



A Review on Modeling of Plant Cell Growth

Jiawei Pei¹ , Changyu Liu² , Aimin Wu³ , Ziwei Cheng⁴ ,
Hansheng Wang⁵ , Jingjing Zheng⁶ , Hao Huang⁷  and Chunyin Wu⁸ 

¹South China Agricultural University, cavey759@gmail.com

²South China Agricultural University, yezhich@gmail.com

³South China Agricultural University, wuaimin@scau.edu.cn

⁴South China Agricultural University, chbbybf@gmail.com

⁵South China Agricultural University, kasoa2419980765@gmail.com

⁶South China Agricultural University, jz146288@gmail.com

⁷South China Agricultural University, huanghaoyeswork@gmail.com

⁸South China Agricultural University, wuchunyin@scau.edu.cn

Corresponding author: Changyu Liu, yezhich@gmail.com,
and Aimin Wu, wuaimin@scau.edu.cn

Abstract. Cell division and growth in plants involve morphological changes as well as the transfer of energy. However, the human eyes couldn't always directly observe these growth processes. Plant growth modeling is a good way to understand the interactions between plant cells. This method quantitatively analyzes the behavior of plant cells, thereby simulating the process of cell growth. Three different kinds of plant growth models are explored in this paper: computer simulation models, biomechanical models, and biophysical models. Biophysical models are designed to study the physical characteristics of plant cells, such as morphology, structure, function and dynamics. These models shed light on various processes, including cell division, the principles behind plant cell development, and morphological alterations brought on by outside factors. Biomechanical models are more suitable for analyzing plant cell movements, mechanics, and other behavioral characteristics, focusing on understanding the spatiotemporal patterns of plant cell behaviors. Additionally, biomechanical models explain the effects of environmental factors inside and outside plant cells on cell motility. Computer simulation models rely on topological retrieval, 3D technology, and genetic algorithms to describe, simulate, and predict plant cells. The three categories of models that were previously discussed offer important insights into the biological and physical workings of plant cells. More effective strategies can be developed to enhance crop yields and the development of plant-based products.

Keywords: Plant growth modeling, plant cell division, biophysics, biomechanics, computer simulation

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1 INTRODUCTION

Plant cell growth modeling is important in the following sectors of agricultural production:

(1) Predicting plant growth [1]: Researchers can accurately predict the growth trend of plant cells through modeling, thereby optimizing plant growth conditions to achieve optimal production efficiency. Plant cell growth modeling also promotes the sustainable development of agriculture by investigating the impact of environmental and stress factors on plant growth.

(2) Plant breeding: Plant cell growth modeling deepens researchers' understanding of genetic characteristics and developmental processes, offering valuable guidance for plant breeding endeavors. Machine learning and gene data analysis techniques were utilized by Niazian et al. [2] to establish a computer simulation model, revealing relationships among plant genes during the breeding process, thereby optimizing the ex-vitro breeding process for plants.

(3) Research on plant stress adaptation: Plant cell growth modeling can explore the physiological, biochemical, and molecular mechanisms by which plant cells adapt to stress, thereby identifying key factors in plant stress adaptation. Modeling for cultivating stress-resistant crops [3] further ensures the security of food production.

(4) Enhance plant disease resistance: Modeling enables the prediction and optimization of interactions between plants and pathogens, thereby improving plant disease resistance [4].

By combining physics, mechanics, and computer technology with knowledge of biology, the researchers aim to create digital twins of plant cells [5], that is, to simulate the growth process of plants in a virtual environment. A useful technique for comprehending the intricate connections between genes and plant growth is plant cell growth modeling.

However, there are still ongoing debates in the field of plant cell growth modeling. One of the debates concerns the integration of modeling methods at different scales to construct a comprehensive, multi-level model of cell growth. Furthermore, another contentious area in modeling revolves around calibrating modeling results with experimental data, as well as optimizing model parameters. This paper's main objective is to summarize important past research results of plant cell growth modeling, integrating biophysics, biomechanics, and computer technology. This review primarily contributes in the following ways:

(1) Providing a systematic summary of related research on plant cell growth models to offer a comprehensive overview.

(2) Pointing out the theoretical standpoint of the models and the evolving direction of research methodologies.

(3) Introducing the utilization of computer technology in models of plant cell growth to showcase its potential and impacts.

(4) Emphasizing the prospective application of these models in agricultural production, encompassing the optimization of plant growth conditions, enhancement of crop yield and quality, and bolstering resistance against diseases and insect pests.

