# Shape Matching of Planar and Spatial Curves for Part Inspection

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## ABSTRACT

Shape matching has many application areas which include manufactured part inspection. This paper deals with shape matching of planar and spatial curves based on the concept of deformation energy. We combine the concepts of differential geometry, geometric modeling, mechanics of deformable bodies and computer aided part inspection to propose a methodology of curve matching for computer aided inspection purpose. In this methodology, one of the two curves being matched is treated as an instance of deformed version of the other. The stretching, bending and twisting transformations associated with this deformation and strain energy necessary to deform is used as a measure of shape matching. The strain energy can be derived directly from intrinsic properties of curves namely arc-length, curvature and torsion. Unlike previous works of curve matching which are restricted to planar curves only, the proposed methodology can be used for shape matching of spatial curves too. Results of shape matching are presented in the context of computer aided part inspection which include inspection of parts manufactured by CNC milling.

**Keywords:** curve matching, intrinsic properties of curve, computer aided inspection, strain energy.

## 1. INTRODUCTION

The problem of shape matching of curves arise in many applications which include character recognition, object recognition, object tracking, medical & biological analysis, cartography, computer games and manufacturing part inspection. In manufacturing part inspection, the design shape is compared with that obtained from a manufactured artifact for the purpose of acceptance/rejection. The acceptance criteria is often specified in terms of dimensional and form tolerances which specify the limits within which the two curves must lie for the part to be accepted.

In the context of part inspection, many curve matching methods have been proposed in the past. A majority of these methods are based on shape localization, in which one curve is localized with respect to the other by means of rigid body transformations [2, 9, 21, 25]. Curve matching problem is formulated as an optimization problem of finding rigid body transformations which align two curves as closely as possible. The objective function in this optimization problem is taken as some measure of the Euclidean distance between two curves being matched [10]. There are two problems associated with this approach. Firstly it is computationally costly as the localization problem has to be solved numerically. Secondly the objective function is a multi-modal function with multiple local and global minima. Searching for a global minimum is always a problem irrespective of the optimization method chosen.

Another approach to curve matching is based on matching intrinsic properties of curve. Intrinsic properties of any curve, specified in terms of arc-length, curvature and torsion, define the shape of curve completely and are independent of their location and orientation in a coordinate system in which they are defined [6, 17]. Most of the work on shape matching of curves by this approach is restricted to planar curves only. A very few attempts have been made in the direction of shape matching of spatial curves. Shape matching of planar curves use only two intrinsic properties namely curvature and arc-length as the torsion being zero for all planar curves. Gope et al. [7] used affine invariant curve matching of 2D curves using curvature information. Bunke and Buhler [3] used string edit approach to match 2D curves. Rrodriguez et al. [16] extended their work to measure similarity between 3D curves. Partial curve matching for spatial curves based on curvature and torsion values has been attempted by Kong and Kimia [11]. The curve matching by use of intrinsic parameters strongly depends on the objective function used. Leaving the arc length as a parameter the other two intrinsic parameters namely curvature and torsion have a strong local dependence.

One way to overcome the above mentioned limitation is to use strain energy of curve as a measure of shape difference. Spline curves which minimize the energy measured in terms of second derivative are widely used in geometric modeling [13]. A spline can be idealized as flexible thin beam or elastica. Strain energy of beam is computed as integral of curvature [5]. Simpler forms of energy models are also used which evaluate strain energy as an integrals of second derivative of curve and not in terms of curvature. Energy minimizing splines have been used in geometric modeling for shape design, interpolation, fairing and smoothing [1, 4, 8, 15, 18, 22, 26, 27].

As energy of curves is a well defined concept it can be used not only for curve design, interpolation and fairing but also for shape matching. Lee et al. [12] have used deformation energy of thin plate splines for shape matching of planar curves with sparse knot sets. They first establish correspondence between two knot point sets by taking into consideration the Euclidian distance and curvature. After that they calculate bending and stretching energy for the comparison of two shapes. Singh et al. [14] used energy based method for planar shape recognition. They selected points with high cornerness or flatness for segmentation of contour. For correspondence they used point morphed to point, point morphed to segment or segment morphed to point.

Wang et al. [23, 24] used energy based shape matching for face recognition. They first used edge detection algorithm to detect edges on image. They used energy minimization method for linking various edges. Tawfik et al. [20] used energy minimization method to detect objects in the image by matching it with the roughly estimated template. Their algorithm consists of two steps. In the first step they use coarse level matching with the help of computing invariant moments. In next step they do fine level matching by calculating deformable energy.

