Research on Post-processing Method of Topology Optimization Model

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Abstract. The boundary diffusion phenomenon often occurs in structural topology optimization-based on the variable density method, and the inevitable jagged boundary also appears in optimized model, which increases the complexity of the structure and the difficulty of manufacturing. Aiming at solving above problems, a post-processing method of the topology optimization model is proposed. The partition weighted sensitivity filtering method is used to remove the gray value to obtain the topology optimization structure with clear boundaries. Then the optimized structure is binarized and the jagged boundary line is extracted to obtain the target discrete angle point set, which is used as sample points for curve fitting to obtain the optimized structure with smooth boundary. The feasibility and effectiveness of the boundary smoothing post-processing method in solving the boundary diffusion and jagged boundary problems are verified by some typical arithmetic examples. This method can effectively obtain a topologically optimized structure with clear, smooth and accurate boundaries, and reduce the difficulty of model reconstruction and fabrication while ensuring the structural performance within the permitted range.

Keywords: topology optimization, boundary diffusion, variable density method, jagged boundary, sensitivity filtering method, boundary smooth post-processing method
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1 INTRODUCTION

Structural topology optimization, as an effective structural optimization method, has become a major research hot spot in the field of structural optimization. It can obtain a best design at the early stage of design, which has reasonable way of material distribution and a stronger and lighter optimized structure [21]. Structural optimization can be classified as shape optimization, size optimization, and topology optimization [14]. Different from shape and size optimization, topology optimization, as a free-form material distribution scheme, enables the creation, merging and splitting of the interior solids and voids during the structural evolution. Therefore, it can explore a larger design space and obtain superior structural performance as can be expected compared to size and shape optimization [8]. Currently, the common topology optimization methods mainly include the variable density method, homogenization method, evolutionary structural optimization and level set method [19]. Among them, the variable density method has gradually become a popular method to solve structural topology optimization problems due to its advantages of less design variables and higher efficiency.

Problems such as boundary diffusion, tessellation lattice and mesh dependence may occur when the variable density method is used for optimization, increasing the difficulty of manufacturing the optimized structure [13]. Among them, the boundary diffusion phenomenon is a common problem affecting post-processing in the topology optimization process, and many intermediate density cells appear in the optimization results. Post-processing by curve approximation or curve fitting without solving the boundary diffusion problem does not result in an ideal and accurate boundary. It also increases the geometric complexity and reduces the efficiency of the optimization. In order to ensure optimization results practicable , scholars at home and abroad have conducted in-depth researches on how to solve the problem of numerical instability arising in the process of topology optimization. Sigmund [12] introduced the minimum filtering radius for distance-weighted averaging of the sensitivity filtering method to effectively solves the problems of tessellation grid and grid dependence, but low optimization efficiency and boundary diffusion are not solved by this method. Guest [5] controlled the boundary diffusion phenomenon by introducing the method of Heaviside function, but the number of iterations is high and the efficiency is low. Yin [18] proposed to establish an efficient topology optimization model based on probabilistic reliability with mass and displacement as objective functions and constraints, which can quickly obtain clear topology optimization boundaries with fast iteration, but individual regions still have gray-scale cells. Lian [7] proposed a sensitivity filtering method considering partitioning to effectively suppress the boundary diffusion phenomenon with low optimization efficiency, while slender rods or porous structures may appear to reduce manufacturing difficulty of the structure.

Besides boundary diffusion will hinder the fabrication of optimized structures, the presence or absence of unit cells is often used to characterize the material characteristics in the continuum structure, and the jagged boundaries appearing in the optimization results also increase the difficulty of fabricating the structure. However, the topology optimization needs to be attached to the mesh so that the jagged boundary of the optimization result is inevitable, which greatly limits the development of the variable density method in the field of structural topology optimization. Therefore, it is necessary to post-process the jagged boundary of the optimized structure in order to improve the smoothness of the topology-optimized boundary and the overall quality of the topology-optimized structure. Liu [9] introduced volume constraints for boundary smoothing, which can effectively prevent the problem of mesh shrinkage. But it is mainly used for smoothing surfaces, and it is rarely used in two-dimensional boundary smoothing. Hashemian [6] proposed a method of curve fitting combined with smooth processing dynamics to achieve the desired results.But it needs to apply the fitting and fairing procedures simultaneously to achieve more desirable results, and the processing process is more complicated. Birk [1] proposed an automatic differentiation method to control the degree of curve smoothing, which has better flexibility in choosing constraints and the smoothing effect is more stable. But it takes a lot of experimentation and data calculation to get the desired shape, and is very labor-intensive. Sun [16] proposed to combine the particle swarm optimization algorithm into B-sample curve fitting, which is able to achieve a smooth boundary with fewer control points and optimization time. However, it is easy to fall into local optimum, while it is difficult to improve the fitting accuracy.

Aiming at solving the problems of boundary diffusion and jagged boundaries which will make structures cannot be fabricated, this paper proposes a post-processing method of topology optimization models based on the variable density method. The method first adopts a partitioned weighted sensitivity filtering method instead of traditional sensitivity filtering method in the variable density method, and introduces a new weighting factor to weight different regions to weaken the boundary diffusion phenomenon. Then the optimization model is binarized and analyzed to extract the serrated boundary lines. Finally, the target discrete angle point set is obtained by numerical calculation which is used as the reference point for curve fitting based on the ordinary least squares method to obtain the model structure with smooth boundaries. The method can effectively solve the problems of boundary diffusion and jagged boundary while ensuring the original design requirements of the optimized structure, and greatly reduces the manufacturing difficulty of the topology optimized structure.

