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An Iterative Generative Design Approach for Multi-Material Components

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Abstract. The lightening of the components is playing an increasingly predominant role in modern industry to create ever more sustainable machines, reducing energy consumption while increasing the performances. Indeed, components and subassemblies are usually made of multiple materials to satisfy the design requirements. The definition of the material lay-out within the component's could be a challenging process and may lead to sub-optimal designs. The proposed Generative Design approach faces this task with an iterative topology optimization based algorithm. Indeed, with the proposed approach it is possible to optimize the materials lay-out and their related inner interfaces within a specified design domain. The approach consists of a two level optimization method and it can consider also non-linear hyperelastic materials as in the case of automotive seats. The production of seats requires a large use of polyurethane foam, combined with plastic and/or metal frames. In particular, the car seats consist of different materials such as steel frame, filled polyurethane foam for seat cushions and vertical backrests, and external textiles. For the considered test case, the method here developed proposes an optimization loop to consider the influence of the polyurethane foam in the topological optimization of the plastic part of the seat, allowing the possibility to modify boundary regions between the different materials. This loop continues optimizing both material regions, achieving an optimized material distribution of the multi-material part.

Keywords: Lightweight Design, Topology Optimization, Design Automation, Generative Design, Automotive
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1 INTRODUCTION

In the recent years, sustainability and attention to energy and material savings have become fundamental in the majority of the engineering fields and industries. With the aim of creating ever more sustainable machines, the lightening of the components is playing an increasingly predominant role to reduce energy consumption, maintaining, or possibly improving, the structural or dynamical performances. To carry it out, the concept of lightweight design is being strongly developed [21, 32, 23], looking also to bio-inspired structures that replicate the natural principle of "putting specific functional material only where is needed". Nowadays, CAD/CAE tools can help designers to obtain this purpose, with methods and procedures that currently can be realized due to the technological and manufacturing levels, in additive or other production processes [12]. By the way, it has to be said that the message "complexity for free" related to the raise of Additive Manufacturing technologies has been discovered as not completely true. In fact, the additive technologies have their own constraints and limitations and the translation of them into design constraints and requirements represents a current challenge for researchers and designers [6]. The research reported in this paper aims to develop, test and assess an iterative and automated Generative Design method based on topological optimization in order to optimize the multi-material distribution of components. Indeed, in the case of multi-material parts, the available commercial tools do not consider the possibility to change (at every iteration) the target volume and, most of all, the space distribution related to each material during the optimization workflow. In particular, the presented methodology has been applied to an automotive case study: the inner portion of a car seat to be composed by different functional materials. In the following sections, it is reported the state-of-the-art, the methodological approach, and the testing and the assessment of the proposed approach onto a case study reporting an approximated 3D model of an automotive seat's base. Conclusions and the possible future developments are reported in the last section of the paper.

2 STATE OF THE ART

Currently, to achieve lightweight designs, three macro-techniques are most widespread due to their effectiveness: Topological Optimization (TO), Lattice Structures (LS) design, and Generative Design (GD). TO, LS design, and GD techniques allow to create free-form organic shapes intended to be additively manufactured [36].

TO is the search for the optimal material distribution changing topology, shape, and size of the part by an iterative removal of ineffective material [1]. This type of study starts with the definition of a target volume (through the division of the entire design domain in design and non-design spaces), loading and boundary conditions, objective function, and design constraint's ones [5, 25]. Researchers have developed several algorithms capable of performing TO. Among them, the most relevant are the Solid Isotropic Material with Penalization (SIMP), and the Level Set Method (LSM) [25]. In the SIMP approach, the relation between the density design variables and the material property is given by the power-law (Eqn. (1)):

$$E(\rho_i) = g(\rho_i)E_0 = \rho_i^p E_0 \qquad with \qquad g(\rho_i) = \rho_i^p \tag{1}$$

where ρ is the penalization parameter and E_0 is the Young's modulus of solid material. For $\rho = 1$ the optimization problem corresponds to the so-called 'variable-thickness-sheet' problem which, for the compliance objective, is a convex problem with a unique solution. However, for the same objective, $\rho > 1$ penalizes intermediate thicknesses or densities and hence favors $0 - 1$ solutions [25].

