



An Objective Weighting Method of Function Requirements for Product Design Using Information Entropy

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Abstract. Weight of function requirements (FRs) plays an important role in product design. Based on weights of FRs, the priority structure and parameters of a product are decided to meet requirements of the product function, cost and lifecycle. The existing methods in defining weights of FRs mainly rely on users' comments and experts' experience, which is subjective. This paper proposes an objective method of weighting FRs for product design using information entropy and benchmarking methods. Initial weights of FRs are assigned by an analytic hierarchy process (AHP) method. The weights of FRs are then improved for objective solutions based on the similarity of benchmarking products using information entropy. Final weights of FRs are decided by the least square method. The proposed weighting method of FRs is verified in a case study of designing an upper limb rehabilitation device. Results show that the proposed method has improved weighting accuracy of FRs in product design.

Keywords: Function requirements (FRs), Information Entropy, House of Quality (HoQ), Analytic Hierarchy Process (AHP), Rehabilitation device.

DOI: <https://doi.org/10.14733/cadaps.2020.966-978>

1 INTRODUCTION

Product design plays an important role in the product development to meet requirements of the product function, cost and lifecycle [17]. Quality function deployment (QFD) and benchmarking methods are commonly used in the product design process. QFD can transform customers' requirements into product function requirements, and then search design concepts of product structures and parameters considering the design priority [24]. The benchmarking method compares different product details to take existing products' advantages in the design solution [23]. By comparing different products' performances in the market, the best design solution from benchmarking products can be adapted in product design, which can reduce cost and improve quality of product design [18].

House of Quality (HoQ) is an analytical tool of the QFD method to transform customer requirements (CRs) into function requirements (FRs) and design specifications [22]. Using HoQ, weights of FRs can be decided with different scores based on relationships of FRs and CRs. Weighting FRs is a very important process to decide the design priority in applying HoQ [5]. Based on weights of FRs, design alternatives of structures and parameters are evaluated and selected in the conceptual search process using HoQ [16].

However, the existing methods of weighting FRs in QFD are subjective and inaccurate because the traditional weighting process uses subjective ranking methods, such as using 9 marks for the most important, 3 for moderation and 1 for weak, to assign weights to FRs [12]. Weighting FRs relies on customers' comments and experts' experience, which generates the subjective solution [6]. Some products such as rehabilitation devices and medical testing equipment are used to meet specific requirements of users. It is very difficult to find enough information in the survey to define CRs and importance rate of CRs accurately, which will affect the solution of weighting FRs [14]. In addition, users may not be able to fully provide their needs to a product, especially for some new products.

For weighting FRs accurately, a benchmarking method can be used for adjusting weights of FRs. Benchmarking products are selected from the best sales products in the market [15]. Through analysis of the benchmarking products, their common FRs can be identified for important rates of functions of the product [25]. If all the benchmarking products provide similar functions, the weight of a FR should be increased. All the benchmarking products with a same function means that this function is attractive and necessary. The weight of a FR should be decreased if this function is rare in the benchmarking products as this function seems less important.

To improve the accuracy and objectivity of weighting methods for FRs, an objective weighting method of FRs is therefore proposed in this research based on the information entropy theory in the evaluation of benchmarking products for FRs. The weighting FRs from the information entropy can adjust the initial weight of a final weight of FRs to eliminate subjectivity of the weighting solution. An upper limb rehabilitation device is designed in the case study based on weights of FRs decided by the proposed method. Performances of benchmarking products and product designed based on the proposed FR weights are compared to verify advantages of the proposed method.

Following parts of the paper are organized as follows. Literature review of the related methods is described in Section 2. An objective weighting method is proposed in Section 3. Section 4 discusses a case study of design for rehabilitation devices using different weighting methods to verify the proposed method, followed by the research conclusion and further work in Section 5.

2 LITERATURE REVIEW

The objective of this paper is improving objectiveness and accuracy in defining weights of FRs for product design. The existing methods for initial weights of FRs are reviewed to select the best method to define initial weights of FRs from experts. Weighting FRs by objective methods are also reviewed to improve objectiveness of weights of FRs.