2 TOPOLOGICAL OPTIMIZATION THEORY OF THE VARIABLE DENSITY METHOD

2.1 Topology Optimization Model of the Variable Density Method

Structural topology optimization is one of the most useful numerical optimization tools to develop new lightweight design concepts by determining the optimum material layout within a given design domain. It is a structural optimization method that seeks the best distribution form of the structure in a given design space with the optimization objective of obtaining greater stiffness and lighter weight [4]. It has more design freedom and can overcome the limitations of parametric processing of structures to obtain a larger design area [3]. The most widely used topology optimization method in engineering applications is the variable density method. The theoretical material interpolation model of the variable density method is to convert discrete problems into continuous optimization problems by introducing intermediate density elements. In reality, the intermediate density cells cannot exist and cannot be manufactured, so it is necessary to penalize the intermediate density value in the design variables to avoid the generation of intermediate density cells. The numerical expression takes the relative density as the design variable and the mathematical expression of the modulus of elasticity and density of the material satisfies:

$$E_i = E_{\min} + (l_i)^p (E_0 - E_{\min}), l_i \in [0,1] \tag{1}$$

Here, the E_i is the modulus of elasticity of the cell after interpolation. The $E_{\min}$ is the modulus of elasticity of the material in the hole part. The l_i is the relative density of the cell, taking a value of 1 means that there is material, taking a value of 0 means that the hole. The P is the penalty factor. The E_0 is the modulus of elasticity of the solid part of the material.

In the topology optimization process, a volume constraint or a mass constraint is set, and the minimum flexibility value is used as the optimization objective to determine the functional relationship between the material cell density and the material properties [17]. The mathematical model of the periodic topological optimization problem based on the variable density method can be expressed as:

$$\begin{cases} \text{find} \quad L = \{L_{1,1}, L_{1,2}, \cdots, L_{i,j}\}^{\mathrm{T}} \in \Omega \\ \qquad\quad i = 1,2,\ldots,m \quad j = 1,2,\ldots,n \\ \min \quad C(L) = U^{\mathrm{T}} K U = \sum_{i=1}^{m}\sum_{j=1}^{n} (L_{i,j})^p u_{i,j}^{\mathrm{T}} k_0 u_{i,j} \\ \text{s.t.} \quad F = KU \\ \qquad\quad V = fV_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} L_{i,j} v_{i,j} \\ \qquad\quad 0 < L_{\min} \le L_{i,j} \le L_{\max} \le 1 \end{cases} \tag{2}$$

Here, the L is the cell relative density vector. The $L_{i,j}$ is the relative density of the j-th cell in the i-th subdomain. The n is the number of cells in the subdomain. The $C(L)$ is the minimum value of flexibility for a given topology. The U is the structural displacement vector of the cell node. The K is the structural stiffness matrix. The $u_{i,j}$ are the cell displacement vectors. The K_0 is the initial cell stiffness matrix. The F is the load vector applied by the element nodes. The V is the volume of the optimized structure. The f is the retained volume fraction. The V_0 is the initial volume. The $v_{i,j}$ is the cell volume. The L_{min} is the minimum value of the design variable. The L_{max} is the maximum value of the design variable.

2.2 Optimization Criterion Method for Topology Optimization

In order to obtain more desirable topology optimization results, a suitable numerical solution algorithm is required. The widely used numerical solution algorithms for topological optimization are optimization criterion method, mathematical programming method and random search algorithm [10]. Among them, the optimization criterion method specifies the optimal criterion that can handle the corresponding constraint from the viewpoint of mechanics principles to build an optimization iterative mathematical model accordingly. The Lagrange equation is obtained by associating the objective function and the predetermined constraints, establishing an optimization iteration based on the Kuhn-Tucker optimization condition, and transforming the combination of the constraints and the objective function into a zero-constraint problem [20]. The iterative convergence of the optimization criterion method is fast, computationally small, and does not become complicated with the increase of structural complexity and design variables. Therefore, the optimal criterion method is chosen as the numerical solution algorithm in this paper. The core idea of the optimization criterion method is to construct the following Lagrangian function by introducing a Lagrangian multiplier:

$$L = C(l_i) + \lambda_1\Big(\sum_{i=1}^{n} v_i l_i - V\Big) + \lambda_2^{T}(KU - F) + \sum_{i=1}^{n} \alpha_i (l_{\min} - l_i) + \sum_{i=1}^{n} \beta_i (l_i - l_{\max}) + \mathrm{C} \tag{3}$$

Here, the λ_1, λ_2, α and β are Lagrangian multipliers. With this model it is possible to transform a nonlinear programming problem with inequality constraints into an unconstrained problem.

The topology optimization of the continuum structure contains more design variables and the unknown variables are very computationally intensive. In order to ensure the stability and efficiency of the entire iterative process, certain limits need to be placed on the amount of variation in the design variables during the optimization process. An improved iterative format of enlightenment, can effectively update design variables, and its iterative expression of design variables can be expressed as:

$$\rho_e = \begin{cases} \max(l_{\min}, l_e - t) & l_e B_e^{\eta} \le \max(l_{\min}, l_e - t) \\ l_e B_e^{\eta} & \max(l_{\min}, l_e - t) \le l_e B_e^{\eta} \le \min(1, l_e + t) \\ \min(1, l_e + t) & \min(1, l_e + t) \le l_e B_e^{\eta} \end{cases} \tag{4}$$

Where

$$B_e = \Big(-\frac{\partial C}{\partial l_e}\Big) / \Big(\xi \frac{\partial V}{\partial l_e}\Big) \tag{5}$$

Here, the l_e is the relative density of cell *e*. The t is the panning limit. The η is the damping factor. The B_e is the intermediate variable. The ξ is a Lagrangian multiplier.

