




## 2D Irregular Optimization Nesting Method based on Adaptive Probabilistic Genetic Simulated Annealing Algorithm

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**Abstract.** In this paper, we present a new optimization nesting algorithm for two-dimensional nesting problems. It is based on a genetic simulated annealing algorithm considering the adaptive probability. We adopt the no-fit polygon (NFP) algorithm and the central expansion strategy to get an initial nesting strategy. Furthermore, we optimize the initial nesting strategy with the algorithm proposed in this paper. The genetic algorithm is used to search the optimal sequence and the angle of the garment piece, the simulated annealing algorithm is used to avoid the genetic algorithm falling into the local optimization. The most advantage of the algorithm proposed in this paper has no requirements for the regular shapes of motherboards and sample pieces and it works well when both shapes are irregular. Our algorithm can increase the speed and efficiency of the nesting and improve utilization of the motherboard. Experimental results show the proposed algorithm is applicable and effective.

**Keywords:** 2D irregular nesting; adaptive probabilistic; genetic simulated annealing algorithm

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### 1 INTRODUCTION

2D nesting problems, also known as optimization or combination optimization nesting, which is defined as a certain number of sample pieces are arranged within given motherboards (e.g., steel sheets, leathers, etc.), so that sample pieces are contained inside the motherboards and do not overlap each other. The objective is usually to minimize the use area and consumption of the motherboards after nesting and to optimize the material utilization or the nesting scheme.

Two-dimensional irregular nesting is widely used in cutting and nesting of garment, leather, glass, wood, metal plate, and so on. With the development of economic globalization and the increasing demand for resources, the manual nesting can no longer meet the needs of enterprises. Computer-aided technology can improve the material utilization of materials and the nesting efficiency. This has

important practical significance for reducing enterprise costs and improving the competitiveness of enterprises. From the perspective of computational complexity theory, the two-dimensional nesting problem is NP-complete, no effective polynomial algorithm has been found. So far, no effective polynomial algorithm has been found for the problem.

The leather product is widely applied in life, such as leather garments, shoes, hats, bags, as well as the car seats, the luxury cruise seats and so on. The characteristics of leather materials are high cost, irregular boundary, and uneven surface quality distribution.

Due to the above characteristics, 2D nesting in leather garment manufacturing is more complex than the traditional 2D nesting problem. Both shapes of the sample piece (leather garment pieces) and motherboard (master leathers) are irregular. In leather garment manufacturing, a leather garment has many sample pieces which need multiple motherboards. When making multiple leather garments, it necessary to layout all sample pieces on many motherboards, and the shapes of the motherboards are not the same. This paper aims at the challenging problem of nesting techniques in leather garment manufacturing proposes a new algorithm to layout all sample pieces into many arbitrary shaped motherboards.

## 2 RELATED WORKS

The existing two-dimensional irregular nesting algorithms can be roughly divided into heuristic algorithms and intelligence algorithms.

(1) Two-dimensional irregular nesting based on the heuristic algorithm. The heuristic algorithm mainly aims at the problem of nesting small sample pieces in one single motherboard and does not consider the quality of the motherboard such as flaws and holes. In the garment industry, a motherboard is big rolls of fabric plate with a total length up to several hundred meters. The goal of the nesting algorithm is usually to use the length of the motherboard as minimal as possible so that the heuristic algorithm has serious limitations in practical application. To reduce the computational complexity, some literature of the heuristic algorithm combined the sample pieces, and then layout them as a complete sample shape. The most common method of heuristic algorithm is solving the minimum external rectangle of the combined sample shape[1]. An approximate method is combining sample pieces into a regular polygon[2][3], such as triangle, quadrangle, pentagon, hexagon to replace minimum external rectangle for layout. In 2002, Gomes and Oliveira [4] used a heuristic algorithm based on the BL (Bottom-Left) strategy to arrange sample pieces in a given order, and each iteration process passed through two adjacent sample pieces exchange strategy to change order. At the same time, the BL strategy is used to achieve optimal nesting of leather products. In 2003, Yong Chen [24] present a heuristic approach to deal with the irregular nesting problem without orientation limitation using Genetic Simulated Annealing Algorithm. In 2006, Burke et al. [5] proposed a new heuristic algorithm combined hill climbing and tabu local search methods and used the BLF (Bottom-Left-Fill) strategy to optimize the nesting problem. The algorithm is also suitable for curved contour with circular arcs and holes. In 2007, Burke et al. [6] proposed a robust NFP (no-fit polygon) generation algorithm based on the track line, the algorithm can be used to the case that the motherboard contains holes. In 2008, Lee WC [7] proposed a QLM (quick location and movement) algorithm to solve the multiple irregular motherboards arrangement of irregular sample pieces. The algorithm approximates the irregular sample pieces to polygons and utilizes the minimum embedding circle center and the gravity center of the irregular sample piece as a reference point to layout the irregular sample pieces on the motherboards. In 2010, Burke [8] proposed a method using NFP for collision detection. In 2014, Baldacci et al. [9] proposed a hybrid heuristic algorithm, which was used to solve the optimization problem of the 2D irregular sample pieces on multiple irregular motherboards, and also considering the motherboard with holes. The sample pieces and the motherboards were converted to raster representation. In 2015, MirHassani [10] proposed the GRASP (Greedy randomized adaptive search procedure) meta-heuristic algorithm to solve the nesting problem with high quality data proper time. This algorithm does not depend on the shape of the sample piece. In 2018 Mundim [22] proposed H4NP for all 2D nesting problems with

limited-size containers, and test it on 1100 instances, H4NP obtained equal or better solutions for 73% of the instances. In 2019 André Kubagawa Sato [23] use the dotted board model to reduce the overlap evaluation to matrix operations, and then it can be accelerated by massive parallelization using the GPU. In 2020, Santiago V. Ravelo [25] shows that multi-objective criteria to classifying solutions may make the results more accurate and they also give a heuristic algorithm and two meta-heuristic approaches to the problem.