In the LSM approach, the boundary of the design is defined by the zero-level contour of the level set function $\phi(x)$ and the structure is defined by the domain where the level set function takes positive values (Eqn. (2)):

$$\rho = \begin{cases} 0 & \forall x \in \phi < 0 \\ 1 & \forall x \in \phi \geq 0 \end{cases} \tag{2}$$

In the past decade, numerous level set methods have emerged which can be classified, for example, in approaches for discretizing the level set function, in approaches for mapping the level set field onto the mechanical model, and approaches for updating the level set field in the optimization process. Most often the level set function is updated via the solution of the Hamilton-Jacobi equation (Eqn. (3)):

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= -V\hat{n} \cdot \nabla\phi \\ \hat{n} &= \frac{\nabla\phi}{|\nabla\phi|} \\ \frac{\partial \phi}{\partial t} &= -V|\nabla\phi| \end{aligned} \tag{3}$$

where t is a pseudo-time representing the evolution of the design in the optimization process and V is the so-called speed function, or velocity field, advecting the level set function. The speed function $V(x)$ at $x : \phi(x) = 0$ represents the sensitivity of moving the interface in the normal direction $\hat{n}$ with respect to some form of merit function, scaled by the spatial gradient of the level set function [25].

Pertaining to Multi-Material Topology Optimization (MMTO), currently, only few works address the problem, and most of them are related to elastic materials [20, 16, 37, 27, 29, 10]. In [20], authors propose a two-elastic-material approach for compliance minimization with volume constraint based on the SIMP method where two pseudo-densities are defined, one for the existence of the material within each element of the mesh, while the second represent the percentage of material 1 with respect to material 2. The authors applied their optimizer to both 2D and 3D problems, using a commercial software for Finite Element pre-processing and analysis, and their own developed software for the TO. In [10], the SIMP interpolation scheme is adopted to compute the loss function of a Feed-forward Neural Network and use its weights and activation functions to predict the pseudo-density value of each point of the FE discretization. Considering non-linear MMTO, in [34], authors report the definition of a Discrete Material Optimization (DMO) interpolating scheme to optimize the 2D lay-out of multi-material structures with hyperelastic behavior.

LS design is based on repeating patterns of cells in the space, maintaining the purpose of supporting loads with the least possible weight, to achieve the optimal material distribution. These structures make possible to lighten the components while maintaining good mechanical characteristics [4, 11]. A lattice structure can achieve up to 70% weight reduction [33], guaranteeing structural performances. Furthermore, TO may optimize LS distribution in the respect of many geometrical aspects (i.e. size and density, distribution of the pattern, orientation with respect to the component's boundary).

GD methodologies exploit algorithmic methods to translate, in an automated way [26, 9, 13], requirements and constraints of the design task into a design domain of possible solutions to be evaluated. GD approaches encompass a wide set of tools and algorithms to generate design variants. These pertain to parametric modelling, Grammars, structural optimization algorithms, and Artificial Intelligence (AI) [22]. The techniques that have been proposed in the scientific literature leverage either one or more of these tools in the generative process. Indeed, in [19], a GD method is provided to let the system develop design concepts based on user defined parameters. Grammars are divided into two main categories: Shape Grammars and L-Systems. Shape Grammars are mostly used in product design, as in the case of the jewelry design method proposed in [18], while L-Systems are often used when performance requirements have to be considered, as in the case of the 3D-printing supports' design method proposed in [35]. In the field of GD methods powered by structural optimization algorithms, researchers are focusing their efforts toward solutions that adopts TO algorithms to generate lightweight, high performance and additively manufacturable conceptual structures [7, 8, 2]. Furthermore, a GD method adopting LS have been proposed in [28]. AI based GD methods pertain either to Evolutionary Algorithms or to Machine Learning techniques, and are usually coupled with TO when dealing with performance-driven solutions. Indeed, in [3], it is provided a GD methodology that couples Genetic Algorithms and TO to derive optimized structures. In [24] it is provided a comprehensive review of Machine Learning Generative models, focusing on Generative Adversarial Networks, Variational Autoencoders, and Reinforcement Learning. These techniques can be used to enhance the capabilities of TO in providing different solutions along the entire design domain [14, 17, 30] and to reduce its computational cost [31].

In the following section (Sec. 3), it is proposed a generative process for lightweight design based on the SIMP interpolation scheme with a reduced number of design variables when compared to the MMTO SIMP approaches available in the scientific literature.

