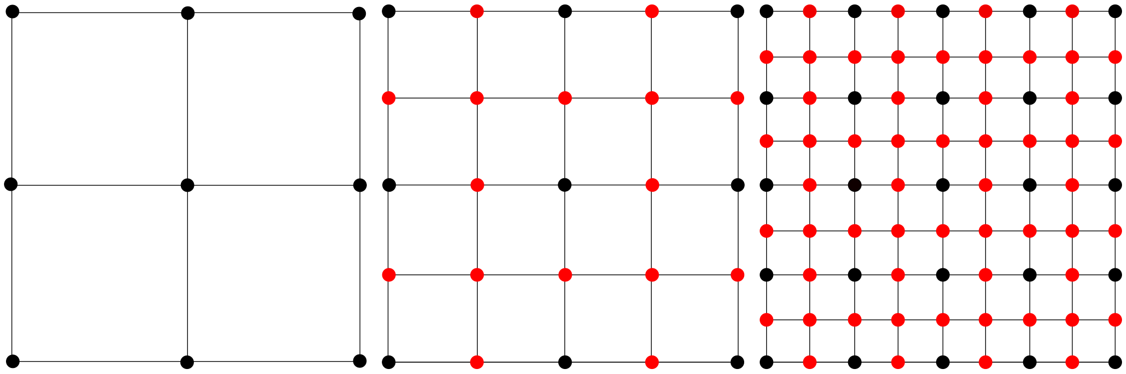


Figure 5: Subdivision of the discrete log-aesthetic surface Left:initial polygon, Middle:n=1, Right:n=2.

6.2 How to determine the initial coordinates of new vertices in each subdivision step

The determination of the initial coordinates of the new vertex is performed in the order of the boundary vertex, the tangent vector designation vertex, and the inner vertex. For a new boundary vertex, the coordinates on the boundary curve of the corresponding position are acquired, and the position is taken as the position coordinate of the mesh vertex. For new tangent specification vertices, the vertices are classified into the following four types, and position coordinates are determined sequentially from the outside of the mesh.

- Type 1: Tangent vector designation vertices at the four corners correspond to type 1. Let $P_{i,j-1}$ be the boundary vertex in the u direction, $P_{i-1,j}$ be the boundary vertex in the v direction, and T_u, T_v be the unit tangent vector to the inside of the surface at each point. Moreover, the closest points of the two straight lines $P_{i,j-1} + t_u T_u, P_{i-1,j} + t_v T_v$ with t_u, t_v as a parameter are calculated, and the middle point is taken as the coordinate value of the new vertex $P_{i,j}$.
- Type 2: Type 2 corresponds to a boundary vertex of the neighborhood and a tangent vector designation vertex when the inner vertex is an existing vertex. Here, a positive tangent vector designation vertex in the u direction is described as an example. Let a adjacent bounding vertex be $P_{i,j-1}$, a tangent vector to the inward direction at the bounding $P_{i,j-1}$ be T_u, a vertex at four corner side be $P_{i-1,j}$, and a vector $(P_{i,j-1} + P_{i,j+1})/2 - P_{i-1,j}$ be T_v. Moreover, the closest points of the two straight lines $P_{i,j-1} + t_u T_u, P_{i-1,j} + t_v T_v$ with t_u, t_v as a parameter are calculated, and the closest point on the straight line $P_{i,j-1} + t_u T_u$ side is taken as the coordinate value of the new vertex $P_{i,j}$.
- Type 3: Type 3 corresponds to the case where all adjacent four neighbors are new tangent vector designation vertices. Here, a positive tangent vector designation vertex in the u direction is described as an example. Let a adjacent bounding vertex be $P_{i,j-1}$, a tangent vector to the inward direction at the bounding $P_{i,j-1}$ be T_u, a vertex at four corner side be $P_{i-1,j}$, and a vector $(P_{i-1,j-1} + P_{i+1,j-1} + P_{i+1,j+1} + P_{i-1,j+1})/4 - P_{i-1,j}$ be T_v. Moreover, the closest points of the two straight lines $P_{i,j-1} + t_u T_u, P_{i-1,j} + t_v T_v$ with t_u, t_v as a parameter are calculated, and the closest point on the straight line $P_{i,j-1} + t_u T_u$ side is taken as the coordinate value of the new vertex $P_{i,j}$.