(2) Two-dimensional nesting problem based on an intelligent algorithm. The emergence of an artificial intelligence algorithm provides new important means for the nesting optimization problem. Since the 1990s, scholars have gradually matured their research on genetic algorithms, simulated annealing algorithm, particle swarm optimization, neural network, ant colony optimization algorithm, artificial immune algorithm, and other intelligent optimization algorithms. People began to study the combination of intelligent algorithms for the nesting process. In 1966, Braglia [11] utilized a simulated annealing algorithm to improve the shortcomings of the local search ability of a genetic algorithm and combined the two algorithms as a hybrid algorithm to solve the single row 2D nesting problem. In 1997, Dagli [12] proposed a hybrid algorithm, which combines artificial neural network, genetic algorithm, and mathematical programming. The hybrid algorithm is used to solve the nesting optimization problem of fixed width and infinite rectangular motherboards. In 2001, Babu [13] proposed using a heuristic algorithm to improve the performance of the genetic algorithm to solve the 2D nesting problem. In 2006, Gomes and Oliveira [14] proposed a hybrid algorithm that combines simulated annealing algorithm and linear programming models to solve the irregular nesting problem of the strip motherboard. In 2010, Sato et al. [15] proposed a hybrid algorithm that combines the simulated annealing algorithm and collision-free region with parallel processing to solve the irregular nesting problem with a strip-shaped motherboard. In the same year, Liang LD et al. [16] used a particle swarm optimization algorithm to solve the irregular nesting problem. Compared with the genetic algorithm and simulated annealing algorithm, this method improves the ability of local optimization and reduces the computational complexity. In 2015, Liu H et al. [17] proposed an improved niche genetic algorithm to solve the irregular layout problem. This method can eliminate the too many similar solutions and avoid the premature convergence of the genetic algorithm.

Aiming at 2D irregular nesting problem in the leather garment manufacturing industry, this paper uses The NFP algorithm [6] to create initial nesting at first and then proposes an adaptive probability genetic simulated annealing algorithm to optimize the initial nesting. In 2017, Xu et al. [18] proposed an optimizing model based on a genetic algorithm to solve the nesting problem of rectangle pieces. This method combines the genetic algorithm and the lowest horizontal line searching algorithm. The algorithm designs and applies three hybrid heuristic strategies, it can solve the irregular nesting problem effectively. Genetic algorithm has a good global search capability, but when the individuals in the population after crossover and mutation operation, it is difficult to producing progeny individuals better than the parent generation, it will prone to premature convergence, and lead the search into a local optimum. The simulated annealing algorithm can jump out the local optimum with a certain jump probability, but the evolution speed is slow, and the global search ability is not strong. Therefore, the two algorithms are combined as a hybrid algorithm, which can not only retain the excellent characteristics of each other but also can compensate for their shortcomings. At the same time, the adaptive crossover probability and mutation probability can be used to control the evolution process of populations.

### 3 DESCRIPTION OF 2D IRREGULAR NESTING PROBLEM

Taking the leather garment manufacturing application as an example, the irregular nesting problem has several main difficulties. Leather motherboards have different shapes and areas. The shapes of leather sample pieces are also irregular and the quantities of them are variable. Irregular nesting of leather sample pieces was studied in this paper, it has the following features.

(1) The shapes of motherboards and sample pieces are irregular. Irregular shape means more parameters for the shapes of the sample piece and motherboard, this feature determines the nesting problem of the irregular sample pieces is high complexity. Because sample pieces placement strategy, collision detection, area calculation these computing steps in nesting are closely related to the shapes of the motherboards and sample pieces.

(2) There are multiple motherboards, the shapes and areas of them are different. Generally, nesting in leather garment manufacturing is on lots of leather sample pieces layout in many leather motherboards, and simple pieces of a leather garment can lay out in different leather motherboards. The goal of nesting optimization is to maximize the utilization rate of each motherboard and to minimize the number of motherboards used. These properties increase the nesting difficulty.

(3) The Position and rotation angle of each sample piece placement in the motherboard are arbitrary. Nesting in fabric garment sample piece layout direction is constrained by the fabric texture. But in leather garment nesting, sample piece layout in motherboards is in arbitrary directions. The traditional fabric garment nesting method is no longer applicable. On the other hand, in sample pieces nesting strategy, each rotation of a small angle will produce a new nesting solution; it will lead to a larger solution space of the nesting optimization. It increases the difficulty of solving the nesting problem. Therefore, it is necessary to according to the actual situation for the rotation angle increased certain constraint conditions.

