

3D Mathematical Modeling and Visualization Application Based on Virtual Reality Technology

Sujing Zhou¹, Ying Zhang² and Junhong Yang³

¹Basic Teaching Department, Zhengzhou Railway Vocational and Technical College, Zhengzhou, Henan 451460, China, <u>zhousujing@zzrvtc.edu.cn</u>

²Basic Teaching Department, Zhengzhou Railway Vocational and Technical College, Zhengzhou, Henan 451460, China, <u>zhangying@zzrvtc.edu.cn</u>

³School of Artificial Intelligence, Zhengzhou Railway Vocational and Technical College, Zhengzhou, Henan 451460, China, <u>yangjunhong@zzrvtc.edu.cn</u>

Corresponding author: Sujing Zhou, zhousujing@zzrvtc.edu.cn

Abstract. With the continuous improvement of information society and productivity, various emerging technologies are constantly emerging in the context of computers and big data. Among them, virtual reality technology and computeraided technology have become popular research topics today. Virtual reality technology utilizes computers to establish multidimensional three-dimensional models, achieving good results in communication and reorganization of various activities from dynamic environments and data numerical analysis. This article starts from the direction of terrain survey and modeling, and uses virtual reality technology to analyze the three-dimensional structure of surface models. Using 3D mathematical modeling to build the basic features of terrain, and combining virtual reality technology to optimize the real-time display and resolution of massive data in 3D modeling. Apply virtual reality technology and computer-aided tool CAD together in the process of graphic generation and rendering. Establish an irregular three-dimensional mathematical hierarchy, generate a continuous, multi resolution visualization model, and achieve the function of dynamically transmitting terrain and landforms. Using numerical simulation 3D visualization method, using ANSYS software for numerical analysis and transmission. The ultimate goal is to build a three-dimensional visualization scene while providing users with an interactive experience. The research results indicate that virtual reality technology and computer-aided CAD tools are effective and practical in the research process of terrain 3D mathematical modeling and visualization. The numerical features also demonstrate strong interactivity and intuitiveness, which has certain promotional value.

Keywords: Virtual Reality Technology; 3D Model; Numerical Simulation; Visualization; Terrain Modeling **DOI:** https://doi.org/10.14733/cadaps.2023.S13.1-13

1 INTRODUCTION

The continuous updates of CAD virtual technology have provided assistance and convenience for optimization and use in various industries. Virtual reality technology, as a product of technologies such as computers, big data, and the Internet of Things, has great effects in industrial manufacturing, medical research, military exercises, product design, and terrain surveying. People can not only obtain information through electronic devices in their daily lives, but also use virtual reality technology to transform a portion of information into non abstract forms. Verifying CAD models from deformable network construction is a current challenge. Augmented reality has become an important technology for modeling reality markers. Akpan et al. [1] constructed a triggering model in a labeled environment. The learning status of the model was evaluated through object enhancement of 3D animated videos. Ben et al. [2] conducted visual analysis of mathematical formulas and 3D scanning of network construction. This new form of transformation and acquisition has helped us gain further advantages in understanding information content and reading data. In modern technology, virtual reality technology utilizes its unique ability to combine electronic information with computer platforms to create three-dimensional virtual spaces. It can provide users with the ability to interact with virtual spaces, enabling them to have a more intuitive experience. By improving virtual reality technology, optimizing both virtual space and 3D environment can enhance the experience of simulation. Brown et al. [3] analyzed students' attitudes towards using virtual reality in chemistry education. The research results indicate that people have a positive attitude towards using virtual reality as an auxiliary tool to help understand chemical concepts. Various perceptions and skills constitute effective interaction with virtual reality space in sensors and IoT devices. In various fields of virtual reality technology application, facing the needs of intuitive natural perception and real-time information transmission, improving user interaction is the main way to change system efficiency and work efficiency. Combining the future urban model in LoD3, Buyukdemirciogly and Kocaman [4] proposed a comprehensive representation framework.