2.1 Existing Methods for Initial Weights of FRs

Initial weights of FRs can be defined by Importance rates (IRs) of CRs and relationships between CRs and FRs, which is mainly based on the opinion of experts. Most of existing methods of defining initial weights of FRs are based on HoQ and decision-making methods. There are some improved HoQ methods proposed to select the most suitable method for defining initial weights of FRs.

Büyükközkın et al proposed an improved HoQ method using a fuzzy analytic network for the quality of the responsiveness to CRs and FRs [1]. Fung et al proposed an asymmetric fuzzy linear regression approach to estimate functional relationships for product planning based on HoQ [7]. Kuo et al applied a fuzzy group method to HoQ in product development planning to reduce the

vagueness and uncertainty in a group decision-making process, which balances the environmental acceptability and overall customer satisfaction [11]. Chuang et al combined the AHP method and HoQ to support a facility location decision from a requirement perspective, which defines weights of FRs by considering the relationship among all FRs [4].

The existing methods in defining initial weights of FRs are mainly used in the industrial product design. Khangura et al applied HoQ method in the design of a high quality and low cost refrigerator according to voice of customer [8]. Korayem et al improved the mechanical robot reliability and quality using HoQ and benchmarking methods [10]. Choi et al improved the design of smart cars for elderly users using a quantitative usability evaluation based on HoQ method [3].

As weight of a FR is an important factor of the design solution, relationships of all FRs should be considered in defining initial weights of FRs. After combining existing methods including HoQ and AHP, the initial weights of FRs can be defined by comparing the importance of FRs for each CR, which can be used as the first step in the proposed method. To reduce the subjective weighting solution based on the experience of experts, an objective weighting method is required to adjust the initial weights to improve the weighting accuracy of FRs.

2.2 Weighting FRs by Objective Methods

For objectiveness of weighting process of FRs in product design, different methods have been suggested such as information entropy, variance and covariance. The information entropy can be used in defining a weight based on a certain or uncertain factor of an event. Chen et al proposed an objective weighting method to solve multiple-attribute decision-making problems using an intuitionistic fuzzy information entropy method [2]. Wang et al developed a fuzzy objective weighting method using the objective weight information entropy theory and closeness coefficient [20]. Zou et al defined weights of evaluating indicators for the water quality assessment in a fuzzy synthetic evaluation using the information entropy method [26].

Variance can be used to define weights according to the degree of dispersion of data. Marín-Martínez et al proposed a weighting method to improve the weight accuracy using the reciprocal of variance and Monte Carlo simulation [13]. Valliant et al adjusted weights by the jackknife variance estimator by dropping groups of units, which has advantages of economizing on the computation time and file size [21]. Covariance has also been used in defining weights by two jointly distributed real-valued random variables. Stip et al applied shared versus unshared covariance to define the optimal weight for the correlation degree of data characters [19].

Comparing with other weighting methods, information entropy can determine weights of FRs based on similarity of functions of benchmarking products for objective solutions.

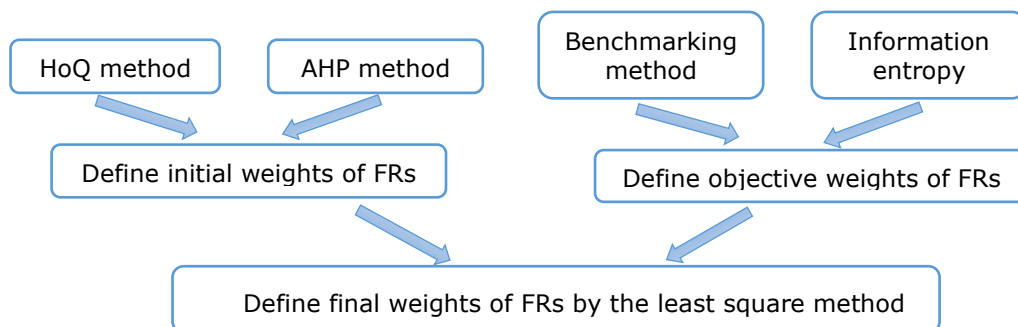


Figure 1: Flowchart of the proposed objective weighting method.