(4) The motherboard is divided into multiple quality regions and sample pieces are divided into more different grades. In the process of production, the leather motherboard is from animals, so leather's quality in the central region is better than the outer region. The outer region in leather has folds, commonly known as "lotus leaf edge" which affects the quality of the garment. Therefore, the outer region of leather cannot be used for important sample pieces of the garment (such as the front part, back part and sleeve, etc.), but can be used in the neckline, nesting inside pocket relatively unimportant piece. Therefore, we divided into different regions of the leather motherboard according to its quality, which the center of leather is a high-quality region and the outer area of leather is a general region. Similarly, we divided all the sample pieces into two grades according to their location importance.

#### **4 GENETIC SIMULATED ANNEALING OPTIMIZATION ALGORITHM WITH ADAPTIVE PROBABILITY**

According to the descriptions above, we illustrated the design of a genetic simulated annealing algorithm with adaptive probability applied for irregular 2D optimization nesting by placing multiple leather sample pieces on multiple motherboards. Compare to other heuristic algorithms, though the fruit fly optimization algorithm (FOA) is a well-regarded algorithm for searching the global optimal solution, it is not competitive in convergence speed, and may fall into local optimum quickly when solving high dimensional mathematical and practical application problems[26]; the differential evolution[27] algorithm is widely used in many fields and is mainly used to solve global optimization problems of continuous variables while GA is mainly used to deal with discrete variables. So, we use an adaptive probabilistic genetic simulated annealing algorithm here. In this paper, the new individual generated by the genetic algorithm is used to replace the old individual with a probability that is obtained by comparing the fitness of the new individual to the old individual. So that the algorithm can jump out of the local optimal and find the global optimal finally. This algorithm is mainly used to adjust the order and angle of the leather sample piece and optimize the nesting. Therefore, we need to get the initial nesting of the garment sample piece and then we use this algorithm to optimize the initial nesting.

In this paper, we use the NFP algorithm [6] to create the initial nesting, which can determine a placement of the sample pieces within the leathers bound polygon. We divided all the sample pieces into two levels according to the importance, so the important pieces of the leather garment were placed in the central region of the motherboard. Figure 1 shows 11 sample pieces layout on a

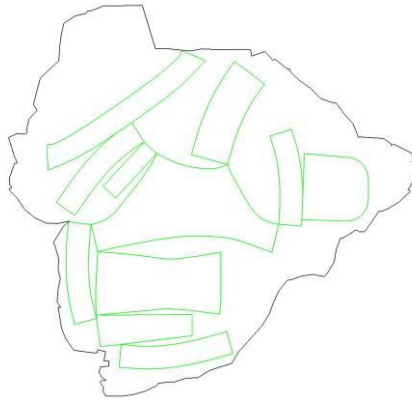
motherboard. The biggest area of the sample pieces is important, so we place those important pieces in the central region of the leather at first.

In this section, the 2D irregular nesting optimization algorithm is introduced in detail.

#### 4.1 Sample Individual Coding

We use the decimal encoding method to encode irregular sample pieces of garments. Since these garment sample pieces are divided into two grades (level 1 and level 2), therefore it needs to be coded the two grades pieces separately.

Let  $n$  garments pieces of importance grade in level 1  $P_{11}, P_{12}, \dots, P_{1n}$ , layout in multiple motherboards of leather garments as pieces set  $\{b_{11}, b_{12}, \dots, b_{1i}, \dots, b_{1n}\}$ . Here the first subscript denotes piece importance grade level. The second subscript denotes the piece number.  $b_{1i}$  represents the garments piece of number  $i$  and garments importance grade in level 1. In addition to sample pieces order, we add an angle variable  $\theta_i$  for each piece. So the individual encoding for sample pieces represented as:



**Figure 1:** Initial nesting result by NFP algorithm.

$$X_1 = \{(b_{11}, \theta_1), (b_{12}, \theta_2), \dots, (b_{1n}, \theta_n)\}$$

Similarly, the individual coding of  $m$  garments pieces  $P_{21}, P_{22}, \dots, P_{2m}$  of importance grade in level 2 is:

$$X_2 = \{(b_{21}, \theta_1), (b_{22}, \theta_2), \dots, (b_{2m}, \theta_m)\}$$

In this paper, we provide group size as  $M = n_1 + m_1$ , where  $n_1$  refers to the garments pieces number of importance grade in level 1, and  $m_1$  refers to the garments pieces number of importance grade in level 2. It indicates that the size of the group will change as some of the number of pieces has been layout

Due to the 2D irregular nesting is without any restrictions to the placement angle of nesting pieces, and all angles within the range of  $0 \sim 360^\circ$  are possible values of the nesting pieces. Therefore, the process of solving the nesting results becomes very complicated. In order to limit the

nesting time to an acceptable range, we define that the rotation angle of the irregular sample piece is an integer multiple of the angle base  $\theta_g$  in the process of nesting.

Researchers show that when  $\theta_g \leq 7^\circ$ , the impact of material utilization in leather nesting is very small. Here we set  $\theta_g = 9^\circ$ , so that the rotation angle of the nesting pieces need to only consider the 40 kinds of rotation possibilities within the range  $[0^\circ, 360^\circ)$ .