It can be seen from the literature that energy based curve matching is primarily restricted to planar curves only. Strain energy calculations based stretching and bending modes of deformation used by researchers can be used for matching plane curves only. The objective of this paper is two fold. First objective is to extend energy based shape matching concept to spatial curves. Spatial curves are characterized by non zero torsion values. In order to compare two spatial curves an additional component of strain energy due to twisting has to be accounted. This has been incorporated in the present curve matching methodology. Strain energy for shape matching consisting of stretching, bending and twisting has been computed based on the principles of elasticity. Second objective of this work is to demonstrate the energy based shape matching concept to manufacturing part inspection. Results of shape matching for two free-form machined components are also presented.

## 2. INTRINSIC GEOMETRY OF CURVES

Figure 1(a) shows segment of a spatial curve in a Cartesian space. Let $\mathbf{r}$ be the position vector of a generic point $P$ on the curve and let $s$, the arc length of $P$ from a reference point, be the parameter describing the curve as $\mathbf{r}(s)$. Using the concepts of differential geometry [6, 17], the unit tangent vector, $\mathbf{t}$, the unit normal vector, $\mathbf{n}$ and the unit binormal vector, $\mathbf{b}$ of the curve at any point $P$ are related through Serret-Frenet equations as follows:

$$\frac{d\mathbf{t}}{ds} = \kappa\mathbf{n} \tag{1}$$

$$\frac{d\mathbf{n}}{ds} = -\kappa\mathbf{t} + \tau\mathbf{b} \tag{2}$$

$$\frac{d\mathbf{b}}{ds} = -\tau\mathbf{n} \tag{3}$$

Where $\mathbf{t} = \dfrac{d\mathbf{r}}{ds}$ and the three vectors are related as follows:

$$\mathbf{b} = \mathbf{t} \times \mathbf{n} \tag{4}$$

These equations describe the location as well as the orientation of moving trihedron consisting of the unit vectors $\mathbf{t}$, $\mathbf{n}$ and $\mathbf{b}$ along the curve. The curvature, $\kappa$ and torsion, $\tau$ of the curve are given by following expressions:

$$\kappa = \sqrt{\mathbf{r}''.\mathbf{r}''} \tag{5}$$

$$\tau = \frac{(\mathbf{r}'\mathbf{r}''\mathbf{r}''')}{(\mathbf{r}''.\mathbf{r}'')} \tag{6}$$

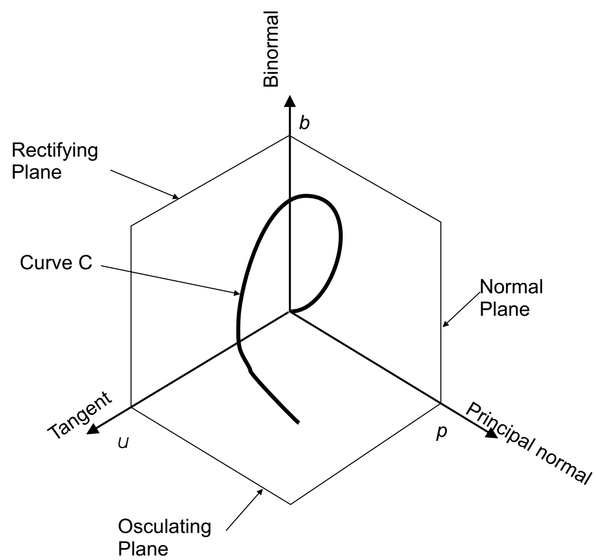



Fig. 1. Intrinsic Geometry of Spatial Curve.

Where $\mathbf{r}' = {d\mathbf{r}}/{ds}$, $\mathbf{r}'' = {d^2\mathbf{r}}/{ds^2}$ and $\mathbf{r}''' = {d^3\mathbf{r}}/{ds^3}$ . The κ and τ are functions of arc-length, *s*. Given curvature κ and torsion τ of curve $\mathbf{r}(s)$ as function of its arc-length, *s*, the curve can be uniquely defined except for its position and orientation in the Cartesian space.

## 3. ENERGY BASED SHAPE MATCHING

As discussed in previous section, arc length, curvature & torsion are the three intrinsic properties of any generic curve. Curve shape can be completely defined with the help of these intrinsic parameters. When curve undergoes any shape or dimension change one or more of these parameters are altered and vice versa. In other words it is not possible for two curves to have same shape with different set of intrinsic properties. If one visualizes curve as a structural member such as *elastica* any change in its intrinsic properties are associated with strain energy required to deform it.

In shape matching of two curves, if one curve is treated as a base curve, the strain energy required to deform this curve to make it identical with the other curve can be used a measure of shape difference. The same approach has been adopted here. For the purpose of part inspection we treat the intended geometry (designed shape) as a base curve and the geometry of the manufactured part (realized shape) as deformed curve. The strain energy required for this deformation transformation is used as a measure of shape matching.