According to the characteristics of virtual reality technology, we can divide it into the following categories: the first feature is the multi perception function of virtual reality technology, which requires perceptual improvement in touch, motion, taste, and other aspects in addition to basic visual and auditory perception. The second characteristic is the presence of the scene, which can directly reflect the details in the real scene. When we use virtual reality technology to restore terrain features, we need to simulate such scenarios to help researchers experience a realistic and immersive experience. At present, research on such characteristics is still in an ideal virtual simulation environment. The third feature is about the interactivity of virtual reality environments. Users can customize a scene environment to achieve a realistic and natural response by manipulating and setting objects within the environment. The last feature is autonomy, which runs objects in virtual simulations according to set rules to ensure a certain level of normal operation. This technical feature is mainly applied in the simulation of dynamic environments. Among them, three-dimensional modeling and mathematical modeling visualization are both the target areas of this study. Automatically form a three-dimensional model in the real environment based on the actual scene, and perform numerical analysis on the data to improve the accuracy of the modeling results.

The development of computer-aided graphic drawing CAD technology, digital processing technology, and other technologies can improve the interactive effect of virtual reality environments. According to the survey results of literature, virtual reality technology, CAD technology, 3D mathematical modeling, visualization technology, etc. are frequently used in terrain detection and geological survey. The three-dimensional scene constructed through the system simulates the real-world experience, and real-time uploads three-dimensional terrain data information as the data source for analysis. CAD technology provides computer-aided drawing assistance for three-dimensional terrain images, combined with the multidimensional spatial data

changes provided by virtual reality technology, ultimately achieving numerical analysis and visual display of terrain.

2 RELATED WORK

Cahyono et al. [5] created an augmented reality mobile math tracking application. This application combines augmented reality technology with mobile devices to help users better understand and master mathematical knowledge by presenting virtual objects and objects in the real environment. In addition, Campi et al. [6] proposed a framework for embedding this method into 3D CAD systems for future development of software tools. The promising 3D nanoprinting methods for depositing nanoscale grid like objects are prone to nonlinear distortion, which limits the complexity and diversity of deposition geometry. Fowlkes et al. [7] conducted computer-aided nanolinear design of nodes. Currently, many manual and software-based tolerance analysis methods are being used. However, compared to software-based methods, manual processes are complex and tedious for complex assemblies. 3D mathematical modeling visualization CAD model is a method of converting mathematical models into visualization graphics using 3D modeling technology. González et al. [8] applied new medical digital algorithms using virtual reality augmentation technology with 3D functionality. It utilized a software system to analyze the structure of the 3D network and constructed a model. The installation of this system in hospitals will provide significant improvements for medical image visualization tools. Goo et al. [9] conducted medical analysis using advanced visualization technologies such as interactive enhancement, hybrid, and virtual reality. The technical advantages of visualization technology were constructed by analyzing the 3D imaging model of patients.

Haghighi et al. [10] transformed mathematical problems into computer graphics problems, and intuitively displayed mathematical relations and change laws by establishing three-dimensional models. Han et al. [11] conducted a visual enhanced virtual reality framework construction test. It has undergone shape transformation in 3D CAD systems through formatting, and combined with digital twin systems to improve product design and production efficiency. Ivson et al. [12] constructed a CAD mathematical model. Establish corresponding mathematical models based on the mathematical problems that need to be solved. This process may require the use of mathematical software, algorithms, and programming techniques. The visual 3D model proposed by Jiang and Zhang [13] has been processed and prepared, including data format processing, outlier elimination and data normalization. Use 3D modeling software to convert mathematical models into 3D models. This process may require the use of basic 3D modeling, level of detail, and other functions. Kounlaxay et al. [14] used graphic visualization software to convert 3D models into visual graphics. This process may require the use of special effects such as lighting, materials, animation, etc. It conducts interaction design and adaptation of visual graphics according to user needs to achieve better user experience. The results indicate that presenting and presenting the results to users facilitates their understanding and mastery of mathematical problems. Lütjens et al. [15] developed a mid-range computer that provides large-scale terrain datasets in VR. An intuitive data model was constructed and analyzed through VR immersive applications of virtual reality. Its CAD program provides the possibility of exploring terrain with or without water surfaces through various motion modes. Oke and Arowoiya [16] analyzed the CAD virtual environment in the architectural environment. By enhancing reality, the application scope of project architecture in the construction industry has been increased. The 3D model assisted by CAD provides a new measurement platform for the construction and protection of intelligent buildings. Templin and Popielarczyk [17] conducted project model construction and implementation for the protection of architectural heritage. Cost saving is achieved through visual architectural heritage and preventive maintenance. Yan et al. [18] analyzed the structural characteristics of CAD under complex and large amounts of data. It develops the characteristics of the project graphics of building visualization, and displays and analyzes the project content of scientific visualization. Yang and Jin [19] conducted feature analysis of CAD 3D modeling under computer graphics analysis. Its expression is concise and easy to understand, in line with readers' habits of receiving information.