- Type 4: Type 4 corresponds to the center of tangent vector designation vertices in each direction. Here, a tangent vector designation vertex in the u direction is described as an example. Let a adjacent bounding vertex be $P_{i,j-1}$, a tangent vector to the inward direction at the bounding $P_{i,j-1}$ be T_u, a negative neighbor vertex in the u direction be $P_{i-1,j}$., and a vector $(P_{i-1,j} + P_{i+1,j})/2 - P_{i-1,j}$ be T_v. Moreover, the closest points of the two straight lines $P_{i,j-1} + t_u T_u, P_{i-1,j} + t_v T_v$ with t_u, t_v as a parameter are calculated, and the closest point on the straight line $P_{i,j-1} + t_u T_u$ side is taken as the coordinate value of the new vertex $P_{i,j}$.

The parameters t_u, t_v for each type above are calculated using the following formula.

$$t_u = \frac{T_u dP - (T_v \cdot dP)(T_u \cdot T_v)}{1 - (T_u \cdot T_v)^2} \tag{30}$$

$$t_v = \frac{-T_v \cdot dP + (T_u \cdot dP)(T_u \cdot T_v)}{1 - (T_u \cdot T_v)^2} \tag{31}$$

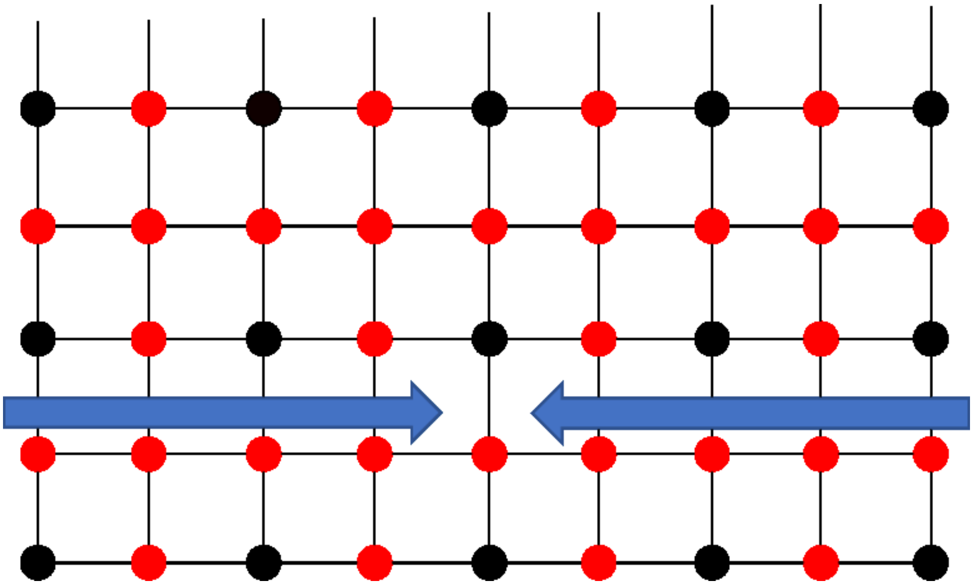


Figure 6: Determine tangent vector designation vertex coordinates.

For new inner vertices, we classify the vertices into the following two types and assign vertex coordinate values.

- Type 1: Type 1 corresponds to the case where two of the four neighboring vertices are existing vertices. Let the average coordinates of two existing vertices be the coordinate values of the new vertex.
- Type 2: Type 2 corresponds to the case where not all vertices in 4 neighborhoods are existing vertices. Determine the coordinates $P_{i,j}$ of the new vertex by $P_{i,j} = (P_{i-1,j-1} + P_{i+1,j-1} + P_{i+1,j+1} + P_{i-1,j+1})/4$.