## 4.2 Fitness Function Design

Fitness function is a standard to measure the quality of the nesting results. In this paper, it is necessary to optimize the nesting pieces of multiple sets of garments in the batch leather motherboards. Compared with the utilization ratio of a single leather motherboard, the number of leather motherboards saved is more in line with the requirements of production. Here use  $M$  to represent the number of motherboards used in nesting, and use  $R$  to represent the utilization rate of the last motherboard in the currently used leathers. Thus for the individual  $X_i (0 \leq i < N)$ ,  $N$  is the group size, which is the number of all simple pieces of garments. The fitness function is as follows:

$$F(X_i) = \alpha * \frac{1}{M_i} + \beta * R_i \quad (1)$$

Where  $\alpha$  and  $\beta$  is the proportionality coefficients, randomly generated but meet to  $\alpha \in (0,1)$ ,  $\beta \in (-1,0)$ . Relatively speaking, the number of leather motherboards is more important than the utilization of the last motherboard. So, it sets  $|\alpha| > |\beta|$ .

## 4.3 Gene Selection Operator

Selecting operation takes the individual fitness of a group as the evaluation criteria. In this paper, the optimal preservation strategy is used to select the individual whose fitness value is largest in the group of the parents to the next generation, and then the remaining individuals are selected by the proportional selection method. Selection operator is to select the superior and eliminate the inferior of the individuals in the group, that is, the individual of large fitness value has a large possibility of being inherited to the next generation, while the individual of small fitness value has a small possibility of being inherited to the next generation.

The probability formula used in the proportion selection method is as follows:

$$f(X_i) = \eta * F(X_i) \quad (2)$$

where  $\eta$  is a proportion coefficient, Its value is randomly generated but meet to  $\eta \in (0,1)$ .

## 4.4 Adaptive Crossover Operator and Mutation Operator

### 4.4.1 Crossover operator

Crossover is the main method for generating new individuals in the genetic algorithm. There are many crossover methods. Single-point crossover changes individual information little, which is not conducive to the search. For multi-point crossover, it is easy to make a better individual gene segment to be destroyed, which affects the search results. In this paper, we use a two-point crossover, that is, in the individual coding string we set two cross points and exchange the gene of the crossover point.

Firstly, according to the individual number,  $M$  individuals in a group of parents can be paired off according to the neighbouring serial number. That is, number 0 and 1 individuals make a pair,

and number 2 and 3 individuals make a pair, and so on. We obtain  $M'$  pairs of individuals,  $M' = \lfloor M / 2 \rfloor$ .

Then, we generate a random number  $R_c$  randomly,  $R_c \in [0,1]$ . If  $R_c$  is bigger than the crossover probability  $P_c$ , then let the paired individuals do cross operation. Otherwise, don't do the cross operation. If the crossover probability is too large, it's easy to destroy individual genes with high fitness value. If the crossover probability is too small, it will make the evolution process to be slow, then affect the final results of the genetic search. Therefore, the adaptive crossover probability is adopted [5], The following is the formula for calculating the adaptive probability.

$$P_c = \begin{cases} \frac{k_1(F' - F_{avg})}{F_{max} - F_{avg}}, & F' \geq F_{avg} \\ k_1, & F' < F_{avg} \end{cases} \quad (3)$$

Where  $F'$  is the larger fitness value of the two individuals who perform the cross operation,  $F_{avg}$  is average fitness function value of each generation group,  $F_{max}$  is the largest fitness function value of the group,  $k_1$  is a constant number, and  $k_1 = 1$ .

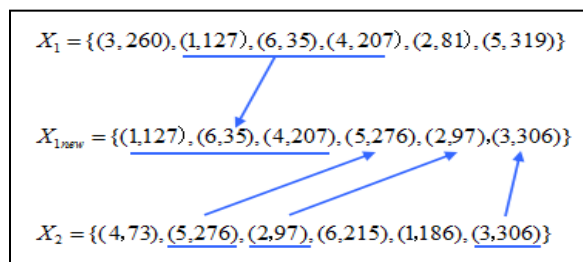
Assume that two individuals that have been paired are  $X_1$  and  $X_2$  respectively, two new individuals after cross operation are  $X_{1new}$  and  $X_{2new}$ . Randomly generate two cross bits in  $1 \sim n$  range  $b_1$  and  $b_2$  which satisfy  $1 \leq b_1 < b_2 \leq n$ , the gene between  $b_1$  and  $b_2$  of  $X_1$  is copied to the front gene of  $X_{1new}$ . The remaining genes are copied to the back of  $X_{1new}$  according to the sequence occurred in  $X_2$ , and the same method can be used to get another individual  $X_{2new}$ .

Suppose the two paired two individuals are:

$$X_1 = \{(3, 260), (1, 127), (6, 35), (4, 207), (2, 81), (5, 319)\}$$

$$X_2 = \{(4, 73), (5, 276), (2, 97), (6, 215), (1, 186), (3, 306)\}$$

Cross bit is  $b_1 = 2, b_2 = 4$ , the generation process of a new individual  $X_{1new}$  is shown in the following figure:



**Figure 2:** Two-point crossover process.

#### 4.4.2 Mutation operator

Mutation operator can maintain the diversity of the population and enhance the local search ability of genetic algorithm. In this paper, the individual encoding includes sequence encoding and angle encoding of garment sample pieces, accordingly, mutation of individual in parents' groups include sequence mutation and angle mutation.