3 METHODOLOGY

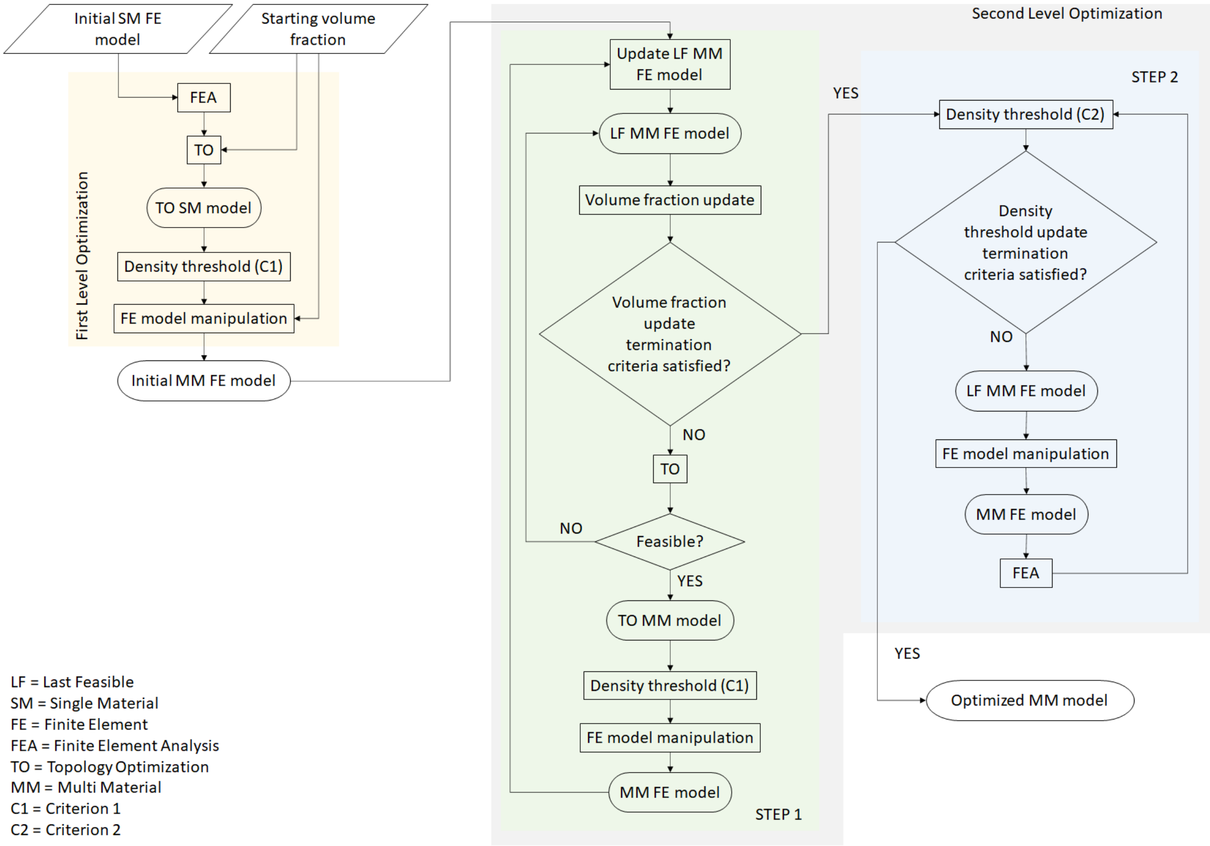


Figure 1: Workflow of the methodology

The goal of the presented methodology is to iteratively optimize the multi-material distribution (using two different materials) within a given design volume through a topological optimization based Generative Design approach. Indeed, with the proposed approach it is possible to optimize the materials distribution and the lay-out inner interface between the used materials. These ones could have different mechanical behaviors, as in the test case reported in Section 4, where 'Material 1' has a linear elastic behavior while 'Material 2' has an hyperelastic behavior.

The proposed methodology has been applied to approximated models of car seats. The production of seats requires a large use of polyurethane foam, combined with plastic and/or metal frames. In particular, car seats consist of different materials such as steel frame, filled polyurethane foam for seat cushions and vertical backrests, and external textiles. The method here developed proposes an optimization loop to consider the influence given by the presence of the polyurethane foam in the TO of the plastic part of the seat, allowing the possibility to have moving boundaries between the different materials. This loop continues optimizing both material regions, achieving an optimized material distribution of the multi-material part.

The workflow of the proposed methodology is presented in Fig. 1 and consists of a two-level optimization process.

3.1 First Level Optimization

The first level of optimization considers as input:

- an initial envelope geometry, i.e. an initial design volume, assigned by default to be totally made by 'Material 1';
- the design structural requirements;
- an initial volume fraction parameter $vfrac_1$ (the amount of material to be retained from the design space when performing a topological optimization).