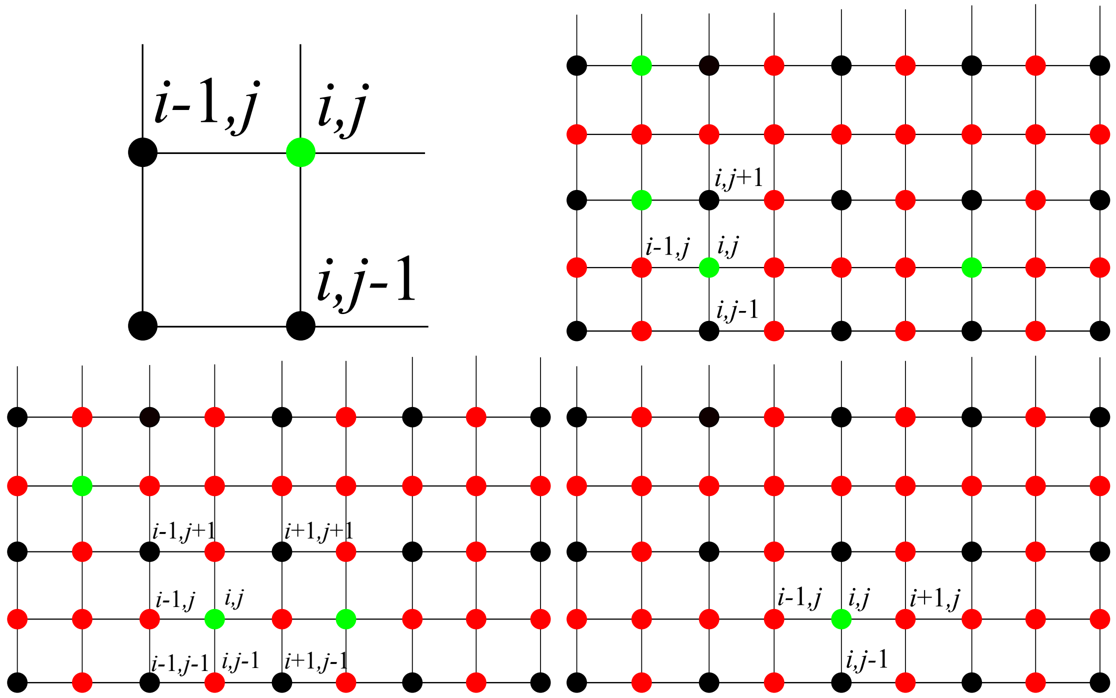


Figure 7: Class of new tangent vector designation vertices(n=2) Black:existing vertices, Red:new vertices, Green:same type vertices, Upper left:type1, Upper right:type2, Lower left:type3, Lower right:type4.

6.3 Optimization

Updating vertices is based on Eq.12 and update the curvature of discrete log-aesthetic surface by following equation.

$$\sum_{j=1}^{2} \sigma_i^j = \frac{1}{8} \sum_{j=1}^{2} \sum_{l=1}^{8} \sigma_{i,l}^j \tag{32}$$

when $\alpha_1 = \alpha_2 = -1$,

$$\sum_{j=1}^{2} \sigma_i^j = \sum_{j=1}^{2} \rho_j^{-1} = \kappa_1 + \kappa_2 = 2H \tag{33}$$

Moreover, calculate the value of ti in Eq.34 below and update the vertex coordinates.

$$Q_i^{k+1} = g_p(Q_i^k + t_i \vec{n}_i^k) \tag{34}$$

$$g_p(Q_i) = \frac{1}{8} \sum_{l=1}^{8} Q_{i,l} \tag{35}$$

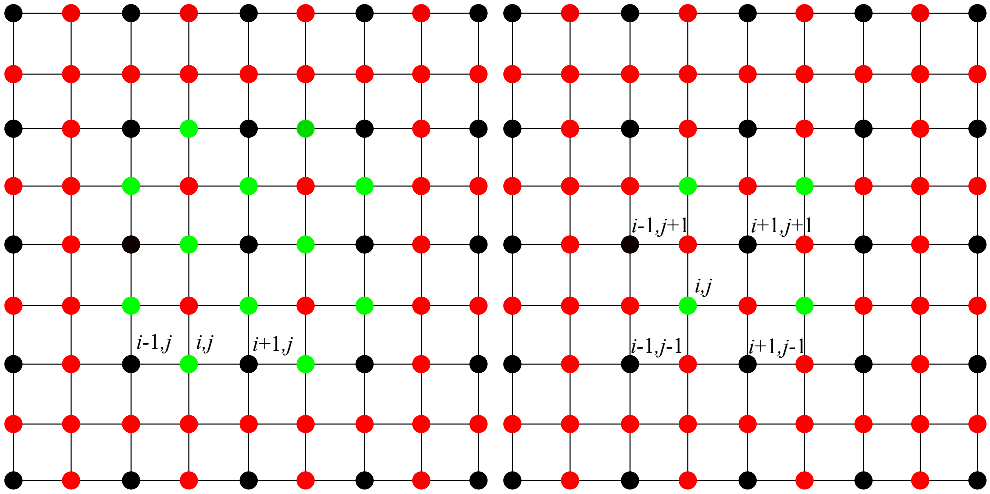


Figure 8: Class of new inner vertices(n=2) Black:existing vertices, Red:new vertices, Green:same type vertices,, Left:type1, Right:type2