The remaining content of each chapter is summarized as follows: Section Two outlines three models of plant cell growth: biophysical models [6-13], biomechanical models [14-15], and computer simulation models [2]. Sections Three, Four, and Five of this review categorize the models into these primary classifications and provide a comprehensive description of each model. Section Six discusses the differences between plant cell and animal cell growth models. Section Seventh summarizes the development prospect of the plant cell growth models.

2 CELL GROWTH MODEL TAXONOMY

In the field of biology, the term model has various connotations. When referring to cell growth models, it can denote a set of equations incorporating multiple variables and parameters that

describe the changes occurring in the system. It can also refer to a series of steps outlining the process of change. All these models represent abstract expressions of the natural changes that occur during cell growth processes in different ways. This paper reviews renowned plant cell growth models sourced from top journals and conferences. These models can be categorized into three main groups as illustrated in Figure 1: biophysical models [6-13], biomechanical models [14-15], and computer simulation models [2].

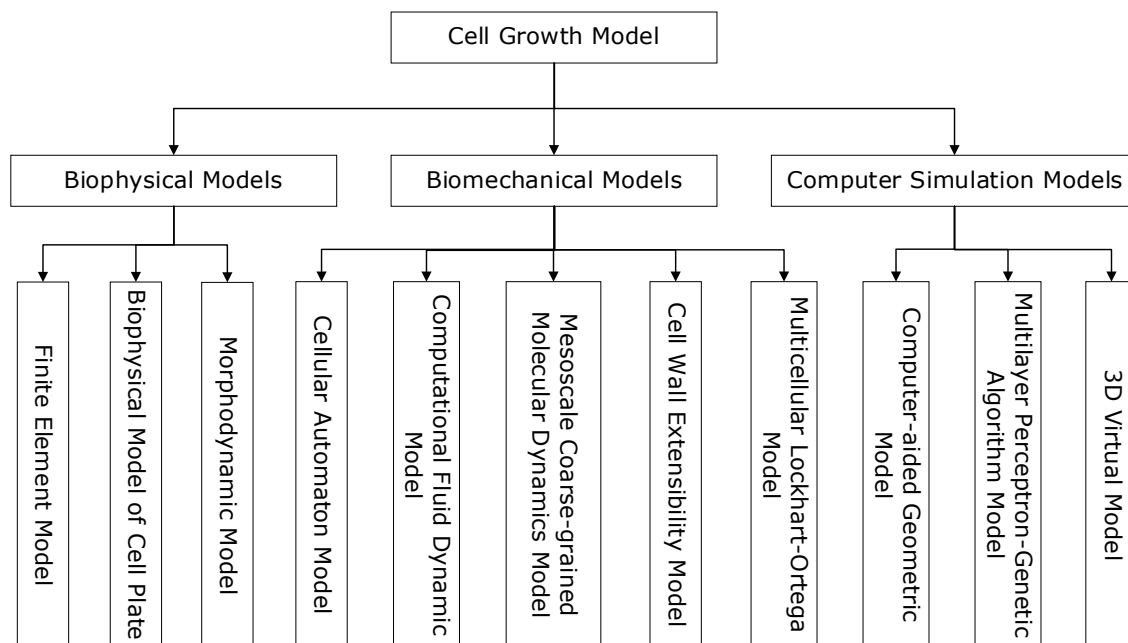


Figure 1: The hierarchical taxonomy of this review.

This paper provides a comprehensive overview of three different biophysical models [6-13]: the finite element model [16], the biophysical model of the cell plate [17] and the morphodynamic model [18]. Five more biomechanical models are included in the review [14-15]: the cellular automaton model [19], the computational fluid dynamic model [20], the mesoscale coarse-grained molecular dynamics model [21], the cell wall extensibility model [22], and the multicellular Lockhart-Ortega model [23]. In addition, the review discusses three computer simulation models [2], namely the computer-aided geometric model [24], the multilayer perceptron-genetic algorithm model [25], and the 3D virtual model [26].

<i>Type</i>	<i>Section</i>	<i>Model</i>	<i>Research Question</i>	<i>Methodology</i>
Biophysical Models	3.1	Finite element model [16]	Investigation of morphological changes [28] and mechanisms of cell membranes [29-31]	Finite element algorithm
	3.2	Biophysical model of cell plate [17]	Study of mechanical properties and growth regulation of plant cells	Experimental measurement, mathematical modeling, parameter estimation, and simulation