3 PROPOSED WEIGHTING METHOD FOR FRs

A flowchart of the proposed weighting method for FRs is shown in Figure 1. Raw data required include CRs, FRs, and IRs of CRs from the market survey and literature. Initial weights of FRs are decided by HoQ and AHP methods. The information entropy and benchmarking methods are integrated to improve the accuracy and objectiveness of initial weighting solutions. The initial weights and objective weights of FRs are then combined using the least square method for the final weights of FRs.

3.1 Initial Weighting FRs by Integrated HoQ and AHP Method

Initial weights of FRs are defined by integrated HoQ and AHP method as shown in Table 1. Absolute and subjective weights of FRs are then calculated using Equation (4). Where f_j is the absolute weight of each FR, c_{ij} is the value of relations between CRs and FRs, d_i is importance of the i th CR. r_j is normalized for a subjective weight of each FR.

AHP derives ratio scales from pairwise comparisons. Based on each CR, a pairwise comparison matrix **A** is built to compare importance between all the FRs to meet a CR in Equation (1). Where matrix **A** is $m \times m$ sized, m is the number of FRs. a_{jk} is an entry in the j row and k column of **A**. The determination for value of a_{jk} is shown in Table 2.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix} \tag{1}$$

Once matrix **A** is built, it is normalized as a pairwise comparison matrix **A**_{norm} by making equal to 1 for the sum of entries in each column using Equation (2).

$$\bar{a}_{jk} = a_{jk} / \sum_{l=1}^m a_{jl} \tag{2}$$

Based on the i th CR, a weight vector c_{ij} is formed by the average of entries in each row of **A**_{norm}, which represents importance of FRs to meet the i th CR in Equation (3).

$$c_{ij} = \sum_{k=1}^m \bar{a}_{jk} / m \tag{3}$$

Results in Equation (3) can be filled in a form as shown in Table 1. The importance result of FRs for meeting each CR can be calculated one by one using the same process for n times.

CRs	IR of CRs	FRs			
		FR ₁	FR ₂	FR _j	FR _m
CR ₁	d ₁	c ₁₁	c ₁₂	c _{1j}	c _{1m}
CR ₂	d ₂	c ₂₁	c ₂₂	c _{2j}	c _{2m}
CR _i	d _i	c _{i1}	c _{i2}	c _{ij}	c _{im}
CR _n	d _n	c _{n1}	c _{n2}	c _{nj}	c _{nm}
Absolute weight by AHP		f ₁	f ₂	f _j	f _m
Initial weight by AHP		r ₁	r ₂	r _j	r _m

Table 1: Definition of initial weights using HoQ and AHP methods.

Value of a_{jk}	Description
1	j and k are equally importance for meeting a CR
3	j is slightly better than k in importance for meeting a CR
5	j is better than k in importance for meeting a CR
7	j is strongly better than k in importance for meeting a CR
9	j is absolutely better than k in importance for meeting a CR
2,4,6,8 reciprocal	Intermediate value of adjacent importance k is better than j in importance for meeting a CR

Table 2: Determination method for value of a_{jk} .

The absolute weights of FRs can then be calculated by Equation (4). Where f_j is the absolute weight of each FR, c_{ij} is the value of relations between CRs and FRs, d_i is importance of the i th CR.

$$f_j = \sum_{i=1}^n c_{ij} d_i \tag{4}$$

r_j is normalized for a subjective weight of each FR as follows.

$$r_j = f_j / \sum_{i=1}^m f_j \tag{5}$$

Results of initial weights of FRs are then obtained as shown in Equation (6).

