Efficient Sizing Optimization Using ANNs Instead of FEM to Evaluate Design Proposals

Masakazu Kobayashi[1] and Yuki Ogawa[2]

[1]Toyota Technological Institute, kobayashi@toyota-ti.ac.jp
[2] Toyota Technological Institute, kametyun@gmail.com

Corresponding author: Masakazu Kobayashi, kobayashi@toyota-ti.ac.jp

Abstract. Sizing optimization is a type of structural optimization and treats structural dimensions, which define the shape of the structure, as design variables. Sizing optimization has a high affinity with 3D CAD that adopts parametric modeling from the viewpoint of the way of defining the shape. In fact, several commercial 3D CADs provide the function of sizing optimization. On the other hand, in sizing optimization, the number of iterations required to reach the optimal solution increases at an accelerating rate as the number of design variables increases. FEM is often used in sizing optimization to evaluate design proposals in each iteration, but it takes a certain amount of time to generate and analyze analytical models that reflect changes in design variables. As a result, computation time becomes so long that the optimal solution cannot be reached in practical time. To overcome this limitation, a new method for efficient sizing optimization using artificial neural networks (ANNs) is proposed. More specifically, networks that infer objective function and constraint conditions from design variables are trained using the training data collected by existing FEM software and the trained networks evaluate design proposals during sizing optimization. It takes a lot of time to collect training data using FEM software, but the time required to evaluate design proposals using the trained networks during sizing optimization is almost 0. Therefore, it is expected that sizing optimization using ANNs can be performed with less time in total. In addition, fine tuning is introduced to reduce the burden of training data collection. By using fine tuning and reusing the network weights from past sizing optimization using ANNs, networks with better accuracy can be obtained with less training data. In the case study, the proposed method is applied to the optimal design of an aircraft wing.

Keywords: Sizing optimization, Structural optimization, Artificial neural network, Fine tuning, Aircraft wing
DOI: https://doi.org/10.14733/cadaps.2023.976-986

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

1 INTRODUCTION

Structural optimization is a type of optimal design problem in which the shape of a structure is determined to minimize or maximize the objective function by using the evaluation characteristics such as weight, stiffness, and natural frequency as the objective function and the shape of the structure as the design variables. Structural optimization can be roughly classified into three types: sizing optimization [10], shape optimization [15], and topological optimization [1], depending on how the shape of the structure is represented and what is treated as a design variable.

In sizing optimization, an optimization problem is formulated using the evaluation characteristics of the structure, e.g., weight, stiffness, and maximum stress, as objective functions while the design dimensions that define the shape of the structure, e.g., beam length and height, plate thickness, and cross-sectional area, as design variables, and mechanical or geometric conditions as constraints, and optimization is performed using an optimization algorithm such as steepest descent, GA (Genetic algorithm) or PSO (Particle swarm optimization). Shape optimization derives the optimal shape of a structure using its outer shape as design variables. To be more specific, the coordinates of the nodes on the boundary of the finite element model representing the initial shape of the structure are used as design variables, and the optimal shape is explored by combining the finite element method, sensitivity analysis method, and mathematical optimization algorithm. Topology optimization is a method of simultaneously optimizing the shape and form of the structure by replacing the problem of finding the optimal shape of the structure with the problem of placing materials in a fixed design domain. Each method has its own characteristics, or advantages and disadvantages, and is used in different ways depending on the situation.