3 TOPOLOGY OPTIMIZATION POST-PROCESSING FLOW

The post-processing of topology optimization can effectively remove gray cells and jagged boundaries, improve the boundary smoothness of the topology structure and the performance related to manufacturing, and then improve the overall quality of structural topology optimization. A topology optimization post-processing method based on the variable density method is proposed in this paper. Firstly, finite element analysis is performed on the target model, and the partitioned weighted sensitivity filtering method is used to remove the gray cells at the boundary to obtain a structure with clear boundaries. Then, the obtained results are subjected to a binarization to obtain optimized results with density values fully distributed as 1 and 0. Next, edge detection is performed to extract the boundaries. Finally, corner points are extracted from the boundary lines to obtain a sample point set, and the point set is used as a benchmark for curve fitting to obtain a topological optimization model structure with clear, smooth and easy manufactured boundaries. The post-processing flow chart of the method is shown in Fig. 1.

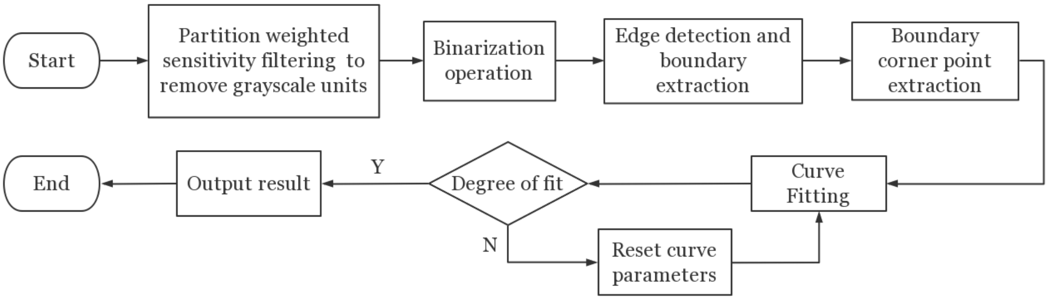


Figure 1: Topology optimization post-processing flow chart.

3.1 Sensitivity Filtering Method

3.1.1 Sigmund sensitivity filtering method

Numerical instabilities such as grid dependencies and tessellation lattices are often observed in topology optimization based on the variable density method. Sigmund [12] proposed a sensitivity filtering method that has become the most widely used class of heuristic algorithms in topological optimization today. It belongs to local constraint methods, which achieve the elimination of problems such as tessellation and network dependencies by filtering the cell sensitivity. The schematic diagram of the Sigmund sensitivity filtering area is shown in Fig. 2. The main idea is to first determine a central cell, and then set a sensitivity filtering area with a minimum filtering radius of r_{min}. Then a weighting factor is introduced to weight the distance of each cell to the central cell, so that each cell close to the central cell gets a higher sensitivity value, while each cell far from the central cell at the sensitivity filtering boundary gets a lower sensitivity value. Replacing the sensitivity value of the central cell with the weighted average of the sensitivities of the cells in the filtered area leads to a general downward trend of the sensitivity value from the central cell to the boundary, preventing drastic fluctuations of the cell density values in the local design area.

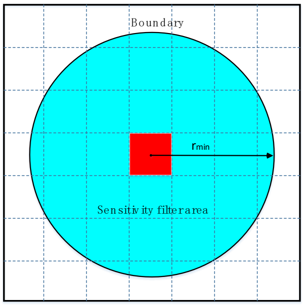


Figure 2: Schematic diagram of the Sigmund sensitivity filter area.

By constructing the average sensitivities of all cells in the sensitivities filtering region, the original cell sensitivities are replaced by the original cell sensitivities for the subsequent iterations. The weighting factor can be used to assign the weight between cells. The smaller the distance between two cells in the filtered area, the greater the weight, the greater the distance, and the smaller the weight. And r_{min} must be larger than 1. When r_{min} tends to 1, the mean value of the constructed sensitivities approximates the original cell sensitivities and cannot eliminate the tessellation problem. The mathematical expressions of the filtered cell sensitivities and the weighting factors are:

$$\begin{cases} \dfrac{\bar{\partial} C}{\partial l_e} = \dfrac{\sum_{i=1}^{n} \tilde{H}_i l_i \dfrac{\partial C}{\partial l_i}}{l_e \sum_{i=1}^{n} \tilde{H}_i} \\ \tilde{H}_i = r_{\min} - dist(k,i), \{i \in N, \ dist(k,i) \le r_{\min}\}, r_{\min} > 1, k = 1,2,\ldots,n \end{cases} \tag{6}$$

Here, the l_i and l_e are the relative densities of the corresponding cells. The n is the number of cells. The $\tilde{H}_i$ is the weighting factor. The $r_{\min}$ is the minimum filtration radius. The $dist(k,i)$ is the center distance between the corresponding two cells.

From equation (6), it can be seen that the weighting factor is taken linearly according to the distance between cells, and the weighting factor tends to decrease linearly from the central cell to the cell where the filtering boundary is located. This ensures that the cells close to the center get higher weight values, but the cells close to the filtering boundary will also have a greater impact on the sensitivity of the center cell, and the topologically optimized boundary will have an overly smoothed optimization result, which may lead to the problem of boundary spreading. Now taking a two-dimensional plane stress structure as an example, the design area is 80mm×80mm, and the mesh division is 80×80. The left end is restrained in full plane, and the upper and lower right corners of the structure are subjected to vertical upward and vertical downward loads respectively, and the model is restrained and loaded as shown in Fig. 3.

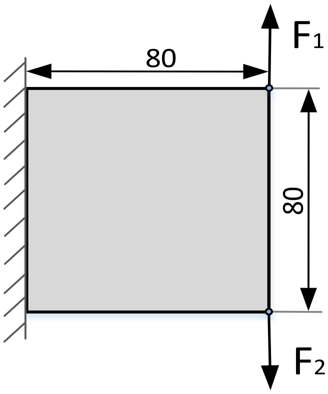


Figure 3: Schematic diagram of the algorithm model.