Zalilov et al. [20] conducted a comprehensive education integration analysis of various design technologies.

Based on the current research and development status, this article explores the application effects of virtual reality technology and computer-aided CAD tools from the perspective of surveying and numerical simulation analysis of terrain images. And, using three-dimensional mathematical modeling to build a virtual environment, real-time transmission of dynamic data forms a visual display and reflects the research results. Mainly exploring methods for drawing irregular terrain images and simplifying 3D models, optimizing the amount of modeling data, and obtaining mathematical models with various accuracies. Display irregular terrain layers in a virtual reality environment to achieve effective control and transmission of dynamic data. Finally, visualization technology and algorithms were used to generate a three-dimensional terrain model, which was compared with real scenes and the research results were verified.

3 RESEARCH ON 3D TERRAIN MATHEMATICAL MODELING AND VISUALIZATION BASED ON VIRTUAL REALITY TECHNOLOGY

3.1 Research on 3D Terrain Numerical Simulation and Mathematical Modeling Based on Virtual Reality Technology

Virtual reality technology is about the plane technology of computers, and it is also a scene environment generated by information technology with multiple functions such as reality and perception. Users in virtual environments can communicate in real time with their natural environment through sensing devices and interactive devices. Essentially, virtual reality technology utilizes computers as interfaces to provide users with a variety of intuitive sensory experiences. We need to capture and transmit images from various dimensions of the physical scene in the research of 3D terrain mathematical modeling and visualization. The use of computer-aided drawing CAD tools can effectively extract and draw image features from multiple planes. CAD tools can complete the editing of image graphics, size labeling, text layer exchange management, data insertion and exchange, and 3D image composition. Utilize the powerful data processing capabilities of computer systems to rearrange and combine the feature components of images, and then use CAD software to simulate the design output results. The auxiliary nature of this visual drawing tool can 3D terrain modeling and drawing, and 2D plane images can also meet the needs of plane angles. The operability of CAD tools is extremely strong, and the combination of mouse and keyboard provides users with a highly smooth experience during the drawing process. We have summarized the research and development trends and CAD technology in recent years, forming an intuitive data graph as shown in Figure 1.

As shown in Figure 1, the source of our data survey is a collection system of relevant domestic and foreign databases. This system utilizes its own retrieval and updates to provide us with a wide range of accurate reference data. In order to make the trend changes of research results more accurate, we also believe that interference data unrelated to the two technologies should be removed. Mainly exploring the changes in the number of research literature on virtual reality technology and CAD technology from 2010 to 2022. From the trend of change, it can be seen that the number of papers on virtual reality and CAD technology has been on the rise, with only slight fluctuations in the early stages of 2010. The average number of papers has shown a significant upward trend in the future. Therefore, we choose virtual reality technology and CAD technology as the research of 3D terrain data modeling to achieve effective results. In the process of generating terrain images, virtual reality technology is the key to maintaining the simulation interface, which can integrate multi-dimensional information space and give the experimenter a realistic display feeling. The most important interface among them is the visual transmission channel, and we also obtain image information features through visual means. In 3D mathematical modeling, any scene that cannot meet the physical and optical requirements will affect the "authenticity" of virtual places.



Figure 1: Development trend chart of two technical research achievements.

If the data transmission is not synchronized, it can also bring external stimuli to the visual interface. Therefore, we need to improve the real-time performance of terrain images, which in traditional processing relies intelligently on pre prepared recordings. However, it cannot meet the user's interactive needs for virtual scenes. In this study, it is necessary to generate virtual images based on real-time changes in terrain image features and sensory viewpoint parameters. That is to say, the simulation generation of images must be dynamic and real-time. This demand has high requirements for image quality and data transmission speed required by data modeling. We will use process visualization to represent the research process, as shown in Figure 2.



Figure 2: Process visualization.