In this research, sizing optimization is focused among the three methods. Sizing optimization treats dimensions that define the shape of the structure as design variables and explores optimal solutions by changing them. Such expression is similar to the way of defining the shape in general 3D CAD that adopts parametric modeling, and thus sizing optimization has a high affinity with 3D CAD. In fact, several commercial 3D CADs [11] has the function of sizing optimization. On the other hand, in sizing optimization, the number of iterations required to reach the optimal solution increases at an accelerating rate as the number of design variables increases. FEM is often used in sizing optimization to evaluate design proposals in each iteration, but it takes a certain amount of time to generate and analyze analytical models that reflect changes in design variables. As a result, computation time becomes so long that the optimal solution cannot be reached in practical time. To overcome this limitation and to handle more design variables within a practical computation time, a new method for efficient sizing optimization using artificial neural networks (ANNs) is proposed. ANN, a method of machine learning, requires a large amount of training data to train networks, but once training is completed, inference can be performed in a small amount of time. In the proposed method, networks that infer objective function and constraint conditions from design variables are trained using the training data collected by existing FEM software and the trained networks then evaluate design proposals instead of FEM software during sizing optimization. It takes a lot of time to collect training data using FEM software, but the time required to evaluate design proposals using the trained networks during sizing optimization is almost 0. Therefore, if the number of evaluations of design proposals performed until reaching the optimal solution in sizing optimization is greater than the number of training data required to train the networks, the total calculation time can be reduced. In order to be applicable to more diverse design problems, the proposed method handles not only structural dimensions such as the length and thickness but also discrete numbers such as the number of ribs and spars of aircraft wings. In addition, to reduce the burden of training data collection, fine tuning is used to reuse the weights of the networks obtained from previous product designs using the proposed method. Using fine tuning, networks can be trained with a smaller number of training data than when network weights are set randomly. This approach is not applicable to completely new designs but is very effective in

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

a corporate environment where the same type of products is designed many times. In the case study, the proposed method is applied to the optimal design of an aircraft wing.

2 RELATED RESEARCH

In recent years, there have been attempts to use machine learning to improve the computational efficiency of structural optimization. When classifying the methods proposed so far, there are two main aspects: the first is whether training processes are done offline or online. The second is the inference target, i.e., whether optimal structures are directly inferred or FEM analysis results and sensitivities required during the optimization iterations are inferred. This section first introduces the sizing optimization researches, which is the subject of this research, followed by an introduction to topology optimization researches, where various methods have been proposed.

As for sizing optimization, Gajewski et. al. proposed a hybrid system combining ANN and FEM and applied it to sizing optimization of an elevator rudder of Bryza, a patrol aircraft derived from a small transport plane made in the former Soviet Union [4]. Yan et al. proposed a method consisting of reinforcement learning and transfer learning [14]. Reinforcement learning is applied to extract the optimization experience from the semi-empirical method DATCOM using deep neural networks while transfer learning is implemented to reuse the experience as priori knowledge in the CFD (Computational fluid dynamics)-based optimization by sharing neural network parameters. They applied it to sizing optimization of a missile wing and found that CFD calls were significantly reduced. Viquerat et. al. proposed the method for generating optimal shapes through deep reinforcement learning (DRL) coupled with CFD without any prior knowledge and in a constrained time [12]. As for topology optimization, Chandrasekhar et. al proposed topology optimization using neural networks named "TOuNN" [2]. This method adopts SIMP method and infers the optimal material layout, or optimal structural topology, by learning the density, a parameter that defines the material layout, using NNs. The method proposed by Chi et. al. [3] adopts an online strategy rather than an offline one. Machine learning model learns the underlying mapping between design variables and their corresponding sensitivities from those history data (e.g., design variables, their corresponding sensitivities, and displacement solutions) during optimization process and derives sensitivities by inference rather than computation. Kollmann et al. employed metamaterials as design targets and attempted to directly infer the optimal topology in the form of images using CNNs (Convolutional neural networks) [6] Wang et. al. proposed a deep convolutional neural network with perceptible generalization ability for structural topology optimization and confirmed that a significant reduction in computation cost was achieved with little sacrifice to the performance of design solutions. [13]. Zhang et. al. did not use machine learning for structural optimization itself, but proposed a new sampling method for airfoils and wings, which is based on a deep convolutional generative adversarial network to improve the efficiency of surrogate-based shape optimization [7]. In addition, commercial engineering optimization software with machine learning functions, such as modeFRONTIER [8] and ODYSSEE [9] is developed.