$$\mathbf{r} = (r_1, r_2, \dots, r_m)^T \tag{6}$$

3.2 Objective Weights of FRs Using Information Entropy

Objective weights of FRs are decided by benchmarking and information entropy methods. For comparing performance of different functions in benchmarking products, matrix \mathbf{X} is built as Equation (7). Where t is the number of benchmarking products selected in the market. Parameters and performances for implementing m FRs in t selected benchmarking products can be determined.

$$\mathbf{X} = (x_{ij})_{m \times t} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1t} \\ x_{21} & x_{22} & \dots & x_{2t} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mt} \end{pmatrix} \tag{7}$$

After normalizing matrix \mathbf{X} , the information entropy of FRs is calculated using Equations (8) and (9). The value of information entropy E_i is calculated using Equation (8) to define the difference of parameters and performances between t selected benchmarking products. The objective weight of FRs can be defined by value of information entropy E_i .

$$E_i = -K \sum_{j=1}^t P_{ij} \ln P_{ij}, \quad K = 1 / \ln t \tag{8}$$

Where, a probability function P_{ij} is determined by parameters in Equation (9) as follows.

$$P_{ij} = x_{ij} / \sum_{j=1}^t x_{ij} \tag{9}$$

For comparing data with different dimensions, results of objective weights of FRs in Equation (8) are transferred into interval from 0 to 1 for normalization using Equation (10)

$$E_{norm} = \frac{E_i - \min_{-1 \leq i \leq n} \{E_i\}}{\max_{1 \leq i \leq n} \{E_i\} - \min_{1 \leq i \leq n} \{E_i\}} \quad (i = 1, 2, \dots, m) \tag{10}$$

Normalized objective weights of FRs are then shown in Equation (11).

$$\mathbf{k} = (k_1, k_2, \dots, k_m)^T \tag{11}$$

3.3 Final Weights by Adjusting the Initial Weights

Initial weights of FRs are defined in Equation (6) using AHP and HoQ methods. Objective weights of FRs are defined using Equation (11). Initial weights of FRs are adjusted by objective weights to define final weights of FRs using the least square method in Equations (12) and (13).

$$\min F(u) = \sum_{i=1}^t \sum_{j=1}^m \left\{ \left[(r_j - w_j) s_{ij} \right]^2 + \left[(k_j - w_j) s_{ij} \right]^2 \right\} \tag{12}$$

$$\mathbf{S} = (s_{ij})_{t \times m} = \mathbf{X}^T, \sum_{j=1}^m w_j = 1, w_j \geq 0 \text{ (j=1,2,\dots,m)} \tag{13}$$

Final weights of FRs are shown in Equation (14) for product design.

$$\mathbf{w} = (w_1, w_2, \dots, w_m)^T \tag{14}$$

Based on the final weights of FRs, the design priority can be decided accurately.

4 CASE STUDY

4.1 Design Process of the Case Study

A case study is conducted for design of an upper limb rehabilitation device to verify the proposed weighting method of FRs. Relations of FRs and CRs of an upper limb rehabilitation device are shown in Table 3. CRs, FRs, IRs of CRs are defined based on literature [9]. 12 CRs in Table 3 include accurate movement CR.1, movement feedback CR.2, automatic CR.3, support the arm CR.4, easy operation CR.5, interesting CR.6, light weight CR.7, reasonable price CR.8, adaptability CR.9, portability CR.10, safety CR.11, and material CR.12. 10 FRs include sensor selection FR.1, motor selection FR.2, interactive function FR.3, adjustable height and length FR.4, suitable material FR.5, flexible movement structure FR.6, portable design FR.7, lightweight design FR.8, displacement limit FR.9, and degree of freedom design FR.10.