	3.3	Morphodynamic model [18]	Integrating single-cell behavior and molecular signaling for plant growth prediction.	Analyzing feedback between growth mechanisms and signaling pathways
Biomechanical Models	4.1	Cellular automaton model [19]	Growth mechanism of the cell membrane [29-31] and distribution of photosynthetic products [32]	Experimental techniques such as spectroscopy, microscopy, and mechanical testing
	4.2	Computational fluid dynamic model [20]	Investigation of plant cell wall [33] growth rate and cell density	Euler Lagrange method, fluid dynamics simulations, data analysis, and validation
	4.3	Mesoscale coarse-grained molecular dynamics model [21]	Experimental measurement of cell wall properties, mathematical modeling, and simulation	Atomistic molecular dynamics simulations and analysis
	4.4	Cell wall extensibility model [22-23]	Investigation of cell wall extensibility and its role in cell growth	Experimental measurement of cell wall properties, mathematical modeling, and simulation
	4.5	Multicellular lockhart-ortega model [24]	Application of the Lockhart-Ortega model in plant cell growth and water flow	Multicellular model, including mechanical equilibrium, geometric constraints, and computational solution
	Computer Simulation Models	5.1	Computer-aided geometric model [25]	Geometric modeling of plant cell shapes using a topological approach
5.2		Multilayer perceptron-genetic algorithm model [26]	Optimization of paclitaxel biosynthesis in plant cell culture using MLP-GA	Data preprocessing, Multi-Layer Perceptron, Genetic Algorithm optimization, training, validation, and testing
5.3		3D virtual model [27]	A 3D Voronoi tessellation	Comparing cell size, shape, and pore generation mechanisms

Table 1: Summary of the cell growth model taxonomy.

Plant cell proliferation is an intricate procedure necessitating the harmonized interplay of diverse molecules and organelles. Additionally, plant cell growth involves intracellular fluid expansion and modifications in cell walls. Table 1 provides a systematic summary of plant cell growth models, research questions, and corresponding methods. This table facilitates the identification of commonalities and differences among these methods, effectively fostering methodological advancements and promoting interdisciplinary collaboration.

3 BIOPHYSICAL MODELS

There are three popular models surveyed for biophysical models [6-13], including the finite element model [16], the cell plate biophysical model [17], and the morphodynamic model [18], as shown in

Table 2. These models describe the morphological changes and growth of plant cells through different focuses and methods.

The finite element model [16] focuses on studying the morphological changes [28] and mechanisms [29-31] of the cell membrane under external forces and pressure. Therefore, the model is relatively simple, without considering other factors such as cell structure and cell growth.

The cell plate biophysical model [17] is more comprehensive as it encompasses the growth, morphological changes [28], and underlying mechanisms of both the plant cell plate [34-36] and cell membrane [29-31].

The morphodynamic model [18] predicts the growth process of plants by capturing changes in plant cell images in real time [1].

<i>Models</i>	<i>References</i>
Finite element model	Bidhendi et al., 2018 [16]
Biophysical model of cell plate	Jawaid et al., 2022 [17]
Morphodynamic model	Chickarmane et al., 2010 [18]

Table 2: Popular approaches and corresponding references for biophysical models.

3.1 Finite Element Model

The finite element model [16] was a valuable tool for studying plant biophysics, simulating cell behavior, and providing insights into growth and development. The model can be utilized to localize and predict pressure changes or movements in single-cell morphology [37]. The finite element model was also utilized by Sayad et al. [38] to observe the phenotypic characteristics of plant cells, thereby analyzing data such as cell size, cell wall thickness, and tissue structure stiffness.

There were multiple important steps were involved [16].

(1) Definition of geometry and dimensions: The model defined the initial geometric shape and size of the plant cell [39-41].

(2) Mesh generation: The model created a mesh structure through discretization [42-44], which involved the decomposition of the geometric shape of the plant cell into smaller finite elements. Typically, the cell surface was divided into small triangular or quadrilateral elements, forming a well-defined grid that facilitates numerical calculations and analysis.

(3) Application of boundary conditions: Boundary conditions were essential for simulating the real environment of cell structures and typically involved considerations of displacement, intercellular interactions, and external forces. Finite element algorithms [16] could define interactions between objects, including boundary conditions such as frictional forces, fluid flow, and other relevant parameters. To characterize the deformation state of plant cells under cell wall limitations, the model incorporated heat maps.

(4) Assignment of a material model: The finite element model linked the material properties of the cells [45-48] with the specified mesh structural elements to simulate the deformation and stress behavior of plant cells under external pressures. Specifically, the stiffness and global matrices were derived using the finite element model based on the geometric shape and composition of plant cells. After boundary conditions were created, equations were solved to find the displacement, strain, and stress caused by the applied loads at each node.