As shown in Figure 2, a preliminary analysis was conducted on the combination of virtual reality technology and CAD technology. In terms of CAD technology, the numerical changes after data dynamic analysis are used to preprocess image features, and the image features are disassembled

and reassembled in a real-time virtual environment. Combining dynamic real-time terrain images processed by CAD technology with virtual reality systems to complete the scene construction of 3D mathematical modeling. Finally, explore the applicability of its algorithm from the visualization technology section, reflect the interactive function of virtual reality scenes, and display the threedimensional terrain model. The dynamic data obtained when using CAD to assist in drawing threedimensional topographic maps must first be parsed in data format before use. Use a triangular network data model for finite element analysis from the method of constructing virtual reality models, and output the data stream through the interface provided by CAD. This article uses the non-mean curve in three-dimensional mathematical modeling for calculation, and the formula is as follows:

$$p(t) = \sum_{i=1}^{n} w_i p_i B_{i,k}(t)$$
(1)

$$p(t) = \sum_{i=1}^{n} w_i B_{i,k}(t) / \sum_{i=1}^{n} w_i Q$$
(2)

In the formula, $B_{i,k}$ represents the initial vertex of a 3D model. w_i represents the weight relationship between vertices and edges. The basic function for calculation is obtained by the following formula:

$$B_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t}$$
(3)

$$B_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t} + \frac{t_{i+k} - t_i}{t_{i+k-1} - t_{i+1}}$$
(4)

In terrain graphics rendering, it is necessary to face multiple angles to capture image features, and at the same time, adjust the projection ratio of the seen graphics on the visual membrane. The user's perspective is defined as:

$$\alpha = 2art\left(\frac{L}{2D_i}\right) \tag{5}$$

In order to generate a three-dimensional perspective in a virtual reality environment, we need to change the user's visual gap and calculate two viewpoints separately. By combining multiple views to form a three-dimensional effect, the projection on a spatial object is defined as:

$$X_{sl} = \frac{X_i \times k + \frac{Z_i \times d}{2}}{k - Z_i} \tag{6}$$

$$Y_{sl} = \frac{Y_i \times k}{k - Z_i} \tag{7}$$

$$X_{sr} = \frac{X_i \times k - \frac{Z_i \times d}{2}}{k - Z}$$
(8)

$$Y_{sr} = \frac{Y_i \times k_0}{k - Z_i} \tag{9}$$

The formula is simplified as:

$$X_{SL} = \frac{X_i + \frac{d}{2}}{1 - \frac{Z_i}{k}} - \frac{d}{2}$$
(10)

In the formula, d represents the distance between different points, k represents the length of the relationship. By using the above calculation methods to process image projections in 3D models, the operational efficiency of virtual reality systems can be improved and image quality can be guaranteed. According to CAD technology, the number of triangular structures in the converted data model is relatively large, which increases the system's memory during the image rendering process. Therefore, we need to simplify the model without affecting the virtual reality effect. This article adopts the algorithm of edge transfer data folding to generate continuous levels of detail to reduce the probability of hollow features. The optimization process of this folding network model algorithm is shown in Figure 3:



Figure 3: Process diagram of feature point detail changes.

From Figure 3, it can be seen that the simplified 3D network model requires error frequency control during the optimization process. This error needs to be based on visual perception rather than terrain structure. The vertex position changes of most illuminated and non-illuminated objects have a more significant impact on vision. Therefore, we label feature points through repeated calculations to reduce the non-real coefficients of the virtual reality 3D model.

3.2 Visualization and Algorithm Research of 3D Terrain Model Based on Virtual Reality Technology

When constructing a virtual environment using virtual reality technology, it is necessary to use real terrain data information and display the real environment based on the data features of the digital model. The most convenient method for terrain generation is to use computer-aided CAD technology to draw adjacent data structures into polygons for projection. This traditional data calculation method has low efficiency and is prone to detail bias when constructing dynamic virtual scenes. In addition to depicting 3D images, real-time rendering of the environment is also required, which requires higher update rates for parameters such as 3D projection. We found from historical literature analysis that the sparse distributions to form a simple data plane, starting from a triangular network model to form a terrain-based framework. This method has a fast-rendering

speed, but the framework is relatively simple and not suitable for virtual reality scenes. This article extracts some calculation formulas from a large number of studies on 3D digital models in terrain rendering, such as:

$$\{V_r, i = 1, 2, \dots, n\}$$
(11)

$$\{V_r = (V_{r1}, V_{r2}, \dots, V_{rm})\}$$
(12)