The Sigmund sensitivity filtering method does not introduce new constraints, making the optimization process relatively simple and easy to apply in practical problems, and it has also become a widely used topological optimization method in the traditional variable density method. However, this method inevitably results in boundary diffusion, where intermediate density cells appear in a laminar distribution at the inner and outer boundaries of the structure. The traditional variable density method is used to optimize the topology of the above example, and the optimization results are shown in Fig. 4.

Figure 4: Topology optimization results of the traditional variable density method.

3.1.2 Partition weighted sensitivity filtering method

There is an obvious and unavoidable boundary diffusion in the results of the topology optimization using the traditional variable density method, and no clear boundary can be obtained. Direct edge extraction is more difficult and will greatly affect the accuracy of subsequent post-processing operations such as boundary extraction and boundary smoothing, while increasing the difficulty of direct fabrication of topologically optimized structures. In order to remove gray cells to obtain clear boundaries, a partitioned weighted sensitivity filtering method is proposed. The main idea is to divide the original sensitivity filtering area into two sub-regions, and use different weighting factors in different sub-regions, so as to increase the influence of cell sensitivity of the cells in the internal sub-region where the central cell is located, and at the same time reduce the sensitivity influence weight of the cells in the external sub-region far from the central cell, which can effectively suppress the phenomenon of border diffusion where a large number of grayscale cells appear. The schematic diagram of the area of the partitioned weighted sensitivity filtering method is shown in Fig. 5.

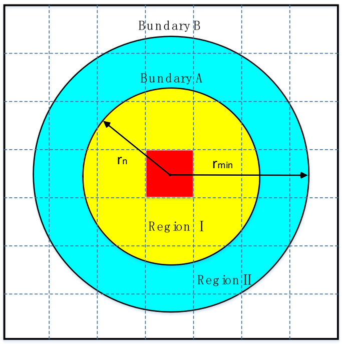


Figure 5: Schematic diagram of the area of the partitioned weighted sensitivity filtering method.

The region I and region II are internal sub-regions and external sub-regions, respectively. By changing the size of filter radius r_n and minimum filter radius r_{min}, the relative positions of boundary A and boundary B can be changed, thus changing the number of cells contained in each region. By repeatedly adjusting the weighting factor within region I for experiments, it was found that the weighting factor of cells within region I can be directly assigned to 1 to ensure that the sensitivity values of cells near the center are not affected by the cells within region II to be in lower weights, and thus increasing the weight of the influence of cells within region I on the sensitivity of the objective function. The weighting factor of region II is determined by the set exponential function, so that the weight within region II can achieve a slow decrease near the boundary A and a significant decrease near the boundary B, thus achieving a further weakening of the influence of the sensitivity of the cell in region II on the central cell. On the premise of ensuring the optimization stability of sensitivity filtering in the traditional variable density method, it can effectively suppress the boundary diffusion problem and obtain a clear topological optimization structure boundary. The mathematical expression for the partition weighted sensitivity filtering weighting factor is:

$$\tilde{H}_g = \begin{cases} 1 & , dist(k,i) \le r_n \\ \eta * r_{\min} * dist(k,i)^{(r_{\min}-\mu)} * e^{\left[-k*dist(k,i)^{r_{\min}}\right]} & , r_n \le dist(k,i) \le r_{\min} \end{cases} \tag{7}$$

Here, the $\tilde{H}_g$ is the modified weighting factor. The η and μ are correction factors for the exponential function. The r_n is the filtering radius of region I. The number of cells contained in each of the two inner and outer sub-regions is changed by setting the size of r_n according to the actual constraint and loading situation.

From equation (7), it can be seen that the maximum value of the weighting factor in region II can be changed by adjusting the magnitude of the correction factor. In order to avoid the boundary diffusion phenomenon due to the excessive difference in the sensitivity value of the transition cell at boundary A, and try to ensure that the maximum value of the weight factor of region II and region I are consistent, so the continuity of the function at boundary A can be ensured by setting the value of 0.7. And the different values of the correction coefficients affect the degree of curvature change of the exponential function. In order to explore the more appropriate value of the correction coefficient μ, the minimum filter radius r_{min} is taken as 2.5 and the filter radius r_n of region I is taken as 1 to analyze the trend of the exponential function image when μ takes different values. The image of the function change is shown in Fig. 6.

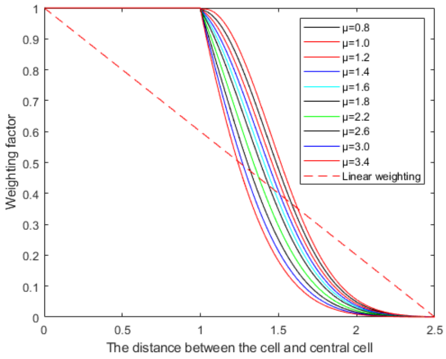


Figure 6: The image of weighted factor functions for different values of μ.

The linear weighting represented by the dashed line in Fig. 6 is the weighting principle image of the traditional variable density method of sensitivity filtering, while the solid line is the function principle image of the weighting factors with different values adopted by the partitioned weighted sensitivity filtering method. When the distance between the optimized cell and the central cell is less than 1, the value of the weighted segmentation function is significantly larger than the value of the linear weighting function, which can effectively improve the influence of cell sensitivity in region I. When the distance between the optimized cell and the central cell is greater than 1, the function changes slowly near the boundary A, while the value of the weighting factor near the boundary B has significantly reduced and tends to be close to 0, further reducing the influence weight of the cell sensitivity near the boundary B. The smaller the value of μ, the slower the change trend of the weighting factor as a function of the boundary A, the weights do not fall rapidly and lead to too small, and the weights near the boundary B are all at a lower value, which meets the optimization requirements of sensitivity filtering. To sum up, the correction factor μ is set to 1 to ensure that the function variation trend satisfies the reasonableness of topology optimization. Of course, the setting of the correction factor is not fixed, but also needs to be adjusted according to the load and constraints of the specific case.