3 PROPOSED METHOD

To enable efficient sizing optimization while simultaneously handling many design variables, the proposed method evaluates design proposals during iterative calculations of sizing optimization using pre-trained ANNs instead of FEM. The proposed method consists of the following four steps:

Step 1: Sampling the design proposals
Step 2: Collection of training data
Step 3: Training the networks
Step 4: Sizing optimization using ANNs

Details of each step are described in the subsequent sections. Then, introduction of fine tuning to further improve the efficiency of the proposed method is described. Finally, considerations for applying the proposed method to practical problems are discussed.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

3.1 Step 1: Sampling the Design Proposals

Design variables for sizing optimization are selected from the parameters that define the design objects. Not only structural dimensions such as the length and thickness but also discrete numbers such as the number of ribs and spars of aircraft wings are handled as design variables in the proposed method. The number of design variables should be as small as possible because increasing the number of design variables dramatically increases the number of training data needed for training. The upper and lower limits of design variables in sizing are then determined. Design proposals are finally sampled by varying the design variables between the upper and lower limits. In order to sample design proposals as uniformly as possible from the design space, Latin hypercube sampling (LHS) is used.

3.2 Step2: Collection of Training Data

The design proposals sampled in Step 1 are analyzed using FEM software to obtain objective function and constraint conditions for sizing optimization in Step 4. Pairs of design variables and the objective function and constraints obtained by the analysis become the training data.

3.3 Step3: Training the Networks

ANNs that infer objective function and constraint conditions from design variables are trained using the training data set collected in Step 2. The design variables selected in Step 1 are handled as input while the objective function and constrained conditions used in sizing optimization of Step 4 are handles as output. Any type of ANN can be used here. For example, the case study uses the ANN where nodes are fully connected across layers and back-propagation is used to adjust connection weights between nodes. As for the configuration of hidden layer, although a single layer is often used in a typical ANN, multiple layers can also be used. The number of nodes in each hidden layer is also an important factor and should be configured according to the design problem. In the case study, hidden layers in various configurations are tested to verify inference accuracy.

3.4 Step4: Sizing Optimization using ANNs

Sizing optimization is performed by evaluating the design proposal using the networks trained in Step 3 instead of FEM software. Any optimization algorithm can be used for sizing optimization, such as the steepest descent, PSO, or GA.

3.5 Fine Tuning for Recycle of the Past Trained Networks

Usually, when training a network, initial values of network weights are set randomly. Fine tuning is a technique for training a new network using the weights of the network previously trained with other training data as initial values. Fine tuning allows for a more accurate network with less training data. In practical product development, since the same type of products are designed many times, The scenario of reusing training data and trained networks for the same type of product obtained in past product development makes sense.

3.6 Practical use of the Proposed Method

This section describes some of the considerations necessary for the practical use of the proposed method.

Setting the appropriate number of training data is one of the most important issues in any machine learning method. If the number of training data is insufficient, the accuracy of the network may be lower than expected. If the number of training data is excessive, the network may be over-fitted, or it may simply be too hard to collect the data. However, the appropriate number of training data cannot be estimated in advance. Therefore, when actually using the proposed method, a certain number of training data should be prepared and trained at first to see if sufficient accuracy can be obtained. If the accuracy is not sufficient, the training data is increased,

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

and training of the network is attempted again. Such a trial-and-error process seems like a waste of time. In practice, however, the time required to collect training data in step 2 is extremely long, but the time required for the other tasks is short. For example, in the case study described in the next section, it takes 40 hours to collect 1000 training data, while sampling of design proposals, training of ANNs, and even PSO takes only a few minutes each. Therefore, it is a good strategy to gradually increase the number of training data while observing the training results of the ANN in order to avoid collecting more training data than necessary.

Configuration of hidden layers of the network also has a significant impact on its inference accuracy. Since the appropriate depth of the hidden layer and the number of nodes cannot be estimated in advance, they must be determined to some extent by trial and error.