Firstly, generate a random number  $R_m$ , and its value satisfies  $R_m \in [0,1]$ . If  $R_m$  is greater than the probability of mutation  $P_m$ , we mutate the paired individuals, otherwise, don't do the mutate operation. Mutation probability should be a reasonable value. If the mutation probability is too large, the genetic algorithm is reduced to a purely random algorithm, if the probability of mutation is too small, the process will be slow, resulting in stagnation. Therefore, according to the actual process, the adaptive mutation probability is adopted as follows[18] [19]:

$$P_m = \begin{cases} \frac{k_2(F_{\max} - F)}{F_{\max} - F_{avg}}, & F \geq F_{avg} \\ k_2, & F < F_{avg} \end{cases} \quad (4)$$

Where  $F_{\max}$  is the maximum fitness function value of the population,  $F$  is the fitness value of the individual performing the mutation operation,  $F_{avg}$  is average fitness function value of each generation group, and  $k_2 \in [0.1, 0.3]$  is a constant valued randomly.

Then we do sequential order mutation. We randomly generate two sequenced order mutation position  $c_1$  and  $c_2$  within ranges 1 and  $n$ , and their satisfaction  $1 \leq c_1 < c_2 \leq n$ , then exchange the  $c_1$  th gene and the  $c_2$  th gene of the individual  $X$ .

Assuming sequence order mutation position  $c_1 = 2, c_2 = 3$ , the individual is:

$$X = \{(3, 260), (1, 127), (6, 35), (4, 207), (2, 81), (5, 319)\}$$

Then the individual after sequence mutation is:

$$X' = \{(3, 260), (6, 35), (1, 127), (4, 207), (2, 81), (5, 319)\}$$

At last, let the individual  $X'$  do angle mutation after sequence order mutation. We randomly generate an angle mutation position  $d$  within ranges 1 and  $n$ , and generate an angle  $\theta$  between 0 and 360 degree. The angle of the  $d$  th gene of the new individual  $X'$  is replaced by  $\theta$ . Then we obtain the new individual  $X_{new}$  after mutation.

Let  $d = 4$  and  $\theta = 60$ , then the new individual  $X_{new}$  will be:

$$X_{new} = \{(3, 260), (6, 35), (1, 127), (4, 60), (2, 81), (5, 319)\}$$

#### 4.5 New Individual Acceptance Probability

After performing the selection, crossover and mutation operation of the genetic algorithm, we execute simulated annealing algorithm (SA) for newly obtained  $M$  individuals in order to avoid the local optimization of the genetic algorithm. The selection of the new individual acceptance probability in the annealing strategy determines the convergence quality of the solution and is very important to the performance of the algorithm.



$$\Delta F = \frac{1}{F(X_{new})} - \frac{1}{F(X_{old})} \quad (5)$$

$$P(X_{new}, X_{old}) = \exp\left(-\frac{\Delta F}{T_k}\right) \quad (6)$$

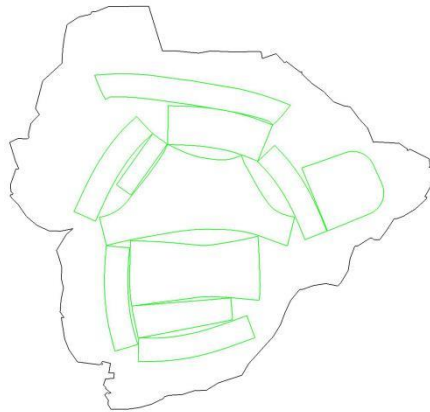
For example, for individual  $X_{new}$  and  $X_{old}$ , calculate their fitness  $F(X_{new})$  and  $F(X_{old})$  according to Eqs. (1) respectively, then calculate the corresponding  $\Delta F$  according to Eqs. (5), when  $\Delta F \leq 0$ , new individual  $X_{new}$  will replace the old individual  $X_{old}$  with a 100% probability. On the contrary, if  $\Delta F > 0$ , new individual  $X_{new}$  will replace the old individual  $X_{old}$  with a probability  $P(X_{new}, X_{old})$ , and we can obtain the probability  $P(X_{new}, X_{old})$  through Eqs. (6).

#### 4.6 Termination Condition of the Algorithm

The whole solving process of the algorithm is an iterative process and finally tends to converge.

In this paper, the termination condition of the algorithm is as follows: if the number of iterations reaches the maximum value 100, the difference of the average fitness function value of the adjacent two generations is less than 0.0005. Then we stop iterating. The corresponding nesting scheme for the individual with the maximum output fitness function value is the final nesting scheme.

Figure 3 shows the result of initial nesting (Figure 1) by our optimization nesting algorithm.



