Generation of κ-Space Curve

Dan Wang[1], Tadatoshi Sekine[2], Shin Usuki[3], Kenjiro T. Miura[4]

[1]Shizuoka University, wang.dan.18@shizuoka.ac.jp
[2]Shizuoka University, sekine.tadatoshi@shizuoka.ac.jp
[3]Shizuoka University, usuki@shizuoka.ac.jp
[4]Shizuoka University, miura.kenjiro@shizuoka.ac.jp

Corresponding author: Dan Wang, wang.dan.18@shizuoka.ac.jp

Abstract. The κ-curve is a recently published interpolating spline which consists of quadratic Bézier segments passing through input points at the loci of local curvature extrema. But their interpolation can only deal with planar curves. Therefore, in this research we propose a method that enables to extend this representation to deal with space curves in a new scheme called κ-space curves.

Keywords: κ-curve, G^2 continuity, Quartic Bézier curve
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1 INTRODUCTION

The κ-curve [3] is a recently published interpolating spline which passes through input points at the loci of local curvature extrema. It is composed of piecewise quadratic Bézier curves. Since the quadratic Bézier curve is planar, the control points of each segment lie in the same plane. However, when it comes to 3D condition, the control points are located in different planes. Although the curvature values at the connection points remain consistent, the curvature directions differ. This deviation in curvature direction disrupts G^2 continuity.

In some applications G^2 continuity is preferable for space curves. For example, when we design a trajectory of a robot, for mechanical smooth movement to avoid an abrupt change of acceleration, G^2 continuity of the trajectory is required.

κ-curve possesses many desirable properties, but there is still significant space for κ-curve to be improved [1]. The following are three important shortcomings of κ-curves:

1. Their curve is almost curvature-continuous everywhere, except at inflection points only G^1 continuity is guaranteed, i.e. the absolute value of curvature around the joints is equal to each other, but the sign is reversed.
2. Since the degree of freedom (DoF) of the quadratic segments is limited, it is impossible to control the magnitudes of local maximum curvature at the input points.

3. Their curve can only deals with planar curves. When it comes to 3D condition, G^2 continuity is broken.

Wang et al. [2] solved the first shortcoming by using log-aesthetic curves instead of Bézier curve. Miura et al. [1] solved the second shortcoming by elevating the degree of the Bernstein functions. In this paper, we propose a new method to solve the last shortcoming by replacing the quadratic Bézier segments by quartic Bézier ones. The length of the replacement curves could be controlled so that we can preserve the locations of the curvature extrema and maintaining G^2 continuity.

The key contributions of our paper can be summarized as follows:

1. Generation of κ-space curves: We introduce a novel methodology for constructing space curves that possess κ-curve's properties. These curves exhibit desirable characteristics, passing through input points at the loci of local curvature extrema.
2. G^2 continuity at connection points: We address the challenge of achieving G^2 continuity between different segments of κ-space curves. While for κ-curve, at inflection points only G^1 continuity is guaranteed. Our approach ensures a smooth transition between quartic and quadratic Bézier curves, guaranteeing G^2 continuity at arbitrary parameter positions.
3. Derivation of connection formula: We have derived a mathematical formula that facilitates the seamless connection between quartic and quadratic Bézier curves while preserving G^2 continuity. This formula provides a practical and efficient solution for connecting curves with different degrees.

2 GENERATION OF κ-CURVE

Yan et al. [3] generate a series of quadratic curves that exhibit G^2 continuity almost everywhere except for the inflection points, and the curves interpolate the input points at the parameter of local curvature extrema by modifying the locations of the middle control points of quadratic Bézier segments.