In order to measure whether the optimization model satisfies the optimization requirements after the partition sensitivity filtering process, measures such as dispersion rate, gray rate and flexibility value are introduced. The dispersion rate reflects the deviation of the density of gray cells from 0 and 1, and is the main basis for judging whether the results after partition sensitivity filtering converge to the discrete solution. When the dispersion rate takes the maximum value, it means that all cells have not been discrete; while when the dispersion rate takes the minimum value, it means that all cells have been discrete. Therefore, in general, the topologically optimized structure should ensure that the dispersion rate is as small as possible, and a better discrete cell structure can be obtained. The gray rate can reflect the degree of the proportion of gray cells in the topology optimization results, and the gray rate is used to quantify the amount of gray cells in the optimization results. When the gray rate is larger, the proportion of gray cells is larger and the boundary diffusion is more obvious. So the optimization result should make the gray rate of the optimization result decrease as much as possible, which can effectively weaken the problem of boundary diffusion. The flexibility value reflects the stability of the structure when subjected to forces. The smaller the flexibility value, the larger the stiffness value, the stronger the ability of the structure to resist elastic deformation when subjected to force, and the smaller the deformation, the better the stability of the structure. So the flexibility value should be as small as possible in the optimization process, so that the optimization results will achieve better.

The partitioned weighted sensitivity filtering method used in this paper is relatively simple to implement through programming. The optimized model changes all the measures in a better direction and can effectively suppress the boundary diffusion phenomenon to obtain a clear boundary structure, which is convenient for the subsequent post-processing operation. The topological optimization of the algorithm model in Fig. 3 above is performed using the partitioned weighted sensitivity filtering method, and the optimization results are shown in Fig. 7.

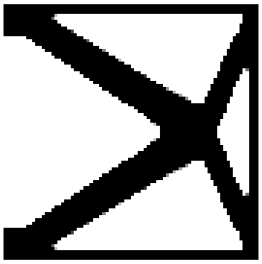


Figure 7: Optimization results of the partitioned weighted sensitivity filtering method.

The optimization results of the partitioned weighted sensitivity filtering method in Fig. 7 are significantly effective in suppressing boundary diffusion and the gray cells are greatly reduced compared with the optimization results using the sensitivity filtering method of the traditional variable density method in Fig. 4, and clearer optimization results can be obtained. Compared with the Sigmund sensitivity filtering method, the main improvement is that the original sensitivity filtering area is divided into two and weighted separately, and the influence of the boundary cell can be effectively weakened through changing the linear weighting factor of the Sigmund sensitivity filtering into the weighting factor of the segment function, so the optimization result with clearer boundary in Fig. 7 can be obtained.

3.2 Binarization Operation

The optimization results of the sensitivity filtering method using the traditional variable density method exist a large number of gray cells and the boundaries are relatively fuzzy. Therefore, the direct binarization operation cannot obtain accurate processing results, and there will be a big error. However, clear boundary optimization results can be obtained after the partitioned weighted sensitivity filtering method optimization and then binarization processing can be performed at this time to obtain the optimization results with completely black and white separation and uniform density distribution, which can avoid unnecessary interference brought by grayscale cells in subsequent topology optimization post-processing. The mathematical expression of the binarization operation process can be expressed as:

$$L = \begin{bmatrix} l_{11} & \cdots & l_{1j} & \cdots & l_{1n} \\ \cdots & & \cdots & & \cdots \\ l_{i1} & \cdots & l_{ij} & \cdots & l_{in} \\ \cdots & & \cdots & & \cdots \\ l_{m1} & \cdots & l_{mj} & \cdots & l_{mn} \end{bmatrix}, \quad l_{ij} \in [0,1] \tag{8}$$

$$g_{ij} = \begin{cases} 1 \quad , \quad l_{ij} \geq \dfrac{\max\{L\} - \min\{L\}}{2} \\ 0 \quad , \quad l_{ij} < \dfrac{\max\{L\} - \min\{L\}}{2} \end{cases} \tag{9}$$

$$G = \begin{bmatrix} g_{11} & \cdots & g_{1j} & \cdots & g_{1n} \\ \cdots & & \cdots & & \cdots \\ g_{i1} & \cdots & g_{ij} & \cdots & g_{in} \\ \cdots & & \cdots & & \cdots \\ g_{m1} & \cdots & g_{mj} & \cdots & g_{mn} \end{bmatrix}, \quad g_{ij} \in \{0,1\} \tag{10}$$

Here, the l_{ij} is the density value of a discrete cell. The g_{ij} is the density value of a particular discrete cell after binarization. The equation (8) first transforms the topology optimization result into a numerical matrix L between 0 and 1. Then the equation (9) compares l_{ij} with the set density threshold value to get g_{ij}, and finally obtains a numerical matrix G with the value of 0 or 1 in the equation (10).

After the binarization, the few existing intermediate density cells disappear completely, and the optimized structure with clear and black-and-white boundaries can be obtained, and the optimization results after the binarization are shown in Fig. 8.

3.3 Edge Detection and Boundary Extraction

There are several edge detection operators in image processing, and the Canny operator is a technique that extracts useful structural information from different visual objects and greatly reduces the amount of data processed. It is able to detect image edges close to the actual edges with a low error rate and is now widely used in various machine vision systems [2]. Therefore, the Canny operator is introduced to extract clear and accurate effective boundaries for edge detection of topology optimization results in this paper.