**Figure 3:** Optimization nesting result from Figure 1.

## 5 THE DESCRIPTION OF 2D IRREGULAR OPTIMIZATION NESTING ALGORITHM

We use the above adaptive probability genetic simulated annealing algorithm to optimize nesting scheme of multiple sets of clothes on the multiple leather motherboards. Garments sample pieces are divided into two grades according to their importance (level 1 and level 2). Each leather motherboard includes one inner contour and one outer contour. Inner contour region is a high-quality region. Concrete steps of nesting algorithm are as follows:

**Step1:** Initialize the population of sample pieces, set the set of irregular sample pieces importance level 1 as  $SP1 = \{P1_i | 1 \leq i \leq n\}$  where  $n$  is the garment sample pieces number of importance in level 1. Set the set of irregular garment importance in level 2 as

$SP2 = \{P2_i \mid 1 \leq i \leq m\}$  where  $m$  is the garment pieces number of importance in level 2. It randomly generates the nesting pieces sequence of SP1 as  $\{b_{11}, b_{12}, \dots, b_{1i}, \dots, b_{1n}\}$ , and randomly rotates some angle  $\theta_i$  for each sample to individualize coding as  $X_1 = \{(b_{11}, \theta_1), (b_{12}, \theta_2), \dots, (b_{1n}, \theta_n)\}$ . Similarly  $SP2$  is coded as  $X_2 = \{(b_{21}, \theta_1), (b_{22}, \theta_2), \dots, (b_{2m}, \theta_m)\}$ .

**Step 2:** Nesting sample pieces with improved NFP algorithm according to the following strategy and procedures, and a flowchart of this step is shown in Figure 4:

①. Set the set of the leather motherboard inner contour as  $SMin = \{MIn_j \mid 1 \leq j \leq k\}$ , the set of the leather motherboard outer contour as  $SMOut = \{MOut_j \mid 1 \leq j \leq k\}$ , and the set of the fusion border as  $SC = \{C_j \mid 1 \leq j \leq k\}$ , where each motherboard outer contour  $MOut_j$  corresponds to an inner contour  $MIn_j$  and a fusion border  $C_j$ ,  $k$  is the number of leather motherboard.

②. Calculate the inner abutment  $NFP_{MInP1_1}$  using the first irregular sample piece to be layout  $P1_1$  in piece set  $SP1$  and the first leather sheet motherboard  $P1_1$  in the leather sheet set.

③. Place the sample piece  $P1_1$  in the central region of  $NFP_{MInP1_1}$ .

④. Subtract the region of  $P1_1$  after positioning from the region of  $MIn_1$ , and remove  $P1_1$  from  $SP1$  and add region of  $P1_1$  to the fusion border  $C_1$  of motherboard.

⑤. Calculate the inner abutment  $NFP_{MInP1_2}$  using  $P1_2$  with the updated  $MIn_1$  and the outer abutment  $NFP_{C_1P1_2}$  using  $P1_2$  with fusion border  $C_1$ . Delete the intersection area from outer abutment  $NFP_{C_1P1_2}$ , and update  $NFP_{C_1P1_2}$ .

⑥. Locate the reference point of  $P1_2$  to  $NFP_{C_1P1_2}$ , similar to processing  $P1_1$ , and implement step ④.

⑦. Until a  $P1_i$  in the set  $SP1$  cannot find the outer abutment in  $NFP_{C_1P1_2}$ , the remaining region of  $MIn_1$  will be added to  $MOut_1$ . Calculate  $P2_1$ ,  $NFP_{MOutP2_1}$  and  $NFP_{C_1P2_1}$ .  $P2_1$  is the first sample piece for nesting.  $NFP_{MOutP2_1}$  is the inner abutment of  $MOut_1$ .  $NFP_{C_1P2_1}$  is the outer abutment of  $C_1$ .

⑧. Delete the outer abutment  $NFP_{C_1P2_1}$  from the intersection and update  $NFP_{C_1P2_1}$ .

⑨. Similar to the positioning method of garment pieces in the set  $SP1$ , repeat step ⑥ until some piece  $P2_i$  in the set  $SP2$  cannot find its outer abutment  $NFP_{C_1P2_i}$ , then select the next motherboard, repeat the above step ② ~ ⑦.

⑩. Until the pieces set of  $SP1$  and  $SP2$  are empty and all garment pieces are completed nesting.

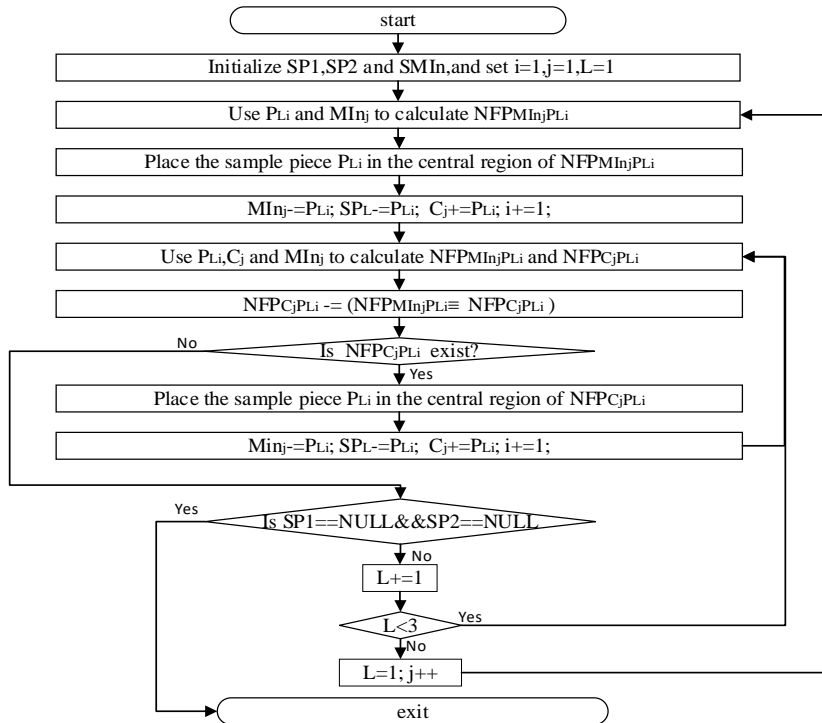
**Step3:** Use the Eq. (1) to calculate the fitness of each individual in the genetic algorithm.