In order to obtain G^1 continuity, as shown in Fig. 1 the control points $c_{i,2}$ and $c_{i+1,0}$ are set with the constant λ_i

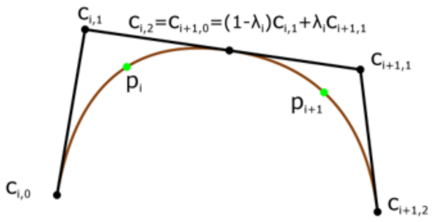


Figure 1: Control points setting of κ-curve [3]

$$c_{i,2} = c_{i+1,0} = (1 - \lambda_i)c_{i,1} + \lambda_i c_{i+1,1} \tag{1}$$

Interpolation of the input points at the local curvature maximum is guaranteed by the condition

$$c_i(t_i) = p_i$$

where p_i is the i^{th} input point. In the quadratic case, we can express the parameter t_i at the point of maximal curvature explicitly, in terms of the Bézier coefficients of the i^{th} quadratic Bézier curve as

$$t_i = \frac{\langle r_i, r_i - s_i \rangle}{\|r_i - s_i\|^2} \tag{2}$$

where $r_i = c_{i,1} - c_{i,0}, s_i = c_{i,2} - c_{i,1}$ and $\langle a, b \rangle$ means the scalar product of vectors a and b. The control points $c_{i,1}$ is derived by

$$c_{i,1} = \frac{p_i - (1-t_i)^2 c_{i,0} - t_i^2 c_{i,2}}{2t_i(1-t_i)} \tag{3}$$

Solving for $c_{i,1}$ and substituting into Eq. (2), we get a cubic equation in t_i that depends only on the end points $c_{i,0}$ and $c_{i,2}$, and the input points p_i.

$$\|c_{i,2} - c_{i,0}\|^2 t_i^3 + 3(c_{i,2} - c_{i,0}) \cdot (c_{i,0} - p_i) t_i^2 + 3(c_{i,0} - 2p_i - c_{i,2}) \cdot (c_{i,0} - p_i) t_i - \|c_{i,0} - p_i\|^2 = 0 \tag{4}$$

The G^2 condition derives

$$\lambda_i = \frac{\sqrt{\triangle_i^+}}{\sqrt{\triangle_i^+} + \sqrt{\triangle_{i+1-}}} \tag{5}$$

by introducing the notations $\triangle_i^+ = \triangle(c_{i,0}, c_{i,1}, c_{i+1,1})$ and $\triangle_{i+1}^- = \triangle(c_{i,1}, c_{i+1,1}, c_{i+1,2})$, and $\triangle$ represents area of the triangle. The generating process of κ-curve is summarized in Algorithm 1.

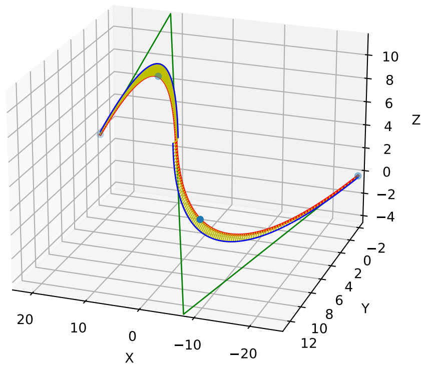
Algorithm 1: The generation of κ-curve [3].

Input: Input points p_i.
Output: Control points $c_{i,k}$ and parameters of maximum curvature t_i.

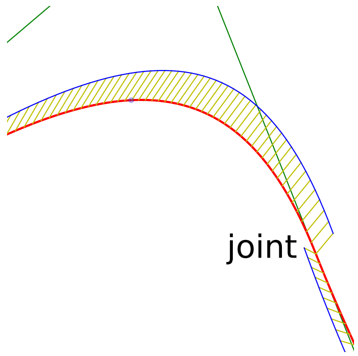
```
Set all λ_i to 0.5;
Compute all c_{i,0} and c_{i,2} by Eq. (1);
while not converged do
    Compute all λ_i by Eq. (5);
    Compute all c_{i,0} and c_{i,2} by Eq. (1);
    Compute all t_i by root finding of cubic polynomial in Eq. (4) ;
    Compute all c_{i,1} by solving the linear tridiagonal system
      p_i = (1 − λ_{i−1})(1 − t_i)^2 c_{i−1,1} + λ_i t_i^2 c_{i+1,1} + (λ_{i−1}(1 − t_i)^2 + (2 − (1 + λ_i)t_i)t_i)c_{i,1};
end
Compute all c_{i,0} and c_{i,2};
```