4 CASE STUDY

In order to confirm the effectiveness of the proposed method, it is applied to aircraft wing design. Aircraft is one of the mechanical products with the most severe weight constraints. By reducing airframe weight, payload and fuel loading can be increased as well as fuel consumption can be reduced. Aircraft are inherently less fuel-efficient transportation machines than ships, automobiles, and railroads, and the recent increase in environmental awareness has made it imperative to improve fuel efficiency of aircraft. In order to reduce the aircraft weight as much as possible, various types of optimizations, including sizing optimization, are used in actual product development [5]. As described later, an aircraft wing consists of internal structural parts called ribs and spars and a skin that surrounds them. Since the number of ribs and spars can be varied in addition to the dimensions of each part, both the structural dimensions and the number of internal structural parts can be used as design variables. In addition, since the cross section of a wing becomes progressively smaller from the root to the tip and their cross sections are similar, a wing can be modeled with a not very large number of design variables. Furthermore, since the basic structure is the same regardless of the type of airplane, fine tuning is easy to apply. For these reasons, the proposed method was used for the optimal design of aircraft wings.

Section 4.1 details the modeling, learning, and sizing optimization of aircraft wings. Section 4.2 describes the results. Section 4.3 describes the verification of the effectiveness of fine tuning. Finally, Section 4.4 provides a summary.

4.1 Details of the Case Study

In the case study, the wings of Hawker Tempest, Mitsubishi A6M Zero, and Kawasaki Ki-45 Toryu, fighters used in World War II, are used as design targets. This is because it is easy to model due to its simple structure and geometry compared to the wings of modern aircrafts, and to collect references describing its structure. From a structural point of view, an aircraft wing is roughly composed of three elements: rib, spar, and skin. Based on the drawings available in the literature, wing models of these aircraft, which consist of three elements, were created. Ribs and spars usually have an I-shaped cross section, but they were simplified to a rectangular cross section. All structural elements are modeled by shell elements. In actual aircrafts, there are cutouts in these structural elements due to the presence of devices inside the wings and moving surfaces such as ailerons and flaps, but these are not considered in this case study. Figure. 1 shows the created models. As for design variables, the thickness of the ribs, spars, and skins, the length of the ribs, and the number of ribs are treated. For the thickness, as also shown in Figure. 1, all the structural elements are divided into 9 groups, and the thickness is configured for each group. The range of thicknesses is shown in Table. 1. For the rib length, the rib length of the original fuselage is considered to be 100%, and the rib length is expressed as X% of that length. All ribs are divided into 5 groups as shown in Figure. 2 and rib length is configured for each group. The range of rib lengths is shown in the Table. 2. The number of ribs varies from 26 to 29, with 27 as the initial value. The total number of design variables is 15: 9 for thickness, 5 for rib length, and 1 for the number of ribs.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

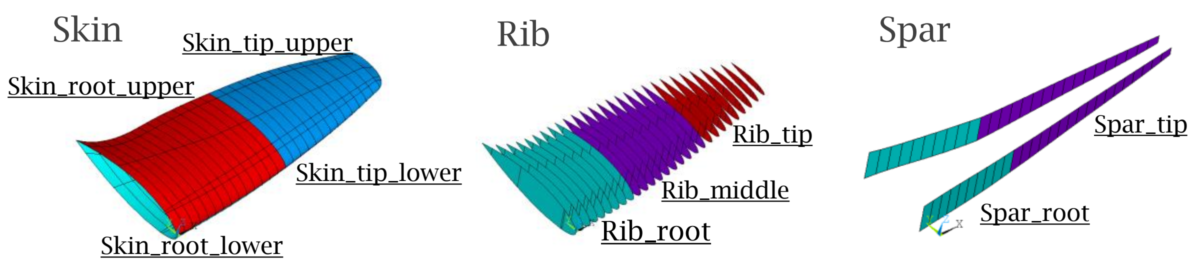

Figure 1: Wing model and design variables concerning thickness.