This level pertains to the definition and validation of the finite element (FE) model to be used to achieve the performance requirements (structural, frequency, etc.). This model is used to perform the static analysis (FEA) under different load cases for the evaluation of the structural requirements. If frequency requirements are present, it is possible to perform the normal mode analysis to evaluate the modal frequencies of interest, as it has been done in the test case reported in the following section (Section 4).

After the validation of the FE model, the following step is represented by the set up of the topological optimization solver deck. The initial design volume of the component is divided into design and non-design spaces. The design objective of the topological optimization is the minimization of the weighted compliance of the overall system (design and non-design spaces). The design constraint is set on the upper limit of the volume fraction ($vfrac_1$) of the design space. Furthermore, other structural and/or frequency requirements could be implemented as design constraints. The volume fraction constraint is the percentage of material to be retained at the end of the TO algorithm. The constraint on this design variable must be carefully chosen to reduce the number of iterations (computational cost) and to provide meaningful designs also with less effort in the post-processing phase. In addition, symmetry and manufacturing constraints as minimum element size and/or overhangs, etc. could be considered in this stage.

The output of the topological optimization with the assumed volume fraction ($vfrac_1$) constraint of the iteration 1 of the workflow is the list of all the *E* design elements of the FE model with the associated element densities of the SIMP algorithm ('dens' file). At this point, the density threshold parameter $density$ is introduced to carry out the further iterations. This parameter is used by the algorithm to update the FE model by switching the material (from 'Material 1' to 'Material 2') for all the elements of the model with an element density lower than the threshold value. It is fundamental to notice that all the elements that are moved from 'Material 1' to 'Material 2' are then set to non-design space and fixed in the further iterations.

3.1.1 Density Threshold Update (Criterion 1)

The criterion 1 for the density threshold parameter $density_j$ update depends on the volume fraction parameter $vfrac_j$ value at the current iteration. The value of the density threshold parameter is carried out by a python script that takes the 'dens' file as input and sorts the E elements in ascending order of the densities. Then, the density value corresponding to the e element (Eq. (4)) of the list is retained as threshold value $density_j$ (Eq. (5)).

$$e = (1 - vfrac_j) \cdot E \tag{4}$$

$$density_j = density(e) \tag{5}$$

3.2 Second Level Optimization

The output of the first level optimization and the input for the second one is the multi-material FE model made by Material '1' and Material '2' using the volume fraction parameter value $vfrac_1$ to compute the density threshold.

The second level optimization is a two-step automated loop for the multi-material lay-out distribution optimization. Step 1 iterates over the volume fraction parameter while step 2 adopts a different density threshold parameter for a further refinement of the optimized model.

3.2.1 Step 1: Volume Fraction Update

The first iteration of the STEP 1 (iteration 2) keeps the initial volume fraction parameter $vfrac_2 = vfrac_1$ to carry the topology optimization and produces an updated 'dens' file as output. The Python script updates the density threshold value $density_2$ according to the initial volume fraction constraint and the FE model for the next iteration is available.

The iterations continue with the initial volume fraction up to the infeasibility of the topology optimization at iteration $i = lnfto$ (last non-feasible topology optimization) due to the violation of at least one of the design constraints. At this point, the algorithm updates the volume fraction value $vfrac_{i+1}$ at the next iteration $i+1$ by considering the average value between the $vfrac_{lnfto}$ at iteration $i = lnfto$ and $vfrac = 1$ (Eq. (6)).

$$vfrac_{i+1} = \frac{vfrac_{lnfto} + 1}{2} \tag{6}$$

The FE model is updated according to the density threshold value and the topology optimization is performed. Based on the result of this optimization, the volume fraction at iteration $i+2$ updates as follows in Equation (7):

$$vfrac_{i+2} = \begin{cases} \frac{vfrac_{i+1}+1}{2} & \text{if (Top. Opt.)}_{i+1} \text{ infeasible} \\ vfrac_{i+1} & \text{if (Top. Opt.)}_{i+1} \text{ feasible} \end{cases} \tag{7}$$

The TO is carried out and, if infeasible, the script updates the $vfrac_{i+2}$ as the average value between $vfrac_{i+1}$ and $vfrac = 1$, meaning the volume fraction is updated towards 1. If the TO carried at iteration $i+1$ is feasible, the algorithm keeps the $vfrac_{i+2}$ at the same value of the previous iteration.

The iteration stops when the change in the volume fraction value with respect to the volume fraction upper bound is lower than a termination threshold value, retaining the last feasible topology optimization.