CRs	IR	FR.1	FR.2	FR.3	FR.4	FR.5	FR.6	FR.7	FR.8	FR.9	FR.10
CR.1	5	0.31	0.09	0	0.10	0	0.10	0	0.04	0.05	0.31
CR.2	3	0.35	0.07	0.26	0	0	0	0	0.10	0.05	0.17
CR.3	5	0.23	0.10	0.13	0.05	0	0.20	0.05	0.12	0.08	0.04
CR.4	4	0.06	0	0	0.14	0	0.49	0.10	0	0.05	0.16
CR.5	3	0.15	0.02	0	0.27	0	0.15	0	0.38	0	0.03
CR.6	2	0	0	0.51	0	0	0.15	0.10	0.11	0	0.13
CR.7	4	0	0	0	0.06	0.06	0.12	0.33	0.38	0	0.05
CR.8	5	0.07	0.03	0.05	0.16	0.02	0.03	0.23	0.33	0.01	0.08
CR.9	3	0.02	0.01	0.05	0.39	0	0.08	0.25	0.05	0.02	0.14
CR.10	5	0	0	0	0.06	0	0	0.61	0.15	0.05	0.06
CR.11	3	0.05	0.02	0	0.12	0	0.02	0.15	0.14	0.33	0.17
CR.12	4	0	0	0	0.03	0.53	0	0.24	0.16	0.01	0.03
Absolute weights by AHP method		5.01	1.47	2.85	5.11	2.44	5.15	8.83	7.59	2.39	5.20

Table 3: CRs, FRs, IRs and weights.

Initial weights of FRs are defined using HoQ and AHP methods. For correlations of FRs and CRs, 10 FRs are compared each other using Equations (1-3) to obtain the initial weights of FRs. 10 FRs to meet CRs are shown in the first row of Table 3. Each row shows the related FR weight to meet a CR. IR of a CR is shown in the second column in Table 3. By combining HoQ method and AHP method, absolute weights of FRs are defined using Equation (4) and shown in the last row in Table 3.

By normalizing absolute weights in the last row in Table 3, initial weights of FRs can be decided using Equation (5) as follows.

$$\mathbf{r} = (0.109, 0.032, 0.062, 0.111, 0.053, 0.112, 0.192, 0.165, 0.052, 0.113)^T \quad (15)$$

To improve the weighting solution of FRs, four popular rehabilitation devices in the market are selected as benchmarking products. Based on the function performance of four benchmarking products, the original matrix \mathbf{X} is formed in Equation (16) using Equation (7). Specifications of FR.4, FR.8, FR.9 and FR.10 can be found from these products directly. For example, details of FR.8 for lightweight design of four benchmarking products are 180, 110, 25 and 150kg, respectively. Details of FR.1, FR.2, FR.3, FR.5, FR.6 and FR.7 can be defined by 5 different levels from 1 to 5 according to the performance of benchmarking products for these FRs. For example, FR.7 for portable design can be evaluated based on product specifications introduced in the product user manual. Benchmark 1 can be moved by wheels (2 marks). Benchmark 2 and 4 cannot be moved without transport vehicles (1 mark). Benchmark 3 can be taken to everywhere easily (5 marks).

$$\mathbf{X} = (x_{ij})_{m \times n} = \begin{pmatrix} 5 & 4 & 2 & 5 \\ 5 & 5 & 2 & 4 \\ 1 & 2 & 4 & 2 \\ 70 & 62 & 61 & 70 \\ 2 & 1 & 3 & 5 \\ 4 & 5 & 3 & 3 \\ 2 & 1 & 3 & 1 \\ 180 & 110 & 25 & 150 \\ 35 & 29 & 35 & 33 \\ 5 & 5 & 3 & 5 \end{pmatrix} \quad (16)$$

Results are normalized to create a normalized matrix \mathbf{X}_{norm} as shown in Equation (17). When values are close to each other in a row of the matrix, the performance of functions in four benchmarking products are similar.

$$\mathbf{X}_{\text{norm}} = (x_{ij})_{m \times n} = \begin{pmatrix} 0.313 & 0.250 & 0.125 & 0.313 \\ 0.313 & 0.313 & 0.125 & 0.250 \\ 0.111 & 0.222 & 0.445 & 0.222 \\ 0.266 & 0.236 & 0.232 & 0.266 \\ 0.182 & 0.091 & 0.273 & 0.455 \\ 0.266 & 0.333 & 0.200 & 0.200 \\ 0.285 & 0.143 & 0.429 & 0.143 \\ 0.387 & 0.237 & 0.054 & 0.323 \\ 0.265 & 0.220 & 0.265 & 0.250 \\ 0.278 & 0.278 & 0.167 & 0.278 \end{pmatrix} \quad (17)$$

Using Equations (8-11), weights of FRs can be normalized and defined using information entropy as follows.