**Step4:** For individual selection, the individual with the maximum fitness value is inherited directly to the next generation, and the Eq. (2) is used to select the remaining individuals.

**Step5:** Generate a new population. Eqs. (3) and (4) were used to cross and mutate the population produced by step4. A new population was generated.

**Step6:** Simulated annealing is applied to the newly generated population to avoid the genetic algorithm falling into a local optimum.

**Step7:** Determine the final nesting scheme. Repeat the above Step 2~Step 6 until the average fitness function value of the second-generation population is less than 0.0005, and the individuals with the maximum fitness value are selected as the final nesting scheme.



**Figure 4:** Nesting sample pieces with improved NFP algorithm.

## 6 EXPERIMENTAL RESULTS AND ANALYSIS

### 6.1 Nesting Experiment without Distinction of Importance Region

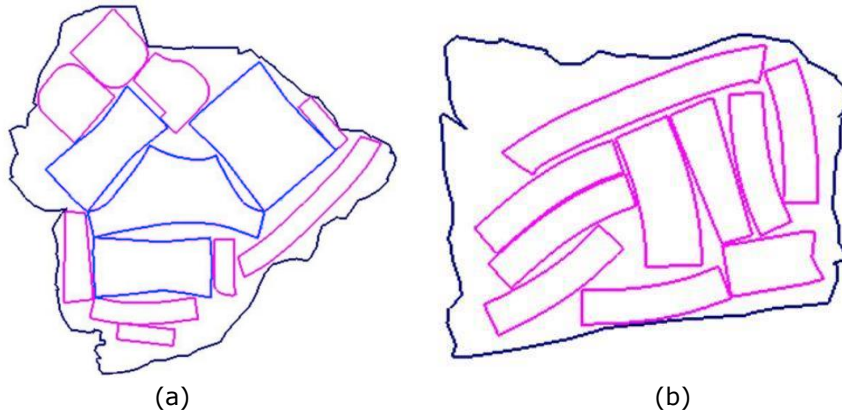
Experiment in Figure 5 is without distinguishing the leather motherboards into the high-quality regions and the general regions. The garment sample pieces also are not classified into different importance grades. In Figure 5, 22 garment sample pieces are laid out on two motherboards.

First, we use No-fit Polygon (NFP) algorithm to generate the initial nesting, and then use the adaptive simulated annealing algorithm proposed in this paper to optimize it. Figure 5 shows the final optimization nesting result.

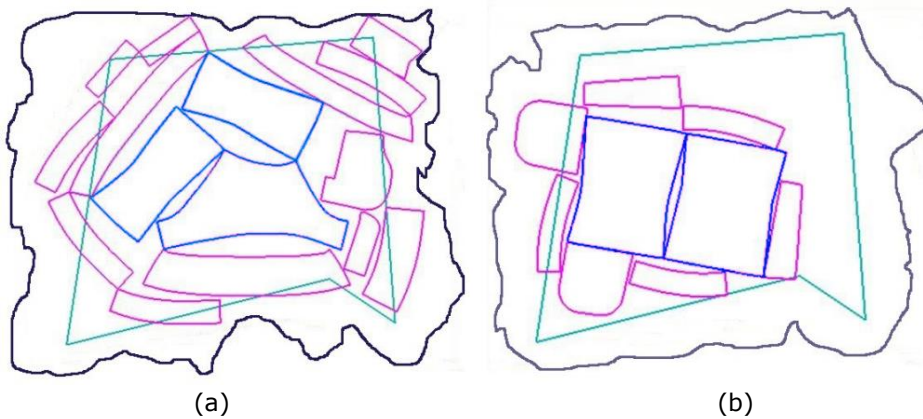
### 6.2 Nesting Experiment of Distinguishing the high-quality Region and the Importance of Garment Sample Piece

In Figure 6, the leather motherboards are partitioned into two grades regions, and the garment sample pieces are also divided into two levels: level 1 and level 2. The region of the leather motherboard within the inner contour represents a high-quality region, which used to layout the importance level 1 pieces (blue part in figure 5 and 6). The region outside the interior contour

represents the general quality region. After the garment sample pieces of level 1 are finished nesting in the high-quality regions, the remaining regions used to nesting the garment sample pieces of level 2 (pink part in figure 5 and 6). The Figure 6 shows 25 garment sample pieces in two leather motherboards.