3 GENERATION OF κ-SPACE CURVE

At first, we use quadratic Bézier curves to form κ-space curves and the control points of a quadratic Bézier curve are on the same plane. Even though the control points are located in 3D space, Eq. (1) and Eq. (3) can be applicable for them. In Eq. (5), we need areas of $\triangle_i^+$ and $\triangle_i^-$, and the control points are on the same plane, respectively. It is straightforward to generate a curve for a sequence of 3D input points as shown in Fig. 2. Since the planes where the control points of consecutive quadratic Bézier curves are located as well as their Frenet frames are generally different, curvature values at the connection point are consistent between the adjacent curve segments, but the curvature directions lie in different planes. G^2 continuity requires not only the matching of curvature values but also the alignment of curvature directions at the connection point. When the curvature directions of the curve segments deviate or lie in different planes, it violates the G^2 continuity condition. This deviation in curvature directions can result in a visual break or an abrupt change in the smoothness of the curve at the connection point (please see the closeup in Fig. 2 (a)). To achieve G^2 continuity, it is necessary to ensure that not only the magnitudes but also the directions of the curvatures align



(a) κ-space curve



(b) Closeup of the joint

Figure 2: Generating κ-space curve with quadratic Bézier curves

properly at the connection point. For this situation only G^1 continuity is guaranteed, although the absolute values of curvature are the same.

For the κ-space curve generated above, we would like to replace two consecutive quadratic Bézier curves with a Bézier curve of higher degree. In order to achieve G^2 continuity, the joint between two consecutive quadratic segments should be replaced with guaranteeing it at both ends of the replacing curve segment of a higher degree.

3.1 Replacement with Quartic Bézier Curves

Typically, the three control points of each of the two quadratic segments are situated on distinct planes, as illustrated in Fig. 3. Therefore, to achieve G^2 continuity, it is necessary to use a curve with a minimum degree of four.

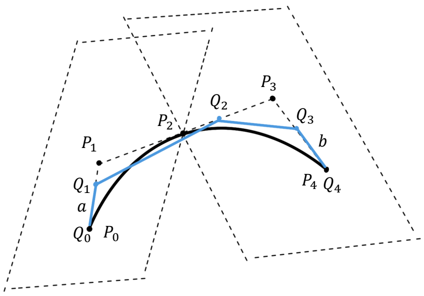


Figure 3: Control points setting of Quartic Bézier curve.

Hence, we use a quartic Bézier curve for replacement. The quartic Bézier curve has five control points and the first three control points should be on the plane of the first quadratic curve segment and the last three ones on that of the second curve segment. This means that the first and second control points are on the first plane, the fourth and fifth ones on the second plane and the third control points must be located on the line of P_1P_3 in Fig. 3. Furthermore, for G^1 continuity at the ends, the second control point is on P_0P_1 and the third one on P_3P_4. Therefore, for the control points $Q_i(i = 0, \cdots, 4)$ of the quartic curve, there exist such a, b and γ that

$$\begin{aligned} Q_0 &= P_0 \\ Q_1 &= (1-a)P_0 + aP_1 \\ Q_2 &= (1-\gamma)P_1 + \gamma P_3 \\ Q_3 &= (1-b)P_3 + bP_4 \\ Q_4 &= P_4 \end{aligned} \tag{6}$$

where P_i $(i = 0, \cdots, 4)$ are defined by the control points $C_{i,j}$ $(j = 0, 1, 2)$ of the quadratic Bézier curve segments

$$\begin{aligned} P_0 &= C_{i,0} \\ P_1 &= C_{i,1} \\ P_2 &= C_{i,2} = C_{i+1,0} = (1-\lambda)C_{i,1} + \lambda C_{i+1,1} \\ P_3 &= C_{i+1,1} \\ P_4 &= C_{i+1,2} \end{aligned} \tag{7}$$