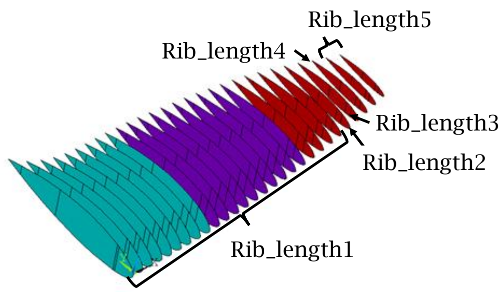

Figure 2: design variables concerning rib length.

	Min (mm)	Initial (mm)	Max (mm)
Skin_root_upper	0.8	2	2
Skin_tip_upper	0.5	1.5	1.5
Skin_root_lower	0.8	2	2
Skin_tip_lower	0.5	1.5	1.5
Rib_root	3	8.9	10
Rib_middle	3	8.9	10
Rib_tip	3	8.9	10
Spar_root	30	55.7	60
Spar_tip	10	24.8	25

Table 1: Range of thicknesses.

Aircraft type	# of ribs	Max/Min	Rib length1	Rib length2	Rib length3	Rib length4	Rib length5
Tempest	26-28	Max	105%	102%	101%	101%	102%
		Min	98%	99%	99%	98%	95%
	29	Max	105%	102%	101%	101%	102%
		Min	98%	99%	99%	98%	95%
Zero	26-29	Max	105%	102%	101%	101%	102%
		Min	98%	99%	99%	98%	95%
Toryu	26-29	Max	105%	102%	101%	101%	102%
		Min	98%	99%	99%	98%	95%

Table 2: Range of rib length.

As for the number of training data, in order to discuss the effect of the number of training data on the inference accuracy and optimization results, the proposed method was run while varying the number of training data from 1000 to 20000. As for the objective function and constraint conditions of sizing optimization, the total weight was used as the objective function, while the

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

maximum stress and natural frequency were used as constraints. In order to collect these values of the design proposals sampled in Step 1, ANSYS was used for modeling and analysis, and MATLAB was used for planning data collection and controlling ANSYS. As for ANNs, the machine learning toolbox in MATLAB was used for network creation, training and inference. A typical ANN in which nodes are fully coupled between layers and back-propagation is used to adjust the network connection weights was used. Three networks were created and trained to infer weight, maximum stress, and natural frequency separately. The inputs are 15 design variables. In order to discuss the effect of network configuration on the inference accuracy and optimization results, networks with two, three or four hidden layers were tested in addition to the standard networks with one hidden layer. Similarly, networks with different number of nodes in the hidden layers were tested. In training, 70% of the training data was used as training data, 15% as cross-validation data, and 15% as test data. As for sizing optimization, PSO, which is implemented as a solver in MATLAB, was used.

4.2 Results of Sizing Optimization

Table. 3 shows the number of hidden layers, the number of nodes in the hidden layers, the number of training data, and the average error for the obtained networks that infer maximum stress, natural frequency, and weight. Here, the hidden layer configuration of 50_60_50_5 means that the network has four hidden layers, and each hidden layer has 50, 60, 50, and 5 nodes. Figure. 3 shows the effect of the number of training data on mean error of the networks that infer maximum stress for Zero. By increasing the number of training data, the average error decreases. The required number of training data depends on how much accuracy is required. Tab. 4 shows the effect of hidden layer configuration on average error of inference. These are the training results using 20000 data. The left table shows the average error of the networks having a single hidden layer while the right one shows the average error of the networks having multiple hidden layers. In this case, the networks with multiple hidden layers generally had better inference accuracy than the networks with a single hidden layer, but in some cases the inference accuracy between them was not significantly different. The number of hidden layers and the number of nodes in each layer also greatly affects inference accuracy. In this case, the inference accuracy was generally better for the networks with a larger number of hidden layers and nodes in each layer. Therefore, it is necessary to configure an appropriate hidden layer that matches the applicable target and evaluation criterion. Next, the optimal solution derived using the learned ANN and PSO and the analytical results of the optimal solution using ANSYS are shown in Table. 4. This result shows that the weight and natural frequency can be inferred with great accuracy, but the maximum stress has an error of several percent in the optimal solution. It is dangerous if the maximum stress is inferred to be lower than it should be. Table 5 shows the estimated computation time for the proposed method and conventional sizing optimization. This estimation of computation time does not include various types of trial-and-error.