FRs	FR1	FR2	FR3	FR4	FR5	FR6	FR7	FR8	FR9	FR10
Weight by traditional method (\mathbf{r})	6	10	7	5	8	4	1	2	9	3
Weight by proposed method (\mathbf{w})	6	8	9	2	10	3	5	7	4	1
Difference	0	+2	-2	+3	-2	+1	-4	-5	+5	+2

Table 4: Design priorities based on traditional and proposed methods.

$$\mathbf{k} = (0.117, 0.117, 0.048, 0.175, 0.011, 0.151, 0.053, 0.000, 0.175, 0.156)^T \quad (18)$$

Final weights of FRs are defined using the least square method in Equation (19).

$$\mathbf{w} = (0.113, 0.073, 0.053, 0.135, 0.033, 0.125, 0.121, 0.081, 0.123, 0.143)^T \quad (19)$$

Weights of FRs from the traditional and proposed methods are shown by Equations (15) and (19). Based on the weight of FRs, design priorities are shown in Table 4. Weights by the traditional method is the initial weights defined using Equation (6) by combining HoQ and AHP methods. Weights by the proposed method is the final weights defined by Equation (14).

According to results in Table 4, weights of FR.2, FR.4, FR.6, FR.9 and FR.10 from the proposed method are increased compared to the weights of FRs by the traditional method. The reason is that different benchmarking products have similar functions for these FRs. For example, adjustment structures in the four benchmarking devices for meeting FR.4 (adjustment height and length) are different. However, different benchmarking products provide a similar function and have similar adjustment ranges for rehabilitation. That means that these kinds of functions are very important and should be met by all the products. Thus, weights for those FRs are increased.

Weights of FR.3, FR.5, FR.7 and FR.8 from the proposed method are reduced because the functions in the benchmarking products related to these FRs are different. For example, all the four benchmarking devices have different function performance for FR.8 (lightweight design). Weights of benchmarking products 1 to 4 are 180, 110, 25 and 150Kg, respectively. Their weights are quite different. Therefore, these kinds of functions are not important and weights for these FRs can be decreased.

Based on final weights of FRs, the design priority can be decided accurately as shown in Table 4 to be used as the design sequence to meet FRs in product design.

4.2 Design Results Based on Different Weighting Solutions

Rehabilitation devices are analyzed using the traditional and proposed weighting methods. 3D models of the four benchmarking products are built for the performance comparison in structures and parameters of rehabilitation devices in Figure 2. Based on the sequence for weights of FRs in Table 4 and benchmarking method, best structures in the benchmarking products are selected to improve the design of rehabilitation devices.

Based on the traditional method weighted FRs and four benchmarking products, a rehabilitation device is designed as shown in Figure 3. According to the sequence of weights of FRs in the first line of Table 4, the structure related to the portable design is defined at first. The design scheme is based on the best portable design structure in benchmarking product 3. A lightweight design is defined by the design scheme in benchmark 3. Then, the degree of freedom is defined as 3 based on benchmark 3. The flexible design is formed using the structure of benchmark 1. The adjustable height and length design is defined by benchmarking products 3. The design of other parts can be completed according the sequence of weights for FRs.

Based on the proposed method weighted FRs and four benchmarking products, a device is designed as shown in Figure 4. According to the sequence of weights for FRs in the second line of Table 4, the degree of freedom is defined as 5 at first. An adjustable structure is then defined by

reference of benchmarking product 4. Then, the flexible movement structure design is completed by benchmarking product 2. Designs of other parts are completed according the sequence of weights for FRs.