For a given γ in order to guarantee G^2 continuity at the end points $Q_0(P_0), Q_4(P_4)$ in Fig 3, let

$$\begin{cases} \kappa_i(0) = \kappa(0) \\ \kappa_{i+1}(1) = \kappa(1) \end{cases} \tag{8}$$

where $\kappa_i(0)$ and $\kappa_{i+1}(1)$ are κ-curve's curvatures at P_0 and P_4, $\kappa(0)$ and $\kappa(1)$ are quadratic Bézier curve's curvatures at Q_0 and Q_4, respectively. The following constraints are derived from Eq. (8) (see details in Appendix A.1):

$$a = \sqrt{\frac{3\gamma}{2\lambda}} \tag{9}$$

$$b = 1 - \sqrt{\frac{3(1-\gamma)}{2(1-\lambda)}} \tag{10}$$

We can adopt λ for γ as an initial value. Fig. 4 shows a quartic Bézier curve with $\gamma = \lambda$. Notice that the shape is approximated well by the quartic curve, but the positions of the curvature extrema are not preserved. The curvature extrema do not generally coincide with the input points.

3.2 Partial Replacement with Quartic Bézier Curves

Here we describe a method to replace the joint part of two adjacent quadratic curve segments with a quartic Bézier curve. We replace the parts of parameter intervals $\{t_1, 1\}$ and $\{0, t_2\}$ from the first and second segments, respectively with a quartic Bézier curve as shown in Fig. 5.

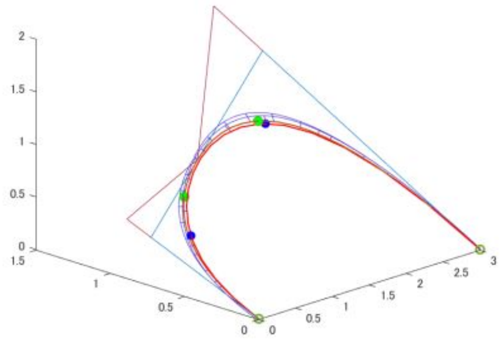


Figure 4: Replacement with a quartic Bézier curve.

The control points of the quartic Bézier curve $\{Q0, Q1, Q2, Q3, Q4\}$ are determined by the auxiliary points $A, B, C,$ and D.

$$\begin{aligned} Q_0 &= (1-t_1)A + t_1 B \\ Q_1 &= (1-a')Q_0 + a'B \\ Q_2 &= (1-\gamma')B + \gamma' C \\ Q_3 &= (1-b')C + b'Q_4 \\ Q_4 &= (1-t_2)C + t_2 D \end{aligned} \tag{11}$$

with

$$\begin{aligned} A &= (1-t_1)P_0 + t_1 P_1 \\ B &= (1-t_1)P_1 + t_1 P_2 \\ C &= (1-t_2)P_2 + t_2 P_3 \\ D &= (1-t_2)P_3 + t_2 P_4 \end{aligned} \tag{12}$$

We would like to apply Eq. (9) and Eq. (10). Hence, we introduce λ' and γ' as follows:

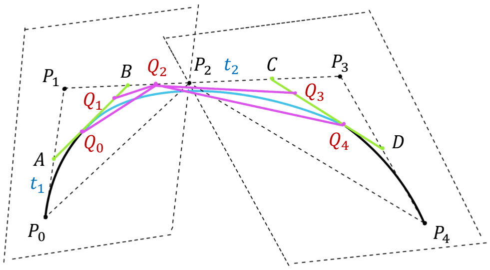


Figure 5: Control points setting of partial replacement with a quartic Bézier curve.