The above formula represents a multidimensional vector sequence in a certain region of virtual reality technology. These vectors can represent variables such as terrain features, resource features, and environmental features. If only the distribution of the mathematical model is considered during the CAD drawing process, the vector sequence is:

$$\{V_r = (X_i, Y_i, Z_i), i = (1, 2, \dots n)\}$$
(13)

Among them, (X_i, Y_i, Z_i) represents the plane coordinates in a virtual reality scene. We perform weighted calculations from different directions of the triangular model:

$$n = \sum_{i=1}^{m} (N \cdot A_i) / \sum A_i$$
(14)

Among them, $(N \cdot A_i)$ represents the distance between a line and a vertex. When the calculated threshold is less than the standard coefficient, it can be successfully marked:

$$N_1 \bullet N_2 = |N_1| \bullet |N_2| \bullet C\theta \tag{15}$$

Based on the above formula, we have preliminarily obtained and calculated the data of the threedimensional mathematical model. Add computational data to a virtual reality system to form dynamically updated image fixed point coordinates. In practical research, we use virtual reality technology combined with CAD to draw the distribution of two viewpoints on a certain terrain. The comparison between the virtual scene and the actual scene is shown in Figure 4:



Figure 4: Comparison of feature point distribution with actual terrain.

From Figure 4, it can be seen that the viewpoint distribution drawn by CAD technology, which are similar to the actual terrain perspective. Therefore, it has been proven that 3D mathematical modeling under virtual reality technology has a certain effect on capturing terrain images. Compared with traditional construction models, the advantage of this study is that it can

significantly preserve the original features when generating dynamic terrain scenes. Among them, CAD technology provides great help for the fitting effect of 3D mathematical modeling. Next, we will deeply explore the modeling language and visualization algorithm in virtual reality technology, and analyze the display process of three-dimensional terrain scene. Add numerical simulation ANSYS software to the visualization process, transform and couple the dynamically obtained data, and use visualization tools to display the simulation results. The VRML language supported by virtual reality technology can establish reality models one-on-one, and has strong 3D rendering effects and user interaction functions in virtual scenes. Using ANSYS to model and process three-dimensional terrain data has various functions such as preprocessing, processing and summarization. Add image parameters to the model unit definition by reading textual commands. During the debugging process of the visual conversion program, the output file format can be directly replaced and browsed in the virtual reality scene. The process of this panoramic interactive technology is shown in Figure 5:



Figure 5: Panoramic interactive technology flowchart.

It can be seen from Figure 5 that all interactive operations are carried out in the virtual space, which can solve the problems of single form and space mismatch in traditional 3D mathematical modeling. Extract features from 3D images drawn using CAD technology, use ANSYS for numerical modeling, and calculate dynamic data results. Read and compile the output format file to complete panoramic browsing and interaction in a virtual reality system.

4 ANALYSIS OF RESEARCH RESULTS ON 3D TERRAIN MATHEMATICAL MODELING AND VISUALIZATION BASED ON VIRTUAL REALITY TECHNOLOGY

4.1 Analysis of Research Results on Three-Dimensional Terrain Numerical Simulation and Mathematical Modeling Based on Virtual Reality Technology

Virtual reality technology, as the core technology in terrain 3D modeling, needs to be combined with CAD drawing tools to achieve hierarchical image reorganization of real scenes. The generated 3D terrain information data plays a role in marking and positioning and improving image quality in computer graphics processing. Through the viewpoint changes of a three-dimensional mathematical model, it is projected onto a virtual reality scene to achieve the visualization of real terrain and landforms on the screen. In this process, triangle basic primitives are used for image rendering, and polygon meshes are used to cover the terrain. This planar modeling method can

provide detailed and accurate data for CAD technology in 3D multi planar modeling. The larger the number of polygons selected during the drawing process, the higher the quality of the image ultimately projected into the virtual reality scene. We will compare the quality of 3D modeling images optimized using virtual reality CAD technology with that of ordinary planar modeling images, as shown in Figure 6:





From Figure 6, it can be seen that as the number of captured images changes, the 3D model using virtual reality CAD technology produces higher pixel values when placed in a simulated environment. With the increase of dynamic image data, the capture of pixels in ordinary planar modeling is not optimistic. Therefore, CAD rendering based on virtual reality technology has good results in 3D image modeling. We further analyze the help of triangular network models in virtual space, where any falling point in this area can form a triangular structure with the edge, improving the distribution accuracy of pixel points through high-frequency data recording. In order to improve the operational efficiency of the virtual system in the future, we used error limitation method to reduce the impact coefficient of image accuracy in the process of reducing the number of drawing facets. The construction of virtual reality scenes also requires the support of computer technology, among which functions such as environmental simulation, natural perception, sensing devices, data storage and processing rely on powerful hardware devices. In this study, a device with a CPU model of FX-8350 and 8GB of memory was used, and the resolution and size of the screen display also need to meet the needs of a panoramic virtual environment. Next, we will analyze the visualization process after building a virtual environment.