To sum up, the computation and examination of these data were essential for developing a better comprehension of the process of plant cell growth, enhancing growth environments, and directing the creation of novel materials.

3.2 Biophysical Model of Cell Plate

Cell plate formation [49] is a crucial process in plant cell division [50] because it ensures the integrity and functionality of daughter cells. The process could be subdivided into multiple stages, as outlined below [51-52]:

The golgi apparatus transported synthesized nutrients to the central position of the dividing plant cell through vesicles. These vesicles connect with the cell membrane to release chemicals and go into the membrane maturation phase. A tubular porous structure was created at the cell division plane by the aggregation of many vesicles, which guaranteed the creation of the plant cell wall. On the cell division plane, polysaccharides such as cellulose, hemicellulose, and pectin were concurrently deposited. At this time, the interplay of stabilizing and expanding forces formed the tubular porous structure, which gradually joined into a porous lamellar structure to produce the cell plate.

A biophysical model of the cell plate was proposed by Jawaid et al. [17], which simulated the shape and structure of the cell plate based on the Helfrich-free energy model [53]. This model predicted the rate of polysaccharide synthesis by calculating the spreading and stabilizing forces [54-57] generated in the cell membrane structure during polysaccharide deposition, thereby determining the maturity of the cell plate. It was essential to note that the spreading and stabilizing forces played a critical role in facilitating cell plate formation during tubular network development. These forces ensured the integrity of the cell plate formation structure [49].

The cell plate was a structure formed during cell division, ultimately transforming into the cell wall of the new cell. The efficient progression of cell division and growth, made possible by the coordinated action of the cell wall and cell plate, retained the structure and functionality of plant cells.

3.3 Morphodynamic Model

Plant cells interact in a complicated spatiotemporal manner during growth and signal transmission processes, which were characterized by Chickarmane et al.'s morphodynamic model [18].

The morphodynamic model [18] not only expressed the plant growth process from single-cell behavior to tissue development [58] but also illustrated the need for feedback between growth mechanisms and chemical or molecular signaling [59]. The model included three features: (1) The model variables were experimentally calibrated. (2) The model was practically parameterized. (3) The model could make key predictions through experiments.

The main challenges and solutions for this model [18] were as follows:

(1) The morphodynamic model solved the transition from single-cell to tissue-scale growth. The solution was to observe the growth of plant cells through real-time imaging technology while ensuring healthy plant growth. Processing methods such as 4D visualization, picture segmentation, and cell tracking were then employed to determine the target cells' size, shape, and relationship to one another, as well as to rebuild the cell geometry.

(2) The morphodynamic model also addressed the relationship between machinery and molecules. The model combined molecular models with growth mechanisms to create self-organizing growth tissues. Furthermore, the model identified key molecular components involved in plant growth, such as relevant genes, hormones, and interaction networks, through experimentation. Then, parameters that affected growth were combined with the molecular network.

4 BIOMECHANICAL MODELS

Biomechanical models [14-15] exhibit a broad spectrum of applications and can be employed in diverse facets of plant biology, encompassing cell morphology, growth, development, and metabolism. Developing such plant models is a challenging task, considering the complexity of plants as systems influenced by various factors such as water, nutrients, and temperature. This task necessitates the integration of interdisciplinary expertise from fields like biology, physics, and

mathematics. There are five popular models surveyed for biomechanical models [14-15], as shown in Table 3.

The cellular automaton model [19], used to simulate carbon source growth and transport within algal biofilms, and the multicellular Lockhart-Ortega model [24], which investigates water flux and cell wall mechanics during plant development, both focus on internal material transport and growth processes within organisms.

The computational fluid dynamics model [20] and the mesoscale coarse-grained molecular dynamics model [21] both involve fluid dynamics simulation. The computational fluid dynamics model [20] assesses the impact of shear stress on safflower cells in a stirred tank bioreactor using the Euler-Lagrange computational fluid dynamic method. The molecular dynamics simulation [21] is used to study the mechanical characteristics of plant epidermal cell walls at the mesoscale level.

The mechanical characteristics of the cell wall are quantitatively analyzed by the cell wall expansibility model [22-23]. The model analyzes the mechanics of water absorption and relaxation in the cell wall, along with the role of hydrogen bonds in the cell wall's network structure.

<i>Models</i>	<i>References</i>
Cellular automaton model	Li et al., 2023 [19]
Computational fluid dynamic model	Liu et al., 2016 [20]
Mesoscale coarse-grained molecular dynamics model	Zhang et al., 2021 [21]
Cell wall extensibility model	Cosgrove et al., 2016 [22]; Yi et al., 2012 [23]
Multicellular Lockhart-Ortega model	Cheddadi et al., 2019 [24]

Table 3: Popular approaches and corresponding references for biomechanical models.