$$(1-\lambda)c_{i,1} + \lambda c_{i+1,1} = (1-\lambda')B + \lambda' C \tag{13}$$

$$(1-\gamma)c_{i,1} + \gamma c_{i+1,1} = (1-\gamma')B + \gamma' C \tag{14}$$

By solving the above equations, we obtain

$$\lambda' = \frac{\lambda t_1 - \lambda}{\lambda t_1 + \lambda t_2 - \lambda - t_2} \tag{15}$$

$$\gamma' = \frac{\gamma - \lambda t_1}{t_2 - \lambda(t_1 + t_2 - 1)} \tag{16}$$

and P_i $(i = 0, \cdots, 4)$ are also defined by Eq. (7). Therefore, a' and b' are derived by G^2 continuity (i.e. Eq. (8)) at the ends of Q_0 and Q_4 (see details in Appendix A.2),

$$a' = \sqrt{\frac{3}{2}\frac{\gamma - \lambda t_1}{\lambda(1 - t_1)}} \tag{17}$$

$$b' = 1 - \sqrt{\frac{3}{2}\left\{1 + \frac{\lambda - \gamma}{(1 - \lambda)t_2}\right\}} \tag{18}$$

Note that a' does not have t_2 and b' does not have t_1 and a' is determined by t_1 and b' is determined by t_2. a' and b' look quite different, but substitute t_2, λ and γ with $1 - t_1, 1 - \lambda$ and $1 - \gamma$, then

$$1 - b' = a' \tag{19}$$

because of symmetry.

The concept of G^2 continuity as well as G^1 continuity is local and G^2 continuity is broken by the replacement with a quartic Bézier curve only at the joint of two consecutive quadratic Bézier curves. Hence, we replace a partial segment at the joint of κ-space curve with a quartic curve to preserve the locations of its curvature extrema.

The generated κ-space curve with the replacement of $C_i(t)$ for $t \in (0.9, 1)$ and $C_{i+1}(t)$ for $t \in (0, 0.1)$ is shown in Fig. 6. Notice that the curvature is continuous at the two joints with the quadratic curves. This replacement does not affect the locations of the original curvature extrema of κ-space curve in this case, and we can make the replacement curve short as much as we like. Therefore, we can preserve the locations of the curvature extrema.

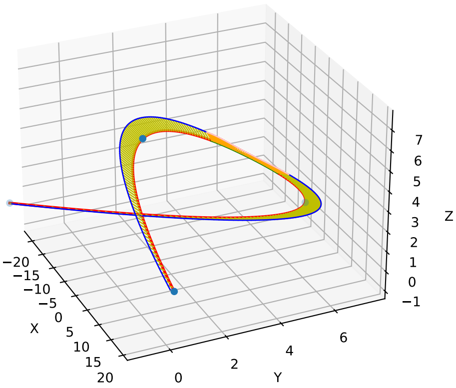
Fig. 7 shows the curvature graph of the replacement curve. Since the curve is of a higher degree than quadratic, it introduces another curvature extremum as shown in Fig. 7. However, the shorter the replacement curve becomes, the closer it approximates a straight line segment, and we can make the value of the newly introduced curvature extremum relatively small. So we can avoid introducing a large curvature extremum by controlling the length of the replacement curve.

4 OPTIMIZATION

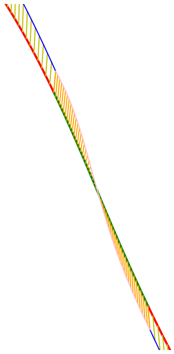
We can optimize γ to minimize, for example, the following objective function:

$$F(\gamma) = \int_0^{\frac{1}{2}} |C(t) - C_i(2t)|\, dt + \int_1^{\frac{1}{2}} \left| C(t) - C_{i+1}\left(2\left(t - \frac{1}{2}\right)\right)\right| dt \tag{20}$$

where $C(t)$ is the quartic Bézier curve and $C_i(t)$ and $C_{i+1}(t)$ are i-th and $i+1$-st quadratic Bézier curves, respectively. We can adopt λ for γ as an initial value for optimization. Figure 8 shows a quartic Bézier curve with the optimized γ. The green and blue points indicate curvature extrema of the original κ-space curve and the quartic Bézier curve, respectively. The difference is subtle, but the curve with the optimized γ approximates the quadratic curves better.