In the case of Zero, the network that infers the maximum stress requires the most training data, which is 14000. The time required to collect one training data set, or in other words, the time required for one analysis using ANSYS, is on average 144 seconds. About 280 hours are required to collect 14000 training data. The time required for sampling, training, and optimization is less than one hour. Therefore, the computation time of the proposed method for Zero was estimated to be 280 hours. On the other hand, 105016 inferences are performed to reach the optimal solution in the sizing optimization of the proposed method. If 105016 analyses were performed during conventional dimensional optimization, it would take 144 seconds multiplied by 105016, so the computation time of the conventional sizing optimization for Zero was estimated to be 4201 hours. This result shows that the proposed method can significantly reduce the computation time in both cases.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

Aircraft type	Objective function / constraint conditions	Configuration of hidden layer	Training data	Training time (s)	Average error (%)
Tempest	Maximum stress	20_30_20_5	7000	329	0.74
	Natural frequency	20_30_20	7000	351	1.09
	Weight	20_30_20	7000	294	0.0061
Zero	Maximum stress	50_60_50_5	14000	844	0.79
	Natural frequency	30_40_30	7000	412	0.73
	Weight	20_30_20_10	11000	374	0.0046
Toryu	Maximum stress	50_70_50_5	9000	764	0.82
	Natural frequency	120_120_50	5000	808	1.16
	Weight	20_40_20	8000	368	0.0048

Table 3: Obtained ANNs that infer maximum stress, natural frequency, and weight.

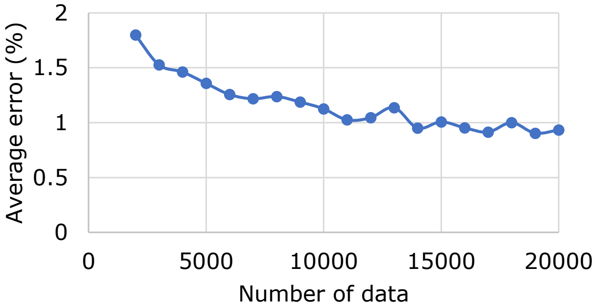

Figure 3: Effect of the number of training data on average error.

Configuration of hidden layer	Average error (%)
10	2.36
20	1.67
30	1.33
40	1.45
50	1.28
60	1.20
70	1.27
80	1.27

Configuration of hidden layer	Average error (%)
10_10	1.27
20_20	0.82
30_30	0.71
40_40	0.65
50_50	0.63
70_70	0.64
100_100	0.88
50_60_50_5	0.55

Table 4: Effect of hidden layer configuration on average error.

Airfraft type	The number of inference	ANN		FEM		Error		
		Maximum stress (Mpa)	Weight (Kg)	Maximum stress (Mpa)	Weight (Kg)	Maximum stress (%)	Weight (%)	Natural frequency (%)
Tempest	57016	10.96	957.1	11.76	957.3	6.83	0.0204	0.76
Zero	105016	6.1	786.6	6.33	786.7	3.62	0.0107	0.58
Toryu	52816	17.34	799.4	18.02	799.4	3.76	0.0005	2.58

Table 4: Optimal solution of sizing optimization.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

Airfraft type	Collection of training data	Sizing optimization
Tempest	280	2281
Zero	560	4201
Toryu	360	2113

Table 5: Calculation time.