4.1 Cellular Automaton Model

A cellular automata model was utilized by Li et al. [19] to simulate the behavior of algal biofilms concerning light and carbon transport [32], as well as growth mechanisms [29-31], thereby improving production efficiency.

The specific steps involved in the cellular automata model [60-63] were outlined as follows:

(1) Establishment of an initial cell model: The initial cell models involve partitioning the biological membrane [29-31] into distinct cells and defining the initial state, encompassing factors such as cell density, light intensity, and carbon dioxide concentration.

(2) Definition of growth and division rules: Rules were established to determine the growth rate of each cell, considering factors [64] such as the photosynthesis rate and the availability of essential nutrients like CO₂ and light. When plant cell growth reached a specific threshold, the resulting daughter cells inherited attributes such as spatial positioning and internal characteristics from the parent cell. The rate equation of cell metabolism was also used to calculate the rate of cell metabolism [65].

(3) Simulation process: Based on the defined growth and division rules, the behavior of algal biofilms under different light intensities and CO₂ concentrations was simulated. The computed parameters were used to update the status of every cell. This iterative process persisted until a designated time point or until the simulation outcomes reached convergence.

(4) Visualization of results: Visualization tools were utilized to exhibit simulation results, analyze the growth and metabolism mechanisms of biofilms [66], and unveil internal patterns and characteristics of biofilms.

By following these steps, the model could effectively simulate the growth and metabolic behavior of algal biofilms under various conditions. Theoretically, this simulation method provided a useful starting point for further research into the fundamental ideas behind plant cell dynamics.

4.2 Computational Fluid Dynamic Model

The computational fluid dynamic model provided by Liu et al. [20] might imitate the mobility and growth processes of plant cells under external influences. The simulation results yielded insights into the influence of shear forces on cell growth, encompassing factors such as cell growth rate and density. The Euler-Lagrangian method [67] was employed to simulate complex multiphase flows, accounting for momentum conservation and continuity and defining the trajectory and forces acting on plant cells. Valuable numerical simulation tools for comprehending the motion and growth mechanisms of plant cells were provided by this research.

The specific calculation steps of the computational fluid dynamic model [68-70] are as follows:

(1) Parameter definition and mesh generation: The model defined the geometry of the mixing tank, including impeller diameter and spacing. The cell shape was divided into mesh cells using software like ANSYS ICEM to calculate fluid flow and mass transfer numerically. Boundary conditions for the mesh cells were defined, including reactor wall characteristics, surface roughness, and stirring speed.

(2) The governing equations' solution: Changes in fluid velocity, concentration, and other parameters were calculated at each discrete point by use of the governing equations, which separated space and time into discrete grids or elements. This could lead to a more accurate comprehension of the dynamic fluctuations and spatial arrangement of plant cells.

(3) Shear stress calculation [71]: Shear stress was the term for the internal tension that results from velocity gradients in a fluid. By altering the mixing tank's speed, the fluidity of the material transfer from the cells might be evaluated. The plant cell motion mechanism was better understood with the help of shear stress computation.

(4) Result analysis and design optimization: The shear stress frequency parameter was introduced by Liu et al. [20], which was the product of shear stress and shear frequency. Evaluating variables like the saffron cell death rate required the use of the Shear Stress Frequency parameter. Experimental processes like particle image velocimetry [72] were utilized to measure the fluid velocity field. Additionally, a comparison of cell growth rate and cell density between particle image velocimetry experiments and computational fluid dynamic simulations [73] was presented by Liu et al. [20]. Such comparisons validated the accuracy of the model and illustrated the hydrodynamic behavior capabilities of plant cells.

4.3 Mesoscale Coarse-Grained Molecular Dynamics Model

The mesoscale coarse-grained molecular dynamics model [21] analyzed the structure and mechanical properties of plant primary cell walls [27], along with external influencing factors. This model's mechanical responses under varied conditions were revealed through simulation and analysis, offering a theoretical foundation and direction for plant cell wall design, engineering modification, and disease treatment.

The following actions were taken to build the Mesoscale Coarse-Grained Molecular Dynamics model:

(1) Definition of initial conditions and force field: The model acquired images of plant epidermal cell walls through microscopy [74-75], thereby defining the positional information, velocity, and interaction forces of plant cell walls [33].

(2) Molecular dynamics simulation: Zhang et al. [21] utilized molecular dynamics simulations to examine the initial state of cellulose within the cell wall. Through simulating the surface deposition guided by plant cells [76] and the formation of wall-like structures by enzyme complexes [77], further exploration of the formation process and biological functions of the cell wall was conducted.