(a) Replacement of $t \in (0.9, 1.1)$



(b) Closeup of the joint

Figure 6: Partial replacement with a quartic Bézier curve.

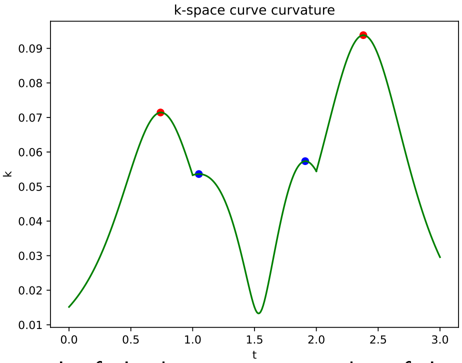


Figure 7: The graph of absolute curvature value of the replacement curve.

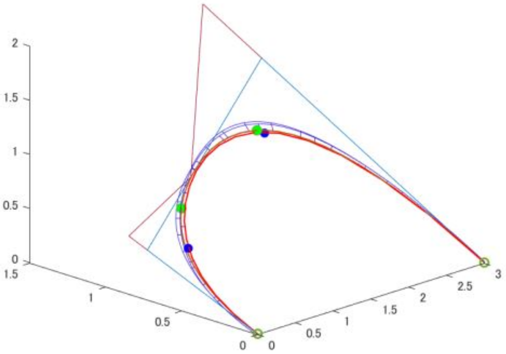


Figure 8: Replacement with a quartic Bézier curve with optimized γ.

5 IMPLEMENTATION

We have already made an application of κ-space curve in Rhino®Plugin program. κ-space curve is a combination of quadratic Bézier curves and quartic Bézier curves, it can be saved as Rhino's curve type: Poly curve. As κ-space curve can be saved in Rhino file formats, the compatibility and seamless integration of our Plugin program with Rhino is ensured. Our curve generation system provides a flexible and intuitive approach to editing and manipulating curves within the Rhino environment, such as rotate, move, split at specific parameter or joint with other curves. Users can easily let it show the control points, the curvature properties, and refine the shape of the curves according to their specific design. As shown in Figure. 9 left the replacement parameter t is set to be 0.01. We can see the curvature extrema are just located on the input points, and the curvature is continuous around the joint. As shown in Figure. 9 right our curves can serve as a fundamental building block for complex 3D modeling tasks within Rhino. They can be used as a basis for lofting, sweeping, or rail-based operations, enabling users to create intricate shapes and forms with precise control over the resulting geometry. Using space curves to generate surfaces offers stronger three-dimensional representation, control over multiple axes, more realistic geometry, and better spatial continuity compared to planar curves. These advantages make space curves an ideal choice in many application areas, particularly when precise modeling and designing complex three-dimensional surfaces are required.

Regarding the applications of using κ-space curve to generate surfaces, our curves can be utilized for a range of common application areas, such as industrial design and robotics: The representation and manipulation of three-dimensional curves are essential in industrial design and robotics applications. κ-space curve can be employed in tasks such as path planning, robot motion control, or shape generation, where maxima curvature property and G^2 continuity are desirable. It also possesses the advantages in terms of precision, manufacturability, or collision avoidance.