$$g_p(H_i) = \frac{1}{8}\sum_{l=1}^{8} H_{i,l} \tag{36}$$

Since the calculated ideal mean curvature satisfies the Eq.37, the objective function F (Eq.39) can be obtained by using the Eq.38, so minimizing this objective function yields ti Find the value.

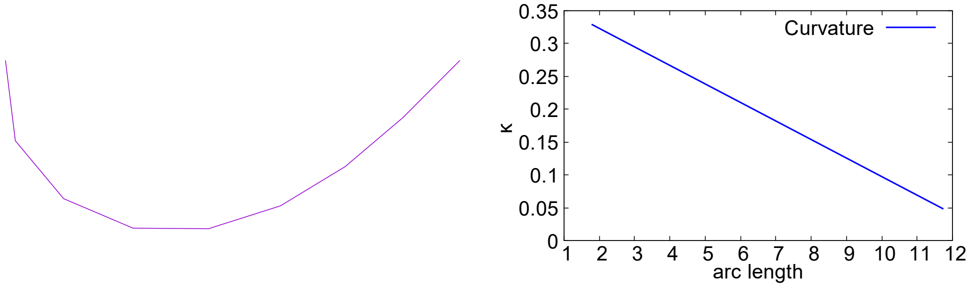
$$\tilde{H}_i^{k+1} = g_h(\tilde{H}_i^{k+1}) \tag{37}$$

$$H_i^{k+1} = \tilde{H}_i^{k+1} \tag{38}$$

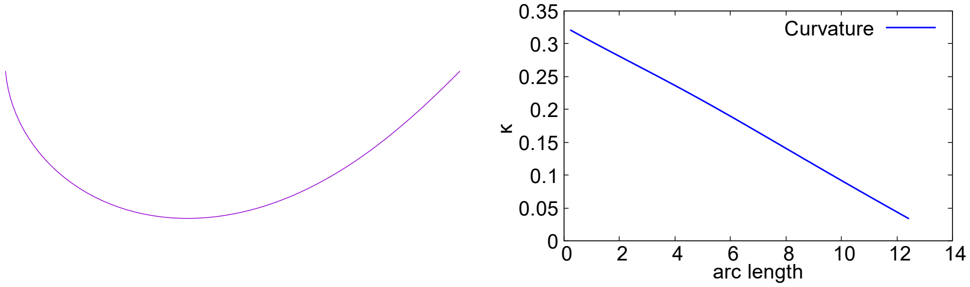
$$F = H(g_p(Q_i^k + t_i\vec{n}_i^k)) - \tilde{H}_i^{k+1} \tag{39}$$

7 RESULT

We show the results of G^1 hermite interpolating with log-aesthetic curve and surface in Fig.9, Fig.10 respectively. Moreover, Fig.11 shows mean curvature distribution of the surface and Fig.12 shows zebra map. For all the following examples, we used a PC with Core i7-8700 3.20GHz CPU. Fig.9 shows the generated log-aesthetic curve($\alpha = -1.0$) and the curvature distribution($\alpha = -1.0$). The number of subdivision is 2 and processing time is 11ms(Fig.9a). Also, 16ms when the number of subdivision is 5(Fig.9b). It can be seen from the curvature distribution that the curvature of the curve monotonically changes. In Fig.10, the α value in the u and v directions is -1.0 respectively and the processing time was 5.1s for 2 divisions and 39.9s seconds for 3 divisions. However, because it is far from the fast processing time required by CAD, it may be necessary to further speed up. Moreover, Fig.13 shows a car model created with the discrete log-aesthetic surfaces



(a) 2times subdivided curve



(b) 5times subdivided curve

Figure 9: G^1 Hermite interpolating with discrete log-aesthetic curve Left:generated curves Right:curvature distribution($\alpha = -1.0$).

proposed in this paper. The generation time of each surface in this model was about 0.004s on average. This result is a considerably short processing time as compared to the case of Fig.10 and Table 1. The reason is that the curves that constract the surface are simple. Table 1 shows the comparison of the processing time according to the difference in the number of divisions in the case of each of curves and surfaces. The rise width of the processing time in the case of curves is not large, but in the case of surfaces, the processing time is greatly increased due to the difference in the number of divisions.