(3) Analysis of simulation results: Top images showing various beginning cellulose orientations were provided, along with side and top views of the equilibrium state as represented by the model. The macromolecular morphology of the plant cell wall [33] was naturally reproduced by the Mesoscale Coarse-Grained Molecular Dynamics model due to its physical interaction mechanisms.

In summary, the mesoscale coarse-grained molecular dynamics model [21] introduced an innovative approach to exploring the mechanical and structural characteristics of plant cell walls [33]. Moreover, this study [21] established the groundwork for the advancement of high-performance materials characterized by outstanding strength and toughness.

4.4 Cell Wall Extensibility Model

A plant cell growth model based on the concept of wall extensibility [78] was proposed by Cosgrove D.J. et al. [22]. The cell wall's irreversible expansion phenomenon was explained by the extensibility model, which characterized the mechanisms of cell wall stress relaxation [79] and the driving force of water absorption [22].

The yield threshold of the cell wall [80-81] could be measured by isolating growing tissues from an external water supply and measuring its turgor pressure when it decayed to a steady value. Regarding plant growth and development, the model presented an in-depth framework for cell wall regulation.

Using cellulose microfibrils and hemicellulose networks, Yi et al. [23] developed a different cell wall model based on the findings of Kha et al. [82]. Cellulose microfibrils were abbreviated as CMF, while hemicellulose is referred to as HC. A joint element, analogous to hydrogen bonding, was introduced to describe the CMF-HC connection, differing from the approach of Kha et al. [82]. The cell wall model primarily focused on describing the weak hydrogen bonding types connecting CMF and HC. At the connection points, where the energy of the hydrogen bonds was transformed into mechanical performance, these hydrogen bonds carried strain energy and intermolecular contact forces. Because of this, the cell wall model's description of hydrogen bonding focused mostly on the kinds of connections that exist between CMF and HC and how those connections affect the mechanical characteristics of the cell wall network.

Overall, these two models delved into the characteristics and growth mechanisms of plant cell walls from different perspectives, providing crucial insights and guidance for understanding plant growth and development.

4.5 Multicellular Lockhart-Ortega Model

The growth of individual elongated cells, water flux, and wall stress was pioneered in modeling by Cheddadi et al. [24], based on the earlier work of Lockhart [83] and Ortega [84]. The comprehensive framework of the Lockhart-Ortega model integrated biophysics and biomechanics to explain the physical mechanisms of water transport across plant cell walls [33].

To explore and quantify the spatial extent, the two-cell model was extended to a chain of $2N+1$ cells by Cheddadi et al. [24], and the focus was on propagation to inhibit the growth of neighboring cells. The N cells were formed into a 2D lattice with a fixed topology, with cells of height h and thickness ω . A correlation between stress and elastic deformation was established in the model, with the elastic modulus representing the stiffness of the material. Cell wall rheology was described in the model using a viscoelasticity law, considering hydraulic and mechanical connections [85] between deformable objects.

Maintaining mechanical equilibrium involved considering the interplay between cell expansion, wall tension, and geometry. Solving for equilibrium at each time step is necessary since it was not possible to obtain an explicit expression for turgor as a function of stress [86-87]. The Lockhart-Ortega model [83-84] ensured compatibility between wall expansion and changes in cell volume [88] through a geometric constraint on unidirectional growth. This correlation was essential for simulating the biophysical interaction between cells and cell walls, providing insights into the

influence of water flow during plant growth and development. However, it was important to note that in a multicellular model, this correlation is no longer valid.

5 COMPUTER SIMULATION MODELS

Computer simulation models [2] are based on various mathematical methods and computational models to simulate plant cell growth, albeit with different areas of research emphasis. There are three popular models surveyed for computer simulation models [2], as shown in Table 4. The computer-aided geometric model [25] emphasizes modeling the shapes of plant cells and developing a topology retrieval algorithm to analyze similarities and differences among various cell types. This approach utilizes computer-aided geometric modeling techniques to construct 3D representations of plant cells.

In contrast, the multilayer perceptron-genetic algorithm model [26] simulates the growth of hazelnut cell cultures and the biosynthesis of paclitaxel. This model enables predictions of growth and paclitaxel production in hazelnut cell cultures, and it provides insights into the impact of external hormone treatment on these biological processes.

The 3D virtual model [27] elucidates fruit tissue microstructure and growth using 3D technology and Voronoi tessellation. By comparing cell size, shape, and pore generation mechanisms, the model understands gas exchange properties, predicts fruit shelf life, and assesses supply capacity.