3.2.2 Step 2: Density Threshold Update (Criterion 2)

STEP 2 is a further refinement of the optimized model. Indeed, it is performed when the termination criterion on the volume fraction parameter is reached. At this point, the volume fraction parameter is set to $vfrac = 1$ and no more topology optimizations are performed. Instead, static and/or modal analyses are used to validate the optimized lay-out. This step considers as input the last feasible topology optimization iteration ($k = lfto$). Indeed, the last feasible topology optimization density threshold value $density = density_{lfto}$ is used to update the FE model by switching the 'Material 1' elements into 'Material 2' ones according to the last feasible topology optimization 'fem' and 'dens' files. If the analyses are feasible, the optimization loop ends and the optimized model is available for the post-processing operations. On the contrary, the script updates the $density_{k+1}$ as the average value between $density_k$ and $density = 0$, meaning the density threshold is updated towards 0 (Eq. (8). This process ends either when a feasible value is found or when the termination criteria is reached.

$$density_{k+1} = \frac{density_k}{2} \tag{8}$$

Instead, while all the iterations from the iteration $k+1$ are feasible, the algorithm updates the density threshold $density$ as the mean value between the last feasible analysis (lfa) and the last feasible topology optimization ($lfto$), thus looking for values closer to the upper bound $density_{lfto}$ (Eq. (9)).

$$density_{k+1} = \frac{density_{lfa} + density_{lfto}}{2} \tag{9}$$

Else, the script computes the next value of the parameter as the mean value between the density value of the last non feasible analysis $density_{lnfa}$ and the last feasible one $density_{lfa}$ (Eq. (10)).

$$density_{k+1} = \frac{density_{lfa} + density_{lnfa}}{2} \tag{10}$$

4 CASE STUDY

The proposed methodology has been applied to a simple case study to optimize the material distribution within the initial design volume. The initial volume consists of a rectangular parallelepiped of 530 mm x 440mm x 120mm which represents a raw approximation of the seat's cushion assembly.

The used materials are Acrylonitrile-Butadiene-Styrene (ABS) for 'Material 1' and Polyurethane Foam (PF) for 'Material 2'. Used individually, they do not guarantee appropriate usability. Indeed, ABS alone is too stiff, and the foam needs a frame. The Ogden's model is used to describe the hyperelastic non-linear and strain-rate dependent behavior of the foam, according to [15]. In the following tables are reported the mechanical properties of the two materials: ABS (Tab. 1) and Foam (Tab. 2).

Table 1: ABS mechanical properties

MODEL	Y (MPa)	ν	$\rho\ (\frac{g}{mm^3})$
LINEAR ELASTIC	2240	0.38	0.00106

Table 2: Ogden model's parameters for the used foam

MODEL	N	ν	$\rho\ (\frac{g}{mm^3})$	μ_1	α_1	μ_2	α_2	μ_3	α_3
OGDEN	3	0.4495	0.000026	2.81	1.66	-2.8	1.61	0.0031	38.28

4.1 Model, Analysis and Topology Optimization

The first level optimization requires to set up the Finite Element model to be used. Since topology optimization requires to divide the initial volume into design and non-design spaces, it has been done once the STEP file has been imported in the solver environment. The above seat plate non-design volume has been retained ensure the presence of the sitting area. The bottom parallelepiped are preserved to consider the interface with the frame (not reported in this preliminary test). Then, the model has been meshed using first order CHEXA elements for the non-design space, and first order CTETRA elements for the design space. Five loading conditions for the seat have been defined and applied. They approximate five different distributions of the same load. As reported in Tab. 3, the loads are applied as uniform pressures on different sectors of the seat top surface, considering a person with mass $m = 100Kg$.

Table 3: Pressure loadcases.