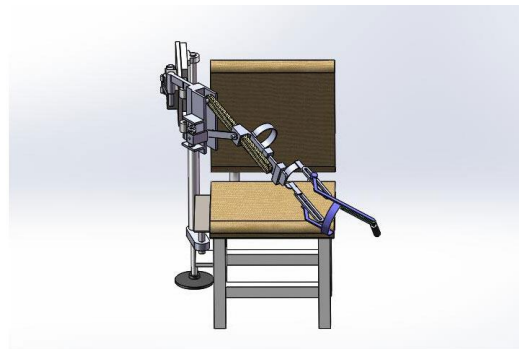
4.3 Evaluation of Results from Different Weighting Methods

For verifying the proposed weighting method, six rehabilitation devices including four benchmarking devices and devices designed based on weights from the traditional and proposed methods are compared for the performance analysis to meet all the FRs.

Information for six rehabilitation devices including the basic characters, adjustment range and function description are shown in Table 5. Basic characters have 4 parameters of the total cost, number of components, weight and degree of freedom. Cost includes costs of raw materials, manufacturing, assembling, packing, distributing of the product. The total number of components is decided based on the structure of the devices. The device weight is calculated using the material density and volume. The adjustment range has 3 parameters of height, upper arm length and lower arm length. Function description has 2 parameters for the injured level and target users.



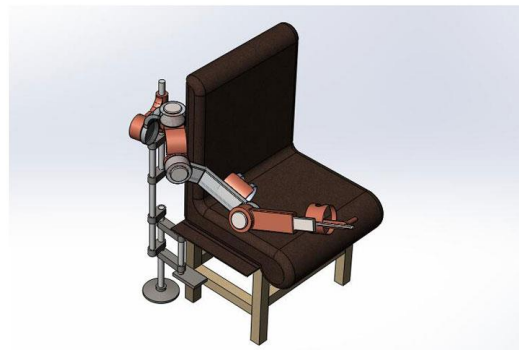
Benchmarking product 1



Benchmarking product 2



Benchmarking product 3



Benchmarking product 4

Figure 2: 3D models of benchmarking products.

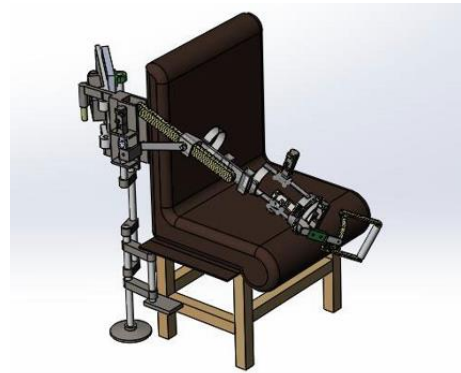
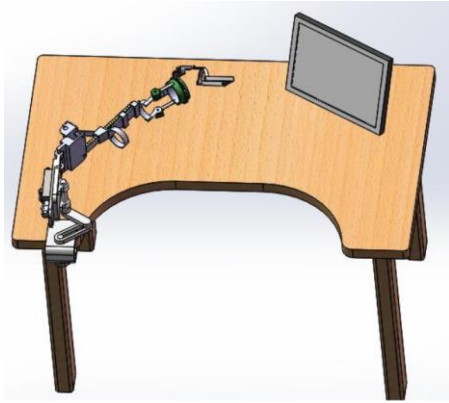


Figure 3: Design by the traditional weights. **Figure 4:** Design by the proposed weights.

Based on parameters of six rehabilitation devices in Table 5, advantages and disadvantages of the devices can be compared accurately. Benchmark 1 has a good performance in structure for a lower arm and bad performance in the interactive and portable functions. Benchmark 2 has a good performance in structure for the upper arm and bad performance in the adjustable structure. Benchmark 3 has a good performance in the portability and interaction and a bad structure in the arm. Benchmark 4 has a good performance in an adjustment structure of height and a bad performance in the arm operation.