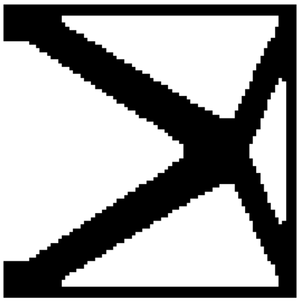


Figure 8: Optimization results after binarization operation.

In order to avoid the negative impact of noise on the edge detection process and results, a Gaussian filter and a model with convolution optimization process are used to smooth the image, which can effectively avoid the interference of invalid noise and reduce the error rate. The expression for a two-dimensional Gaussian filter can be expressed as:

$$f(x,y)=\frac{1}{(\sqrt{2\pi}\sigma)^2}e^{-\frac{(x-ux)^2+(y-uy)^2}{2\sigma^2}} \tag{11}$$

Here, the (x,y) is the coordinate of any point inside the mask. The σ is the standard deviation, (ux,uy) is the coordinate of the center point inside the mask.

The results of the topology optimization model are extracted, and the gradient direction and amplitude of the smoothed image are calculated after the Gaussian filter smoothing and noise removal processing, and the mathematical expressions are shown below:

$$\theta(x,y)=\arctan(\frac{\partial f}{\partial y}/\frac{\partial f}{\partial x}) \tag{12}$$

$$M(x,y)=\sqrt{(\frac{\partial f}{\partial x})^2+(\frac{\partial f}{\partial y})^2} \tag{13}$$

The gradient direction can be generally divided into four directions: horizontal, vertical, 45° and 135°. The larger the gradient amplitude, the greater the rate of change of the pixel value and the more obvious the change.

In practical applications, the points with gradient amplitude greater than the threshold value are marked as edges. In order to realize that edges in an image are marked only once and ensure that possible image noise is not marked as edges, non-maximal suppression of gradient magnitudes along the gradient direction is required to find local maximal points in the image gradient and to obtain refined edges by setting the non-maximal points to zero. Finally, the double-threshold algorithm is used to detect and determine the potential boundaries and eliminate the weak boundary connection edges. The results after boundary extraction are shown in Fig. 9.

3.4 Boundary Corner Point Extraction

After the topology optimization model extracts the effective boundary, the individual corner points of the boundary need to be determined. The Harris corner point detection algorithm is introduced to extract the corner point coordinates, and its basic idea is to imagine a sliding window in the image, so that the sliding window is moved in all directions in a small range, and the point in the window area with a large change in gray level is the desired corner point [15].

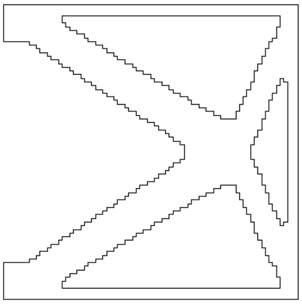


Figure 9: Boundary extraction results.

The expression for the change in the average pixel gray value within the window is shown below:

$$E(u,v)=\sum_{(u,v)\in W(x,y)} w(u,v)\left[I(x+u,y+v)-I(x,y)\right]^2 \tag{14}$$

Here, the u and v are the window deflections in the horizontal and vertical directions. The $W(x,y)$ is the window centered on the point (x,y). The $w(u,v)$ is the Gaussian weighting function. The I is the gradient value of the corresponding point.

The quadratic term function is essentially an elliptic function, and the flatness and size of the ellipse are determined by the eigenvalues of the local matrix M. The eigenvalues and response function of the matrix are calculated and zeroed when the response value is less than the set threshold, and the response function expression is as follows:

$$R=\det(M)-k*\mathrm{trace}^2(M) \tag{15}$$

Here, the R is the Harris response value of the pixel point. The $\det(M)$ and $\mathrm{trace}(M)$ are the determinant and trace of the matrix M, respectively. The k is an empirical constant.

The above optimization results are locally non-maximal suppressed in the preset domain, and when the response value of a point is the neighborhood maximum and greater than the set threshold, the point can be confirmed as the desired corner point. The coordinate origin is further determined, and the coordinates of each corner point are confirmed to form a point set using a single discrete cell boundary dimension as the coordinate reference cell. The result of corner point extraction is shown in Fig. 10.

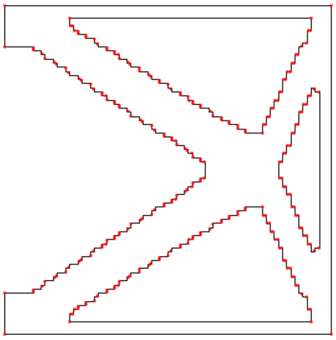


Figure 10: Boundary corner point extraction results.

3.5 Smooth Boundary Treatment

The ordinary least squares is widely applied as the standard regression method for analytical calibrations, and it is usually accepted that this regression method can be used for quantification starting at the limit of quantification [11]. It is used to fit the set of boundary corner points in this paper, and the optimized model can obtain a smooth boundary, which greatly reduces difficulty of the optimized model. It can find the most suitable matching function for the data by minimizing the sum of squares of errors, and get the position data with the smallest sum of squares of errors between the actual data to achieve curve fitting. The mathematical expression for the ordinary least squares method is shown below:

$$\begin{cases} S = \sum_{i=1}^{n} \delta_i^2 = \min \sum_{i=1}^{n} [f(x_i) - y_i]^2 \\ f(\mathbf{X}) = \alpha_1 r_1(\mathbf{X}) + \alpha_2 r_2(\mathbf{X}) + \ldots + \alpha_m r_m(\mathbf{X}), m < n \end{cases} \tag{16}$$

Here, the δ_i is the error of the data point (x_i,y_i). The n is the number of data points. The $f(\mathbf{x})$ is the fit function. The α is the coefficient to be determined. The $r(\mathbf{x})$ is the conversion function that converts the nonlinear part into the linear part.