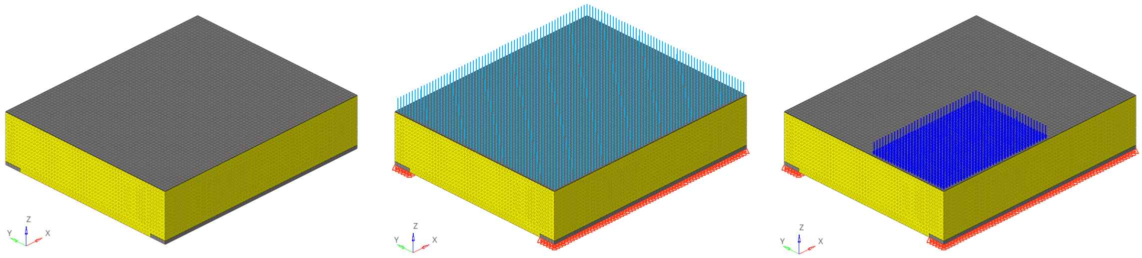
Loadcase	Pressure ($\frac{\mathrm{N}}{\mathrm{mm}^2}$)	Area (mm^2)	Depth (mm)	X (mm)	Width (mm)	Y (mm)
1-Full Surface	4.29E-03	2.33E+05	530	0	440	0
2-Corner	1.73E-02	5.78E+04	289	0	200	0
3-Front Middle	1.73E-02	5.78E+04	289	0	200	120
4-Side Middle	1.73E-02	5.78E+04	289	120	200	0
5-Center	1.73E-02	5.78E+04	289	120	200	120

These load collectors are associated to linear static analysis load cases only for the First Level Optimization (iteration 1), while in the Second Level Optimization (starting from iteration 2) they are associated to non-linear static analysis ones due to the presence of the hyperleastic material. In addition, it has been set a load case to analyze the first $N = 3$ natural frequencies. To simulate the anchorage of the seat to the seat frame, the elements of the non-design space region located in the bottom part of the model have been constrained locking all their DOFs, along the entire depth in X direction, for sake of simplicity, as reported in Fig. 2.

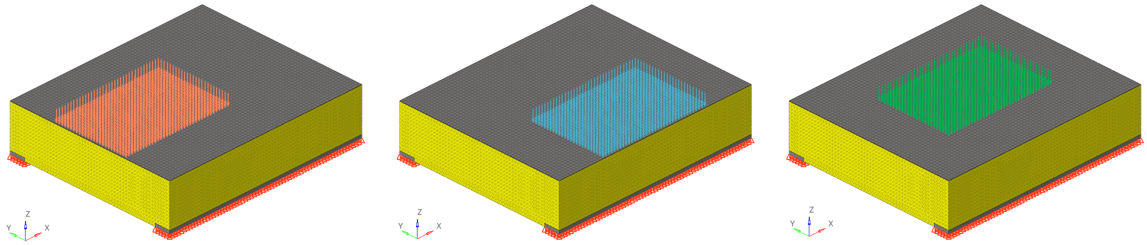
According to the design variables for the TO defined in the "methodology" section, the objective function is to minimize the weighted compliance of the overall component, while the constraints are on the upper limit of the volume fraction of the design space and on the lower limit of the first $N = 3$ natural frequencies. The last constraint is set according to standards and comfort requirements, also considering a factor of safety. Indeed, the first $N = 3$ natural frequencies of the system have been constrained to be above $90Hz$ to prevent dangerous frequency ranges for the human body. Four test cases are considered for different values of the initial volume fraction, to show the relevance of this parameter on the result in terms of both the computational cost and post-processing operations (Tab. 4).

Table 4: Test cases definition: design objectives, constraints, initial volume fraction.

Test case	Objective	Freq. constr.	$vfrac_1$
1	Min. WCompl	$> 90Hz$	< 0.95
2	Min. WCompl	$> 90Hz$	< 0.80
3	Min. WCompl	$> 90Hz$	< 0.70
4	Min. WCompl	$> 90Hz$	< 0.50



(a) FE model. (b) Pressure load 1 - Full Surface. (c) Pressure load 2 - Corner.



(d) Pressure load 3 - Front Middle. (e) Pressure load 4 - Side Middle. (f) Pressure load 5 - Center.

Figure 2: FE model, pressure load collectors and boundary constraints.