4.3 Effectiveness of Fine tuning

This section verifies the effectiveness of fine tuning. The final weights of the network trained for Tempest are used as the initial values for Zero. Multiple training runs are performed while varying the number of training data from 1000 to 15000, and mean error and MSE (mean square error) of inference of the trained networks is evaluated. For comparison, a network is also trained using randomly set initial weights. Figure 4 shows the effect of the number of training data on mean error and MSE with and without fine tuning. These figures show that fine tuning improves the inference accuracy of the network regardless of the number of training fata.

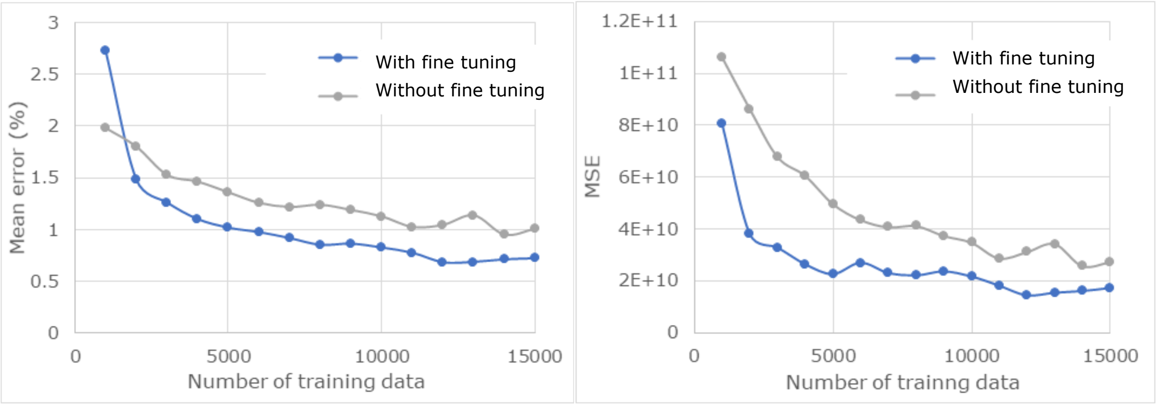

Figure 4: Effect of fine tuning.

4.4 Discussion

The proposed method has succeeded in significantly improving the efficiency of dimensional optimization. In addition, by reusing weights from previously trained networks using fine tuning, a network with high inference accuracy can be obtained with less training data. On the other hand, precautions are as follows. Since the inference error of the trained network cannot be zero, the results of sizing optimization also contain errors. Therefore, verification of the optimal shape is essential. Since the number of training data depends on the problem, it must be determined by trial and error. However, the time required for sampling, training, and optimization of design alternatives is very short compared to the time required to collect training data, so the number of training data required to obtain a network with sufficient inference accuracy can be minimized by increasing the training data incrementally. Since configuration of hidden layers of the network also depends on the problem, trial and error is also required.

5 CONCLUSION

In this paper, an efficient sizing optimization method using artificial neural networks is developed in order to handle more design variables within a practical computation time. The features of the

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

proposed method are that structural topology, e.g., the number of ribs, can be treated as a design variable, and that the number of data required to train networks can be reduced by reusing data collected in the previous optimal designs using fine tuning. As confirmed in the case study, collection of training data takes an enormous amount of time, while training of the network and sizing optimization using the trained network are completed in a very short time. Therefore, a realistic use is shown, i.e., the number of training data is gradually increased while checking the inference accuracy of the network and the optimal results of dimensional optimization. In the case study, the proposed method was applied to the optimal design of three World War II fighter wings, and it was confirmed that optimal results could be obtained in a significantly shorter time than traditional sizing optimization. In addition, networks were trained with different configurations of hidden layers and different numbers of training data, and the results show that those factors have a significant impact on the inference accuracy of those network. It was also confirmed that the network with good accuracy can be obtained with a small amount of training data by conducting fine tuning using the training data collected for other fighters. In the case study, the proposed method was applied to the wings of World War II fighters. However, the proposed method can be applied not only to classic aircraft wings as shown in the case study, but also to modern aircraft wings, and even to various types of structures other than aircraft wings, if they have structural dimensions and thickness whose values can be changed as design variables. Also, if multiple design targets can be modeled using the same design variables, like the three types of aircrafts in the case study, the training data used in previous optimal designs can be reused to reduce the number of training data to be newly collected. This streamlines the development of product variations and design based on existing products, which is common in many companies.