<i>Models</i>	<i>References</i>
Computer-aided geometric model	Yi et al., 2020 [25]
Multilayer perceptron-genetic algorithm model	Salehi et al., 2020 [26]
3D virtual model	Abera et al., 2014 [27]

Table 4: Popular approaches and corresponding references for computer simulation models.

5.1 Computer-Aided Geometric Model

The research on individual plant cells and the abstraction of the plant cell wall [33] into a geometric structure resembling a hollow cuboid were focused on by Yi et al. [25]. The computer-aided geometric model simulated the elastic-mechanical structure of plant cells using topological and geometric methods and enabled automatic topological retrieval of cells.

3Gmaps [89-90], proposed by Yi et al. [25], referred to three-dimensional generalized maps. 3Gmaps provided an accurate geometric representation of the structure of plant cells. The hollow cuboid, representing the cell wall, was decomposed into six faces, with each face composed of four sides. In zero-dimensional space, each side was further divided into two half-edges. To better describe the geometric shape and topological structure of the cell wall, these half-edges were combined as the smallest topological units to form 3Gmaps: (W, t_1, t_2, t_3) . Therefore, the plant cell was represented as a geometric shape with 8 vertices, 12 edges, and 6 faces. Each vertex consisted of 6 half-edge structures. A topology retrieval algorithm [91] was then devised based on these half-edges, and the cyclic combination operator $\langle t_i, t_j \rangle$ was employed to iterate the components of the 3Gmaps structure. Eight components constituted the topological unit's vertex data structure: the vertex's Euclidean coordinates, topology data pointing to its edges, and specifics regarding fixed points or involution operators. The algorithm was implemented using C++ and OpenGL.

The flexibility to obtain precise geometric units of cell shapes was offered by the computer-aided geometric model. Unlike conventional numerical modeling techniques that depend on curves and surfaces [92-95], the model made it possible to locate and analyze geometric fragments inside cell shapes precisely.

5.2 Multilayer Perceptron-Genetic Algorithm Model

The multilayer perceptron-genetic algorithm model was applied by Salish et al. [26] to optimize the biosynthesis of paclitaxel, a secondary metabolite [96-99]. Combining the Multi-Layer Perceptron [100-101] with the genetic algorithm [102-105], the model captured the intricate relationships between input factors and the desired output in plant cells. By preprocessing experimental data, establishing the model, and optimizing parameters, the biological synthesis process of the target product in plant cells could be more accurately predicted.

The operation steps of the multilayer perceptron-genetic algorithm model were as follows:

(1) Data preprocessing: Experimental data were processed, transformed into input and output formats, standardized, and split into training, validation, and testing sets.

(2) Multi-layer perceptron modeling: Complex interactions between different input parameters and the intended output could be captured by applying artificial neural networks [26].

(3) Genetic algorithm [102-105]: In order to optimize the biosynthesis of paclitaxel, the genetic algorithm changed the model parameter configuration and searched the solution space for the best possible solution. This process was based on concepts like natural selection, crossover, and mutation that are inherent in plant cells [106-110].

(4) Model training, validation, and testing: Three stages were undergone: training to improve predictive accuracy, validation to assess performance on new data, and testing to verify generalization.

In summary, the utilization of the multilayer perceptron-genetic algorithm model [26] optimized the biosynthesis of paclitaxel [96-99], which not only enhanced production efficiency but also propelled the production and application of secondary metabolites in plant cells. The model [26] provided crucial scientific foundations and technical support for the production and application of secondary metabolites in plants.

5.3 3D Virtual Model

A 3D virtual model [27] defined the microstructure of fruit tissue through 3D technology. This model demonstrated the biomechanical process of the expansive growth of plant cells. The model used a 3D Voronoi tessellation [111] to generate the topology of the cell. It explained the structural differences between cells through Hooke's law [112]. And, the model described differences in cell shape across plant species.

The 3D virtual model [27] observed the different microstructures of plants by comparing the size and shape of cells, the mechanism of pore generation, and the virtual and real tissues, and inferred the growth state of plants under conditions such as transportation or deformation. The model used X-ray tomography to scan images of apples and pears to distinguish between cells, voids, and cell walls. The growth model regulated cell size by adjusting the ratio of the cell's maximum resting length to its initial value. In addition, calculating different cell wall stiffnesses adjusted for cell shape differences.

The 3D virtual model analyzed the gas exchange rate and characteristics of internal fruit cells, understanding the freshness and variation trends of fruits during storage, thereby predicting the shelf life and supply capacity of the fruit consumption market.