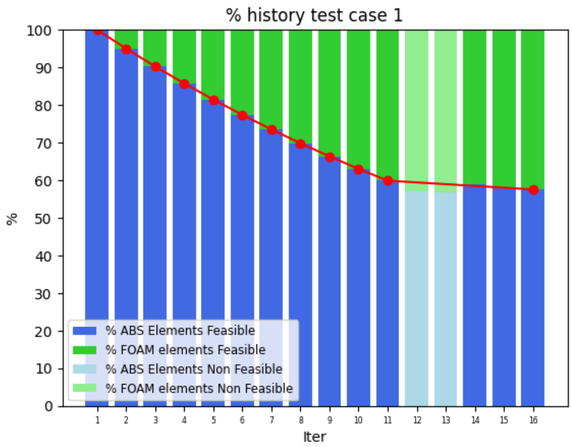
4.2 Results

Test case 1 has run 16 iterations, 14 of which are feasible. Of the initial 'Material 1' design elements, the 57% are retained while the remaining are moved to 'Material 2'. Test case 2 has run 46 iterations, 39 of which are feasible. Of the initial 'Material 1' design elements, the 10% are retained while the remaining are moved to 'Material 2'. Test case 3 has run 12 iterations, 9 of which are feasible. Of the initial 'Material 1' design elements, the 15% are retained while the remaining are moved to 'Material 2'. Test case 4 has run 11 iterations, 4 of which are feasible. Of the initial 'Material 1' design elements, the 52% are retained while the remaining are moved to 'Material 2'.

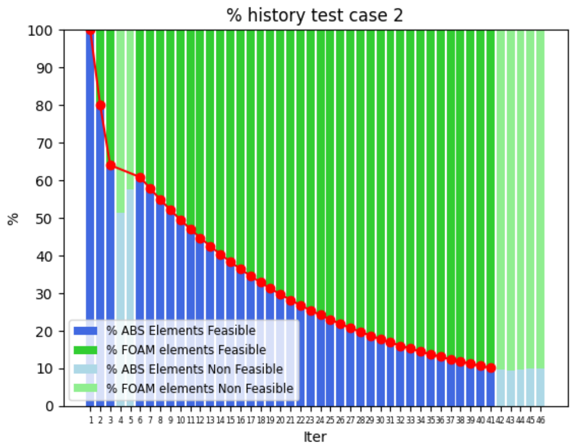
All the solutions satisfy the design constraints, however, the test cases 2 and 3 reached lighter configurations. Furthermore, test case 3 performed the iterations in 2 hours 27 minutes while test case 2 took 9 hours 52 minutes. All the iterations have run on an Intel Core i7-1185G7 CPU laptop equipped with 32 GB RAM.

In Figure 3, it is reported the summary for each test case. In case of test 1 (Fig. 3(a)), it is possible to notice that even if iterations 14 and 15 are feasible, they are related to the step 2 of the second level optimization process where normal mode analyses are performed and only the last feasible one is retained.

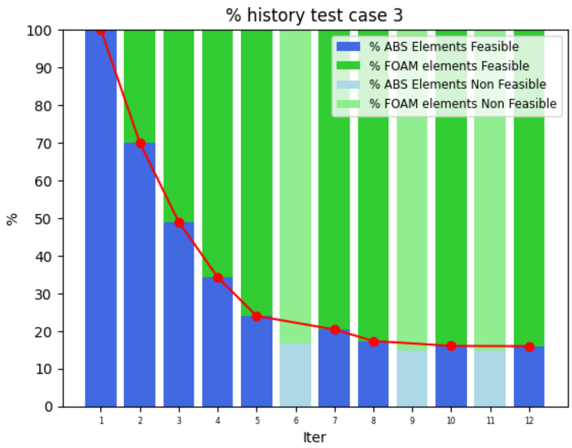
Based on these results, it is possible to evaluate visually the results by slicing the solutions for each test case at different sections. Test cases 1 (Fig. 4(a)) and 4 (Fig. 4(d)) both retained over the 50% of the starting ABS elements. They result in bulk, rigid and heavy configurations. Among these ones, test case 1 doesn't have any infeasible closed infill of foam elements while test case 4 does. However, test case 1 has sparse ABS elements in the foam matrix, which must be filtered out in the post-processing operations. In conclusion, test case 1 performs better in terms of feasible iterations with respect to test case 4, and they have comparable computational costs. Test cases 2 (Fig. 4(b)) and 3 (Fig. 4(c)) both retained under the 15% of ABS elements; indeed, these solutions are reasonably light when compared to a traditional seat. Both these solutions doesn't have many sparse ABS elements in the foam matrix. However, test case 2 performs 46 iterations with a four times higher computational cost than test case 3.



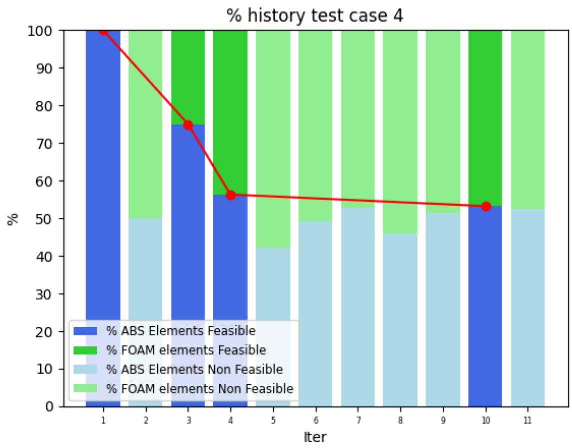
(a) Test case 1.