4.2 Visualization of 3D Terrain Models Based on Virtual Reality Technology and Analysis of Algorithm Research Results

We use details such as terrain image point nodes and surface nodes to define shape models in scenes constructed using virtual reality technology. Select the features from various scenarios one by one, and ultimately complete the construction of the virtual environment. Among them, CAD technology has provided significant assistance in 3D digital modeling. In addition to processing dynamic data, real-life images were also drawn from a multidimensional perspective. As the numerical simulation results used in this article are more suitable for 3D visualization, there are several steps in the processing of the visualization interface. Firstly, ANSYS is used to calculate the dynamic changes of terrain, and the results are input into the CAD drawing system as format files

for use. Secondly, visualization software and virtual reality technology are used to read the sequence of image patterns after positioning, and an interface program is used to export a 3D model. Finally, the virtual reality technology compiler is used to automate the construction and update of the panoramic view. We compared the 3D scenes supported by virtual reality technology with ordinary mathematical modeling scenes in terms of user interactive surveys, as shown in Figure 7.



Figure 7: Virtual Reality Technology Supported 3D Scene and General Mathematical Modeling Scene from User Interactive Survey.

From Figure 7, it can be seen that the 3D panoramic venue formed using virtual reality technology has significant advantages in interactive experience. As the number of user samples increases, the interactive evaluation coefficient gradually increases. Next, we will further analyze the resolution of CAD technology in terrain rendering. Resolution not only reflects the effectiveness of virtual scenes, but also directly affects the operational efficiency of virtual reality systems. Due to the continuous generation of dynamic data in terrain detection, the expression of image resolution is also extremely important in visual browsing. Based on the dynamic changes in browsing information, we select resolutions for different terrain angles in a virtual environment and adjust the resolution size in real-time. We have also established a database in the terrain classification of the data module, which adaptively adjusts the model level according to users' needs for virtual reality scenes, reducing the power and burden of real-time virtual environment system operation.

5 CONCLUSION

The combination of computer-aided tools has made this research a hot topic in multiple fields. We explore the application effects of virtual reality technology and CAD technology in 3D modeling from the perspectives of terrain image rendering, real-time scene updates, and virtual construction. This article first conducts data analysis on the development process of virtual reality technology and computer-aided CAD technology, and applies them to three-dimensional mathematical modeling and visualization construction of terrain. Improving the feasibility of this technology through manual data intervention. Using CAD technology to extract graphic and image features from three-dimensional and multi angle terrain, and dynamically recombine feature information. Use data analysis and triangular network models for finite element analysis to enhance the positive impact of dynamic data on virtual scene construction. Using non mean curves to calculate the viewpoint positions of images in physical scenes in different environments. Using folding model algorithm to improve the quality of 3D images. Combine the dynamic real-time

terrain image after data processing with the virtual scene constructed by virtual reality technology. Use ANSYS software to calculate three-dimensional dynamic terrain data, convert it into a format file, and add it to the virtual reality system operation. An error limitation method has been proposed to ensure the resolution of the 3D model while maintaining the authenticity and computational power of the virtual reality scene. We also optimized the fast-rendering capability in the visualization interface display, and accelerated the visualization speed by planning the multi angle view area. The research results indicate that terrain 3D modeling supported by virtual reality technology and CAD technology can achieve real-time updates and interactions. And the visualization of virtual places ensures the user's sense of experience and authenticity.

6 ACKNOWLEDGEMENTS

This work was supported by Henan Province education science "fourteen Fifth Plan" general subject, Research on "Double promotion" mode of Mathematical contest in modeling and Mathematics course teaching (No.: 2021YB0661).

Sujing Zhou, <u>https://orcid.org/0000-0002-6039-6380</u> Ying Zhang, <u>https://orcid.org/0009-0008-6365-899X</u> Junhong Yang, <u>https://orcid.org/0009-0005-0438-8713</u>

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