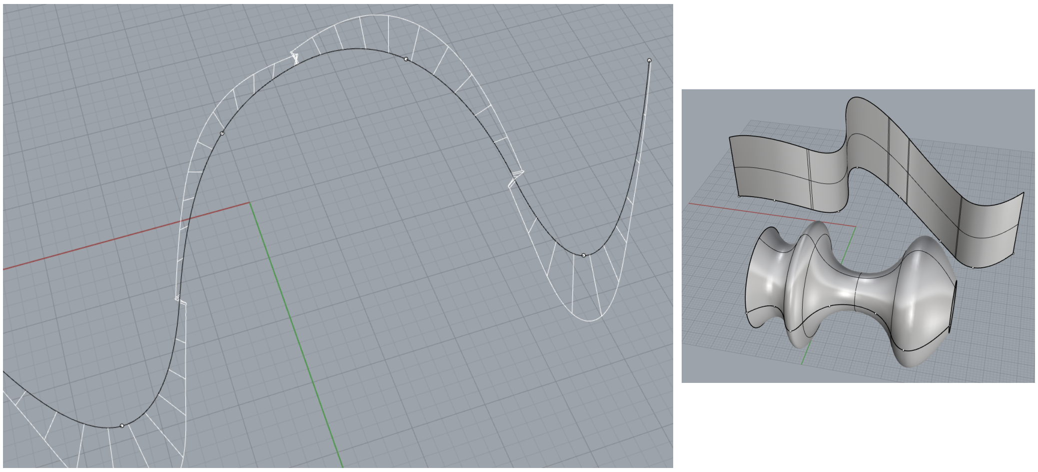


Figure 9: κ-space curve Rhino plugin.

6 CONCLUSIONS AND FUTURE WORK

This paper proposes a new method that enables to extend κ-curve to κ-space curve and maintain G^2 continuity by replacing quadratic Bézier curves with quartic Bézier ones. In addition, we have derived a formula that enables to connect quartic Bézier curves to quadratic Bézier curves at arbitrary parameter positions with G^2 continuity. Although the new replacement part introduce additional points of maximal curvature, by controlling the length of the replacement curve, we can make the value of the newly introduced curvature extremum relatively small. This replacement does not affect the locations of the original curvature extrema of κ-space curve. They serve the purpose of ensuring a smooth transition and continuity between the curve segments. In the future, we will extend κ-curve to κ-surface that could be applied in complex geometric modeling. We also would like to compare κ-surface with subdivision surface.

ACKNOWLEDGEMENTS

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A Proof of constraints to guarantee G^2 condition

A.1 Replacement with Quartic Bézier Curves

$$a = \sqrt{\frac{3\gamma}{2\lambda}}$$
$$b = 1 - \sqrt{\frac{3(1-\gamma)}{2(1-\lambda)}}$$

Proof. The derivatives of quartic Bézier curves at two end points Q_0 and Q_4 in Fig 3 are given by

$$\begin{aligned} C'(0) &= 4(Q_1 - Q_0) \qquad & C''(0) &= 4 \times 3 \times (Q_2 - 2Q_1 + Q_0) \\ C'(1) &= 4(Q_4 - Q_3) \qquad & C''(1) &= 4 \times 3 \times (Q_4 - 2Q_3 + Q_2) \end{aligned}$$

And the curvatures of quartic Bézier curves at two end points are given by

$$\kappa(0) = \frac{\|4(Q_1 - Q_0) \times 12(Q_2 - 2Q_1 + Q_0)\|}{\|4(Q_1 - Q_0)\|^3} = \frac{3}{2}\frac{\triangle(Q_0, Q_1, Q_2)}{\|Q_1 - Q_0\|^3} \tag{21}$$

$$\kappa(1) = \frac{\|4(Q_4 - Q_3) \times 12(Q_4 - 2Q_3 + Q_2)\|}{\|4(Q_4 - Q_3)\|^3} = \frac{3}{2}\frac{\triangle(Q_4, Q_3, Q_2)}{\|Q_4 - Q_3\|^3} \tag{22}$$

The curvatures of κ-curve at two ends P_0 and P_4 in Fig 3 are given by

$$\kappa_i(0) = \frac{\triangle(P_0, P_1, P_2)}{\|P_1 - P_0\|^3} \tag{23}$$

$$\kappa_{i+1}(1) = \frac{\triangle(P_4, P_3, P_2)}{\|P_4 - P_3\|^3} \tag{24}$$

To guarantee G^2 continuity at the ends,

$$\begin{cases} \kappa_i(0) = \kappa(0) \\ \kappa_{i+1}(1) = \kappa(1) \end{cases} \qquad \Rightarrow \qquad \begin{aligned} a^3 &= \frac{3}{2}\frac{\triangle(Q_0, Q_1, Q_2)}{\triangle(P_0, P_1, P_2)} \\ (1-b)^3 &= \frac{3}{2}\frac{\triangle(Q_4, Q_3, Q_2)}{\triangle(P_4, P_3, P_2)} \end{aligned}$$