Masakazu Kobayashi, http://orcid.org/0000-0002-1212-2879
Yuki Ogawa, http://orcid.org/0000-0002-7071-3239

REFERENCES

[1] Bendsoe, M. P.; Kikuchi N.: Generating optimal topologies in structural design using a homogenization method, Computer Methods in Applied Mechanics and Engineering, 71(2), 1988, 197-224. https://doi.org/10.1016/0045-7825(88)90086-2
[2] Chandrasekhar, A.; Suresh, K.: TOuNN: Topology Optimization using Neural Networks, Structural and Multidisciplinary Optimization, 63, 2021, 1135–1149. https://doi.org/10.1007/s00158-020-02748-4
[3] Chi, H.; Zhang, Y.; Tang, T. L. E.; Mirabella, L.; Dalloro, L.; Song, L.; Paulinoa, G. H.: Universal machine learning for topology optimization, Computer Methods in Applied Mechanics and Engineering, 375, 2021. https://doi.org/10.1016/j.cma.2019.112739
[4] Gajewski, J.; Golewski, P.; Sadowski, T.: Geometry optimization of a thin-walled element for an air structure using hybrid system integrating artificial neural network and finite element method, Composite Structures, 159, 2017, 589–599. https://doi.org/10.1016/j.compstruct.2016.10.007
[5] Grihon, S.; Krog, L.; Bassir, D.: Numerical Optimisation applied to structure sizing at AIRBUS: A multi-step process, International Journal for Simulation and Multidisciplinary Design Optimization, 3(4), 2009, 432 – 442. https://doi.org/10.1051/ijsmdo/2009020
[6] Kollmann, H. T.; Abueidda, D. W.; Koric S.; Guleryuz, E.; Sobhc, N. A.: Deep learning for topology optimization of 2D metamaterials, Materials & Design, 196, 2020. https://doi.org/10.1016/j.matdes.2020.109098
[7] Li, J.; Zhang, M.; Martins, J. R. R. A.; Shu, C.: Efficient Aerodynamic Shape Optimization with Deep-Learning-Based Geometric Filtering, AIAA journal, 58(10), 2020, 4243–4259. https://doi.org/10.2514/1.J059254
[8] modeFRONTIER, https://engineering.esteco.com/, ESTECO.
[9] ODYSSEE, https://www.mscsoftware.com/, MSC Software.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net

[10] Schmit L. A.: Structural design by systematic synthesis, Proceeding of Second Conference on Electronic Computation ASCE, 1960, 105-122.
[11] SolidWorks, https://www.solidworks.com/, Dassault Systems.
[12] Viquerat, J.; Rabault, J.; Kuhnle, A.; Ghraieb, H.; Larcher, A.; Hachem, E.: Direct shape optimization through deep reinforcement learning, Journal of Computational Physics, 428, 2021. https://doi.org/10.1016/j.jcp.2020.110080
[13] Wang, D.; Xiang, C.; Pan, Y.; Chen, A.; Zhou, X.; Zhang, Y.: A deep convolutional neural network for topology optimization with perceptible generalization ability, Engineering Optimization, 2021. https://doi.org/10.1080/0305215X.2021.1902998
[14] Yan, X.; Zhu, J.; Kuang, M.; Wang, X.: Aerodynamic shape optimization using a novel optimizer based on machine learning techniques, Aerospace Science and Technology, 86, 2019, 826–835. https://doi.org/10.1016/j.ast.2019.02.003
[15] Zienkiewicz, O. C.; Campbell, J. S.: Shape optimization and sequential linear programming, Optimum Structural Design, John Wiley, New York, NY, 1973.

© 2023 CAD Solutions, LLC, http://www.cad-journal.net