6 DISCUSSION

Discussing the differences between plant cell and animal cell growth models is meaningful. Even though they are the fundamental units of all life, the characteristics and behaviors of plant and animal cells vary widely in the fields of biology and ecology.

Firstly, plant cells possess a cell wall and large vacuoles [113], while animal cells typically lack a cell wall and exhibit more complex organelle organization [114]. Consequently, this paper lists

several biomechanical models [6–13] that highlight the significance of the cell wall in plant cell shape and mechanical structure [115].

Secondly, plant cells and animal cells acquire nutrients differently. For instance, the cellular automaton model [19] in the text investigates the distribution of photosynthesis products in plant cells, thereby analyzing the energy transmission mechanism of plant cells. In contrast, animal cells obtain energy and nutrients by ingesting organic substances. As a result, standard animal cell growth models are typically not readily transferable to the study of plant cells.

Lastly, due to differences in cell structure, plant and animal cells have distinct growth-regulating systems. In addition to studying the cell cycle and signal transduction pathways seen in animal cell models [116], plant cell growth models also look at how water, light, temperature, and the availability of nutrients affect plant cells [117]. The cell plate biophysical model [17], the morphodynamic model [18], and the multilayer perceptron-genetic algorithm model [26] in the paper all comprehensively analyze the growth mechanisms and signal feedback mechanisms of plant cells. These kinds of studies help to improve plant cells' development efficiency.

Comparative discussions between plant and animal cell growth models can provide important guidance and insights for related fields of research and applications. Modeling plant cell growth helps improve crop output and quality predictions in the agricultural production sector [118], hence improving agricultural ecosystems. In the field of bioengineering technology [119], modeling plant cell growth aids in designing more efficient plant gene editing techniques and identifying plant compounds with medicinal and industrial value.

7 CONCLUSIONS

The plant cell growth model offers a practical and straightforward approach to comprehending the intricate growth mechanisms of plants, which may be challenging to observe and forecast with precision under natural conditions. By quantifying and calibrating biophysical parameters at the cellular level, the model demonstrates the growth mechanism of plant cells, including expansion, division, and chemical transport processes. Therefore, this review classifies plant cell growth models into three types: biophysical models, biomechanical models, and computer simulation models. These models use mechanical, biological, chemical, mathematical, and computer methods.

Plant cell growth models have found extensive applications in agriculture, forestry, and education for addressing diverse, complex, large-scale problems. These models have been utilized to predict crop varieties, estimate yield, and manage growth density and planting environments for field crops.

In the field of modeling plant cell growth, several challenges and opportunities for improvement can be identified:

(1) Integration of various plant cell growth models and research methods: Plant cell growth involves multiscale processes, from molecular interactions to tissue-level mechanics. Developing models that effectively capture and integrate these different scales is challenging but crucial for gaining a comprehensive understanding of growth processes.

(2) Refinement of modeling: Most existing plant cell growth modeling research is based on macroscopic features, but the modeling of microscopic details is still relatively insufficient. To achieve a more accurate description of plant cell growth processes, a combination of data-driven approaches is necessary.

(3) Data-driven approaches: With the development of high-throughput data technology, there is a tremendous opportunity to gather a wealth of data about the growth process of plant cells. Therefore, integrating and analyzing big data helps to describe the process of plant cell growth more accurately, thereby uncovering the potential value of plants.

(4) Verification and application: Ensuring the accuracy and reliability of plant cell growth models is of paramount importance. Models should be connected to agricultural production methods and evaluated in both lab and field environments with the aim to improve their efficacy and quality. Furthermore, continuous evaluation and improvement of the plant cell growth model can be achieved

through on-site testing and collaboration with agricultural technology experts, ensuring its practical application in agricultural production.

The overarching goal of future research in plant cell growth modeling should be to enhance precision and utility. By continuously refining models, integrating diverse research approaches, and leveraging emerging technologies, the understanding of the fundamental mechanisms governing plant cell growth can be deepened.

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Jiawei Pei, <http://orcid.org/0000-0003-4751-6592>

Changyu Liu, <http://orcid.org/0000-0002-1407-8103>

Aimin Wu, <http://orcid.org/0000-0001-8322-8833>

Ziwei Cheng, <http://orcid.org/0009-0003-0978-5473>

Hansheng Wang, <https://orcid.org/0009-0008-3353-6019>

Jingjing Zheng, <https://orcid.org/0000-0002-4650-3493>

Hao Huang, <https://orcid.org/0009-0002-5622-9165>

Chunying Wu, <http://orcid.org/0009-0008-5091-1407>

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