$$\because \quad \begin{aligned} \frac{\triangle(Q_0, Q_1, Q_2)}{\triangle(P_0, P_1, P_2)} &= \frac{a\gamma}{\lambda} \\ \frac{\triangle(Q_4, Q_3, Q_2)}{\triangle(P_4, P_3, P_2)} &= \frac{(1-b)(1-\gamma)}{(1-\lambda)} \end{aligned}$$

$$\therefore \quad \begin{aligned} a &= \sqrt{\frac{3\gamma}{2\lambda}} \\ b &= 1 - \sqrt{\frac{3(1-\gamma)}{2(1-\lambda)}} \end{aligned}$$

□

A.2 Partial replacement with Quartic Bézier Curves

$$\begin{aligned} a' &= \sqrt{\frac{3}{2}\frac{\gamma - \lambda t_1}{\lambda(1-t_1)}} \\ b' &= 1 - \sqrt{\frac{3}{2}\left\{1 + \frac{\lambda-\gamma}{(1-\lambda)t_2}\right\}} \end{aligned}$$

Proof. The curvatures of quartic Bézier curves at two end points Q_0 and Q_4 in Fig. 5 are the same with Eqs. (21) and (22) Appendix A.1. The curvatures of κ-curve at two points Q_0 and Q_4 in Fig 5 are given by

$$\kappa_i(t_1) = \frac{\triangle(P_0, P_1, P_2)}{\|(1-t_1)(P_1 - P_0) + t_1(P_2 - P_1)\|^3} \tag{25}$$

$$\kappa_{i+1}(t_2) = \frac{\triangle(P_4, P_3, P_2)}{\|(1-t_2)(P_3 - P_4) + t_2(P_4 - P_3)\|^3} \tag{26}$$

Similarly to guarantee G^2 continuity,

$$\begin{cases} \kappa_i(t_1) = \kappa(0) \\ \kappa_{i+1}(t_2) = \kappa(1) \end{cases} \quad \Rightarrow \quad \begin{aligned} (a'(1-t_1))^3 &= \frac{3}{2}\frac{\triangle(Q_0, Q_1, Q_2)}{\triangle(P_0, P_1, P_2)} \\ ((1-b')t_2)^3 &= \frac{3}{2}\frac{\triangle(Q_4, Q_3, Q_2)}{\triangle(P_4, P_3, P_2)} \end{aligned}$$

$$\because \quad \begin{aligned} \frac{\triangle(Q_0, Q_1, Q_2)}{\triangle(P_0, P_1, P_2)} &= a'(1-t_1)^3\frac{\gamma'}{\lambda'} \\ \frac{\triangle(Q_4, Q_3, Q_2)}{\triangle(P_4, P_3, P_2)} &= (1-b')t_2^3\frac{(1-\gamma')}{(1-\lambda')} \end{aligned}$$

where

$$\begin{aligned} \frac{\gamma'}{\lambda'} &= \frac{\gamma - \lambda t_1}{\lambda(1-t_1)} \\ \frac{(1-\gamma')}{(1-\lambda')} &= 1 + \frac{\lambda - \gamma}{t_2(1-\lambda)} \end{aligned}$$

$$\therefore \quad \begin{aligned} a' &= \sqrt{\frac{3}{2}\frac{\gamma - \lambda t_1}{\lambda(1-t_1)}} \\ b' &= 1 - \sqrt{\frac{3}{2}\left\{1 + \frac{\lambda-\gamma}{(1-\lambda)t_2}\right\}} \end{aligned}$$

□

Dan Wang, http://orcid.org/0000-0003-1906-4998
Tadatoshi Sekine, http://orcid.org/0000-0003-1813-822X
Shin Usuki, http://orcid.org/0000-0003-1165-5507
Kenjiro T. Miura, http://orcid.org/0000-0001-9326-313

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