8 CONCLUSIONS

In this research, we propose a G^1 Hermite interpolation method based on a discrete log-aesthetic curve aiming at speeding up curve generation based on point sequence interpolation based on discrete clothoid curve, and generated log-aesthetic plane curves. Also, by extending the method used on the curve to a surface, we propose

Figure 10: G^1 Hermite interpolating with discrete log-aesthetic surface Left:input Middle:2times Right:3times.

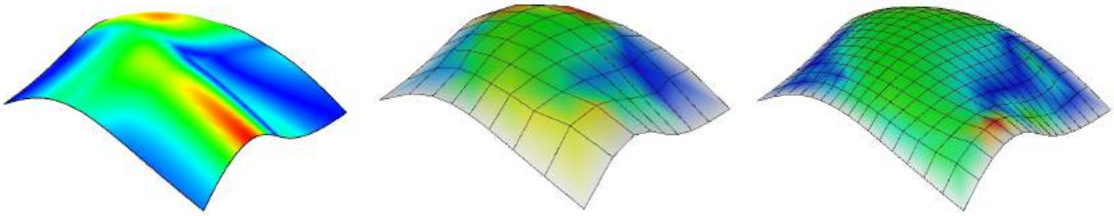


Figure 11: Mean curvature distribution Left:input Middle:2times Right:3times.

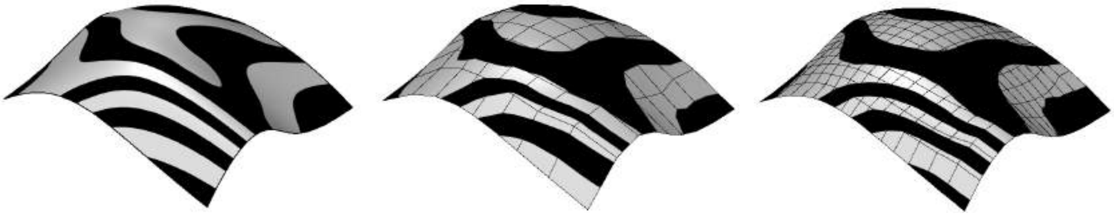


Figure 12: Zebra map Left:input Middle:2times Right:3times.

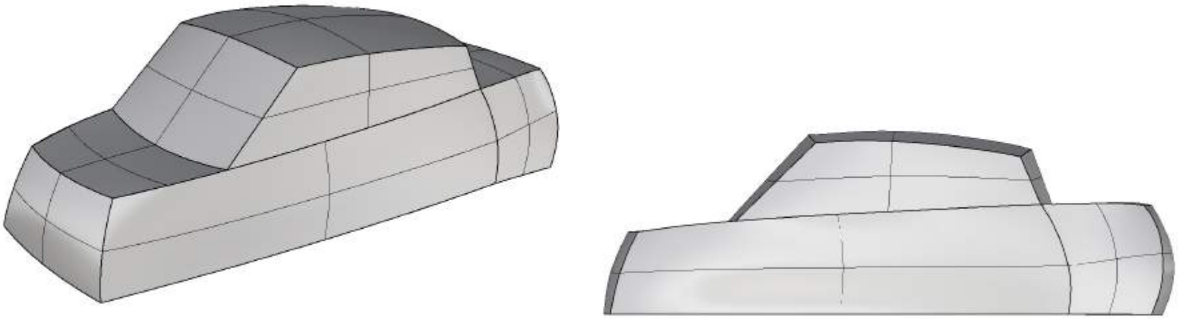


Figure 13: Car model composed of discrete log-aesthetic surfaces.

	2times	3times	5times
Curve	11ms	12ms	16ms
Surface	5.1s	39.9s	1229.3s

Table 1: Comparison of processing time by difference of division number

a G^1 Hermite interpolation method based on a log-aesthetic surface, and generated log-aesthetic surfaces. In the future, we would like to compare processing time and accuracy with existing methods, or create more practical examples.

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