The curves processed by boundary smoothing need to be discriminated by the degree of smoothing, and the fit between the detection curve and the model boundary are used as a measure. The smaller the value of the fit is, the more the curvature change tends to be linear and the more uniform the change is, the better the curve fit is. The relevant expression for the degree of fit can be expressed as:

$$C = \frac{\sum_{i=1}^{n} (f_i - \widehat{f}_i)^2}{\sum_{i=1}^{n} (f_i - \bar{f}_i)^2} \tag{17}$$

Here, the f_i is the measured value of the corner point. The $\widehat{f}$ is the theoretical value of the fitted curve. The $\bar{f}$ is the mean value. The smaller the C value of the fit, the better the curve fit.

By merging the independent smoothing curves, the closed area of the curve with smooth boundary can be obtained, and the final topology optimization structure with clear and smooth boundary can be obtained. In this way, the jagged boundary existing in topology optimization can be effectively removed, and the optimization structure is more convenient to manufacture at the same time. The final topology optimization results after the boundary smoothing treatment are shown in Fig. 11.

Figure 11: The results of the boundary smoothing treatment.

4 NUMERICAL ALGORITHM ANALYSIS AND VERIFICATION

The feasibility and effectiveness of the partitioned weighted sensitivity filtering method and the subsequent boundary light-smoothing post-processing method are verified by topology optimization algorithms in the MATLAB-2019a software environment. In the calculations of each algorithm, a planar four-node quadrilateral cell is used for discrete processing, and the elastic modulus of the material is set to 1 with Poisson's ratio in 0.3 and penalty factor in 3.

4.1 Single Load Model and Post-processing Results Analysis

The two-dimensional planar stress structure design area for the single load model is set to 90mm ×60mm, with a full-plane fixed restraint at the left end of the structure and a vertical downward load applied at the middle node of the structure’s right end. The optimization area is divided into 90×60 rectangular four-node cells, and the allowable material volume fraction of the optimized structure is set to 0.3. The schematic diagram of the model is shown in Fig. 12.

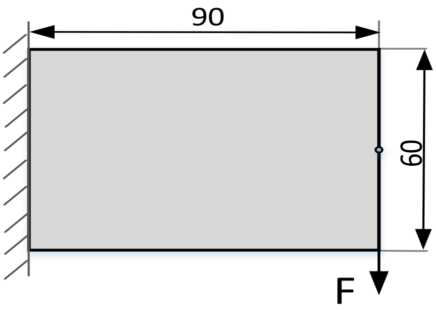


Figure 12: Schematic diagram of single load model.

The model is subjected to a single load. By comparing the topology optimization results under the traditional variable density method with the topology optimization results of the partitioned weighted sensitivity filtering method, and the optimized structure after the boundary smoothing post-processing, the feasibility and effectiveness of this paper's method in solving the boundary diffusion problem and removing the jagged boundary when subjected to a single load are verified. The model was programmed and implemented with the traditional variable density method, the partitioned weighted sensitivity filtering method and the boundary smooth post-processing method, and the optimization results are shown in Fig. 13.

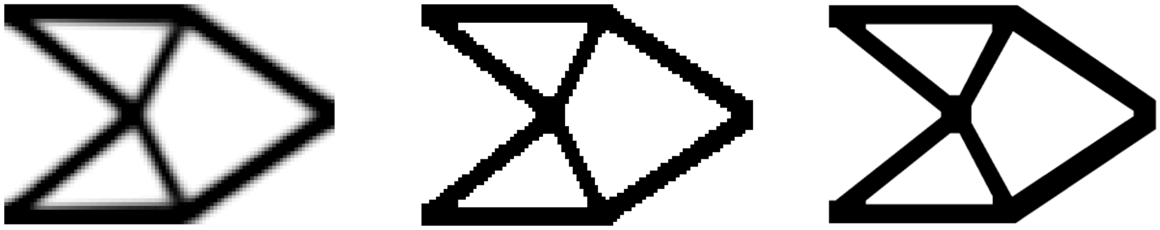


Figure 13: Left：Optimization results of traditional variable density method；Middle：Optimization results of partitioned weighted sensitivity filtering method ; Right : Optimization results of boundary smooth post-processing method.

By comparing the optimization results of the traditional variable density method and the partitioned weighted sensitivity filtering method in Fig. 13, it is obvious that the partitioned weighted sensitivity filtering method can solve the boundary diffusion problem existing in the traditional variable density method. In order to quantitatively assess the differences between these two methods, three major measures are introduced to compare these two methods, and the comparison of the optimization data with the two methods in this model is shown in Table 1.

Optimization methods	Dispersion rate(%)	Gray rate(%)	Flexibility value
Traditional variable density method	18.269	43.705	64.430
Partitioned weighted sensitivity filtering method	1.069	3.458	56.436
Reduction rate	94.1%	92.1%	12.4%

Table 1: Comparison of optimization data for different optimization methods.

As can be seen from Table 1, the percentage of gray cells of the model optimized by the traditional variable density method is up to 43.705%, and the boundary diffusion is serious, which greatly affects the accuracy of the optimized model manufacturing. The reduction rate of both dispersion rate and gray rate by using the partitioned weighted sensitivity filtering method is more than 90% compared with the traditional variable density method, and the flexibility value is also reduced, which is significantly improved. It not only solves the boundary diffusion, but also improves the stiffness of the structure and facilitates the post-processing of topology optimization.

In summary, it can be seen that the partitioned weighted sensitivity filtering method is significantly more effective than the traditional variable density method in solving the boundary diffusion. The boundary smoothing post-processing method can effectively process the jagged boundary optimized by the partitioned weighted sensitivity filtering method, and finally obtain the topological optimization model with smooth boundary with a good feasibility.