(b) Test case 2.

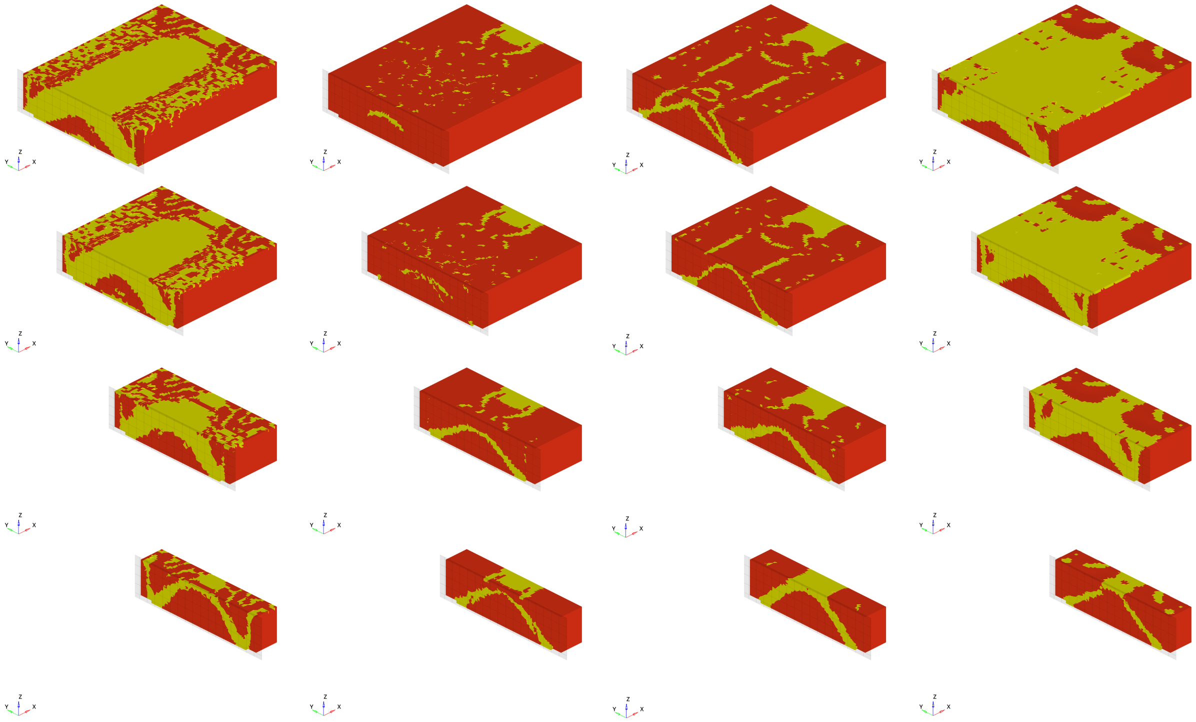


(c) Test case 3.



(d) Test case 4.

Figure 3: Test cases history bar plots.



(a) Test case 1. (b) Test case 2. (c) Test case 3. (d) Test case 4.

Figure 4: Material distribution visual inspection (yellow = 'Mat. 1') at sections $x = 0, 150, 350, 450$.

5 CONCLUSIONS AND FUTURE WORKS

This paper presents a novel methodology for considering the multi-material design lay-out optimization by an iterative Generative Design algorithm based on topology optimization. The methodology automatically updates the FE model to fill non-structural elements with the other material, which could also be modeled with a hyperelastic behavior. The methodology has been applied to a simple case of an automotive seat whose frame is intended to be additively manufactured to test its effectiveness in the optimization of the boundaries between the two materials (ABS and Polyurethane foam). Since the volume fraction parameter influences both the computational cost and the necessity for post-processing operations, the future works will pertain to the definition of an automated procedure to properly select this key parameter and the process for achieving the optimized CAD model, possibly considering a further shape optimization. It will be considered a criterion to avoid closed volumes filled with the other material. In addition, it will be possible to consider also Lattice Structures for intermediate densities. Moreover, to analyze a real scenario of the seat test case, the future steps of this study will include the reduction of the accelerations for passengers, the random solicitations from the road, other structural requirements (displacement, etc.) and the complete model of the seat.

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