**Figure 5:** Optimization nesting result without distinction of importance region.



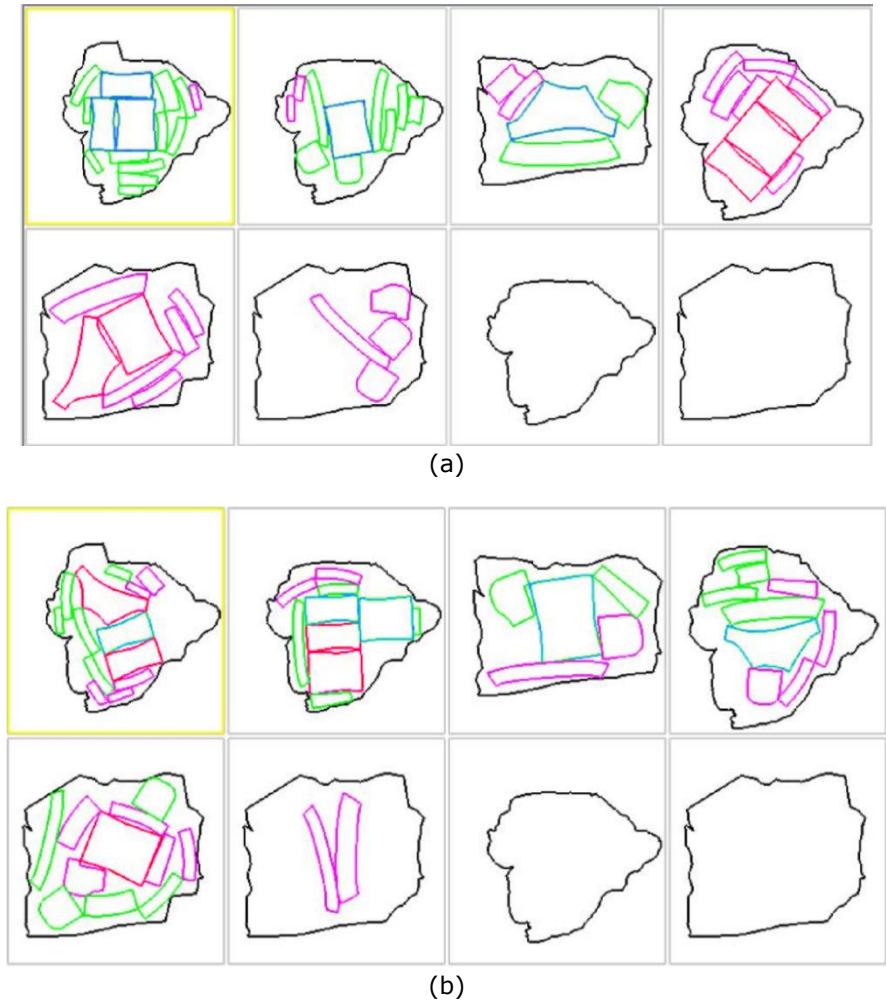
**Figure 6:** Optimization nesting result.

### 6.3 Nesting Experiment of Multiple Sets of Clothes

In the application of leather product nesting, multiple sets of garments are usually laid out in multiple leather motherboards. The minimum number of leather used is the goal of nesting. The following experiment tested the nesting of different sets of garments on multiple leather motherboards. For the nesting problem of multiple sets of garments in multiple motherboards, we compared the NFP algorithm and the adaptive simulated annealing algorithm proposed in this paper. Table I shows the number of leather motherboard used for different sets of garments using different algorithms. A set of garments includes 25 sample pieces.

Figure 6 shows the nesting results of 2 sets of clothes in 8 leather motherboards. We use different colors to distinguish different garment sample pieces, green and blue sample pieces belong to a set of clothes, pink and red sample pieces belong to another. The central region of the leather motherboard is nesting for the important sample pieces with level 1 (blue and red sample pieces in figure 7). The region near the edge of the leather is nesting for the pieces with level 2 (green and

pink sample pieces in figure 7). Figure 7 (a) is the initial nesting result using NFP algorithm, Figure 7 (b) is the optimized nesting using our algorithm. As the figure shows that the utilization of leather is improved by using our algorithm.



**Figure 7:** Multiple sets of clothes nesting.

As shown in Table 1: after using the optimization algorithm in this paper, the more sets of nesting garments the more leather masters will be saved, and it meets the actual demand of leather nesting.

Algorithm	1 set of clothes	3 sets of clothes	5 sets of clothes	10 sets of clothes
NFP	3	8	14	27
GSAO	3	8	13	24

**Table1:** Number of leather used before and after optimization.

## 7 CONCLUSIONS

Aiming at the problem of optimal nesting for irregular sample pieces of leather garment, this paper has completed the following major works:

(1) The leather motherboards are divided into high quality and general regions, and the sample pieces are divided into different importance levels. Sample pieces in high importance level (level 1) will be placed to the high-quality region of the leather motherboard.

(2) A genetic simulated annealing algorithm with adaptive probability is proposed to find the optimal order and rotation angle of each piece in the leather nesting.

(3) According to the optimal order and rotation angle of the sample piece, the central expansion strategy and the positioning algorithm based on NFP can be used to layout multiple sample pieces on multiple motherboards.

Using the above methods, an optimal sequence is obtained within an acceptable time range. The algorithm proposed in this paper has no requirements for the regular shapes of motherboards and sample pieces. The algorithm also works when both shapes are irregular. For a large number of sample pieces, our algorithm can increase the speed and efficiency of the nesting, save the cost of production, improve utilization of the motherboard. Therefore, the algorithm has a significant economic benefit in actual production.

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