4.2 MBB Beam Model and Post-processing Results Analysis

The MBB beam model has a geometry of 240 mm×40 mm and is subjected to a vertical downward external load at the middle of the top. In order to simplify the optimization, the 1/2MBB beam structure is meshed to obtain an optimized model with geometry of 120mm × 40mm. The schematic diagram of the MBB beam model is shown in Fig. 14.

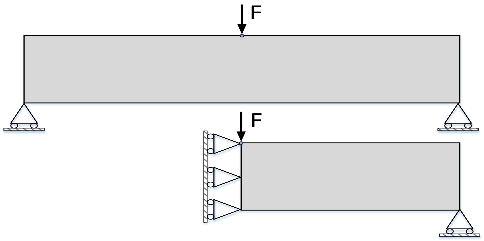


Figure 14: Schematic diagram of the MBB beam model.

By setting different optimization parameters, whether the partitioned weighted sensitivity filtering method and the boundary smooth post-processing method are feasible at different grid densities and volume ratios is analyzed and verified. The optimization results with different optimization parameters are shown in Table 2.

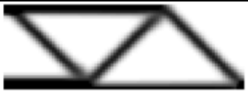
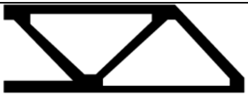
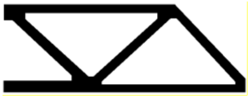
Experiment number	Grid densities	Volume ratios	Traditional variable density method	Partitioned weighted sensitivity filtering method	Boundary smooth post-processing method
1	90×30	0.3			
2	120×40	0.3			
3	150×50	0.3			
4	120×40	0.4			

Table 2: Comparison of optimization results of MBB beam with different optimization parameters.

As can be seen from Table 2, Experiments 1, 2 and 3 show the comparison of optimization results under different grid density conditions. By analyzing the different experimental results, it can be seen that the partitioned weighted sensitivity filtering methods can all obtain the ideal topology optimization structure with well-defined structural boundaries, and all of them have only a very small number of gray cells. The boundary smooth post-processing method has good optimization results for the models optimized by the partitioned weighted sensitivity filtering method in different grid densities. Experiments 2 and 4 show the optimization results under different volume ratio conditions, and it can be seen that the partitioned weighted sensitivity filtering method and the boundary smooth post-processing method also have good optimization effects under different volume ratio parameters.

4.3 Multiple Load Model and Post-processing Results Analysis

The multi-load model is a conventional Michelle-type structure with solid support at both ends, with a design area of 150 mm × 50 mm and fixed constraints at the lower left and lower right nodes. Vertical downward loads are applied at the upper quarter and three-quarter nodes of the model, while vertical downward loads are applied at the middle node at the structure’s lower end. The grid is divided into 150×50 four-node planar stress cells with equal size, and the allowable material volume fraction of the optimized structure is set to 0.3. The schematic diagram of the model is shown in Fig. 15.

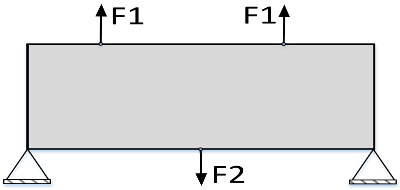


Figure 15: Schematic diagram of the multiple load model.

By comparing the topology optimization results of the traditional variable density method with the topology optimization results of the partitioned weighted sensitivity filtering method and the optimization results after the boundary smooth post-processing, the feasibility and effectiveness of the method in this paper can be verified when subjected to multiple loads. The optimization results of different methods for this model are shown in Fig. 16.

Figure 16: Left：Optimization results of traditional variable density method；Middle：Optimization results of partitioned weighted sensitivity filtering method； Right： Optimization results of boundary smooth post-processing method.

From Fig. 16, it can be seen that the partitioned weighted sensitivity filtering method is equally feasible in the case of multiple loads and can suppress the boundary diffusion. A comparison of the optimized data from the traditional variable density method and the partitioned weighted sensitivity filtering method in this model is shown in Fig. 17.

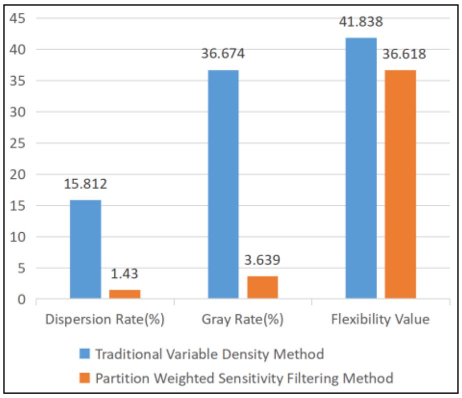


Figure 17: Histogram of the optimization data of the model by different methods.

As can be seen from Figure 17, the partitioned weighted sensitivity filtering method has improved in terms of dispersion rate, gray rate and flexibility value compared to the traditional variable density method. In particular, the values of gray rate and dispersion rate are greatly reduced, and the optimization effect is remarkable. At the same time, by comparing the optimization results of the conventional variable density method with those of the post-processing, it is verified that the use of the boundary smooth post-processing method is equally feasible when subjected to multiple loads.

5 CONCLUSIONS

The boundary diffusion and the unavoidable jagged boundary in the optimization of the variable density method will bump the reconstruction and manufacturing of the optimization model. To address the above problems, this paper proposes a post-processing method of topology optimization models based on the variable density method. Some typical arithmetic models are optimized and post-processed in the case of single loads, multiple loads, and different mesh densities and volume ratios, respectively. The experimental results show that the method can effectively avoid boundary diffusion and jagged boundaries, while ensuring that the structural performance is within the permitted range and the optimized structure can be manufactured more easily. At present, the method is only applicable to the post-processing of topology optimization of two-dimensional planar structures, and its application scope is somewhat limited. In the future research, its expansion and application in 3D spatial structures need to be further explored to enhance the effectiveness and applicability of the method in 3D topology optimization post-processing.

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