	Basic characters				Adjustment range			Functions description	
	Total cost (US Dollar)	Number of components	Weight (Kg)	Degree of freedom	Adjusted range of height for the whole device (cm)	Adjusted range of upper arm length (cm)	Adjusted range of lower arm length (cm)	Whether can be used for serious injured patients?	Whether can be used for children?
Benchmarking product 1	1586	115	180	5	0-40	0-10	0-20	Yes	Yes
Benchmarking product 2	1321	89	110	5	0-32	0-20	0-10	Yes	Yes
Benchmarking product 3	2500	34	25	3	0-41	0-10	0-10	No	No
Benchmarking product 4	1875	98	150	5	0-60	0-5	0-5	Yes	Yes
Design by traditional method	2950	45	35	3	0-41	0-10	0-10	No	No
Design by proposed method	1290	83	145	5	0-60	0-20	0-20	Yes	Yes

Table 5: Comparisons of four benchmarks and two proposed devices.

Compared to two proposed rehabilitation devices in Figures 3 and 4, the device by the proposed weighting method has a better performance in rehabilitation for many FRs including adjustment height, adjustment length, degree of freedom and flexible movement structure, which can meet requirements for most users in rehabilitation to improve the competitiveness of the product in the market. The device by the proposed weighting method has a large adjustment ranges which can be used for both adults and children. However, the device by the traditional weighting method only can be used for adults. The device by the proposed method has 5 motors for the passive exercise of serious patients. As limitations in degrees of the freedom and flexible movement structure, the device by the traditional weighting method cannot be used for serious injured patients.

Comparing with the rehabilitation functions in Figures 3 and 4, the only advantage of the design by the traditional weighting method is that the device can meet the portable requirement. However, the function of portability may not be very useful for most of patients if they complete the rehabilitation in hospitals and therapy centers. The improvement of devices using the traditional weighting method cannot meet requirements for most of users as weighted FRs mainly focused on subjective matters. Performances of functions in benchmarking products are ignored in weighting FRs. Therefore, the proposed weighting method can improve the design solution using improved weights of FRs in product design.

4.4 Discussion of the Proposed Weighting Method

Comparing with the existing methods in defining weights of FRs, the proposed method can provide subjective and accurate weights of FRs for product design. The existing methods in defining weights of FRs are only based on the experience of experts. For example, IRs of FRs are assigned marks by experts. Without enough data, experts cannot give the weight accurately, especially for new products.

The proposed method combines attitudes of experts and characters of current popular products in the market, which can define objective and accurate weights of FRs. By combining HoQ and AHP methods, relationships of all FRs can be considered to define the initial weight accurately. For reducing the influence of subjectivity from experts, the objective weights of FRs are defined by characters of benchmarking products in the market. When all the benchmarking products have a same function, this function is attractive and necessary. The weight of this FR should be increased. Information entropy can describe the similarity of functions in benchmarking products. The value of information entropy is increased if a function in benchmarking products is similar. Therefore, the objective weight defined by information entropy and benchmarking products is used to adjust weights of FRs, which improves the objectiveness for weights of FRs. Compared to the existing methods, the proposed method can consider relationships of all FRs and characters of popular products in the market, which provides more objective and accurate weights of FRs.

5 CONCLUSIONS

This paper presented an objective weighting method for FRs using the information entropy, least square and benchmarking methods. The similarity of functions in benchmarking products is considered to adjust weights of FRs, which can improve the accuracy and objectiveness of weighting FRs. Design of rehabilitation devices in the case study verified advantages of the proposed method compared to solutions from the traditional weighting method.

Further work will consider other factors for weighting FRs including the future trend of customers and environmental requirements. In addition, a fuzzy weighting FRs and fuzzy weighted average method will be considered for some uncertain factors for weighting FRs of product.

6 ACKNOWLEDGEMENTS

The authors wish to acknowledge that this research has been supported by the Discovery Grants (RGPIN-2015-04173) from the Natural Sciences and Engineering Research Council (NSERC) of Canada, the Graduate Enhancement of Tri-Council Stipends (GETS) program from the University of Manitoba, the Leading Talent Project of Guangdong Province and National Key R&D Program of China (2018YFB1701701).

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