Change in three intrinsic properties of curve namely arc-length, curvature and torsion have direct correspondence with three possible modes of deformation namely stretching, bending and twisting. Strain energy associated with these three modes of deformation can be expressed mathematically in terms of their respective intrinsic parameters as shown in Table 1. The mathematical expressions for three types of strain energies namely $U_{stretching}$, $U_{bending}$ and $U_{twisting}$ shown in the table has direct analogy with those derived from theory of elasticity [19].

| **Intrinsic property of curve** | **Mode of deformation** | **Deformation Energy** |
|---|---|---|
| Arc-length (*s*) | Stretching | $U_{stretching} = \alpha \int \left( \frac{ds_1}{ds} - 1 \right)^2 ds$ |

| | | |
|---|---|---|
| Curvature (κ) | Bending | $U_{bending} = \beta \int (\kappa_1 - \kappa)^2 ds$ |
| Torsion (τ) | Twisting | $U_{twisting} = \gamma \int (\tau_1 - \tau)^2 ds$ |

Tab. 1. Intrinsic Parameters of a Curve and Corresponding Strain Energy.

The total strain energy of deformation can be calculated by adding individual energy components:

$$U_{\text{stain energy}} = U_{\text{stretching}} + U_{\text{bending}} + U_{\text{twisting}} \tag{7}$$

where $\alpha, \beta$ and $\gamma$ are constants associated with three modes of strain energy. These constants have physical significance in mechanics of deformable bodies and are derivable from geometric and material properties of the structural member. In case of shape matching these constants can be visualized as weightages associated to different shape variables.

Any deviation in shape of two curves being matched can be measured in terms of $U_{\text{stain energy}}$. In case of matching two planar curves $U_{\text{twisting}}$ term is absent. In matching two plane curves of equal arc-length only $U_{\text{bending}}$ term is used.

## 4. RESULTS & DISCUSSION

The shape matching methodology for curves discussed above has been implanted in a MATLAB environment. Many computational experiments were carried out to assess the potential of proposed method and also to validate results with those existing in the literature. Figure 2 shows shape matching scenarios for two free-form plane curves. Strain energy here has two components one due to stretching and another due to bending. For the purpose of strain energy calculation both the curves are first represented in a parametric form and are descretised. Discretisation is carried out on the basis of curve parameter value. After disceretization, arc length, curvature and torsion values at discrete location were used to calculate strain energy in stretching, bending and twisting. The strain energies for the two cases were found to be 0.0148 and 0.0106 respectively for values of $\alpha = 1, \beta = 1$. Figure 3 shows shape matching of two spatial curves. The strain energy here includes a component due to torsion also. The strain energies for the two cases were found to be 0.0017 and 0.0007 respectively for values of $\alpha = 1, \beta = 1, \gamma = 1$.

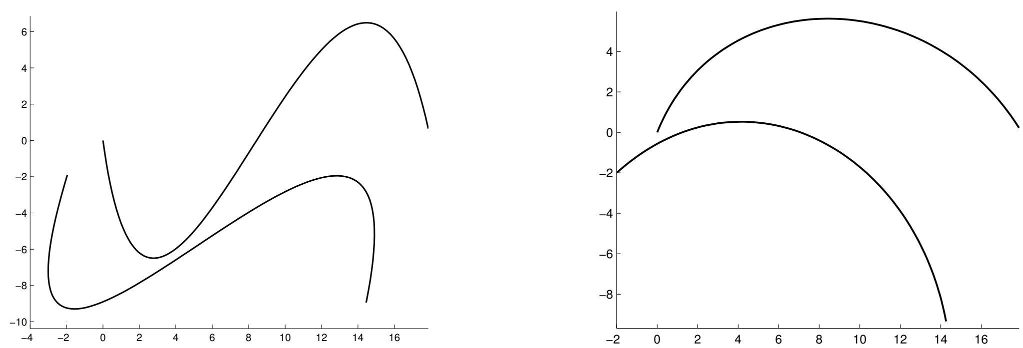


Fig. 2. Shape Matching of Planar Curves.

Based on computational experiments with many free-form curves, it was realized that there are a few issues which need to be addressed before energy based curve matching becomes a practical tool for part inspection. Firstly, it is necessary to define energy tolerance which can used for part acceptance/rejection. This is analogous to form tolerance used for part acceptance/rejection. Secondly, the concept of using strain energy for part acceptance/rejection seem will work only if the two curves being matched are smooth. Presence of noise or discontinuity in part shape though may not result in part being rejected by traditional inspection methods but results in large strain energy. Unlike other application areas of shape matching, curves being matched are usually smooth in most part inspection scenarios. Lastly, selection of correct weights ( $\alpha, \beta$ and $\gamma$ ) for three energy components used for strain energy calculations is also an issue. Computational experiments carried out here reveal that energy based inspection is somewhat sensitive to selection these values.

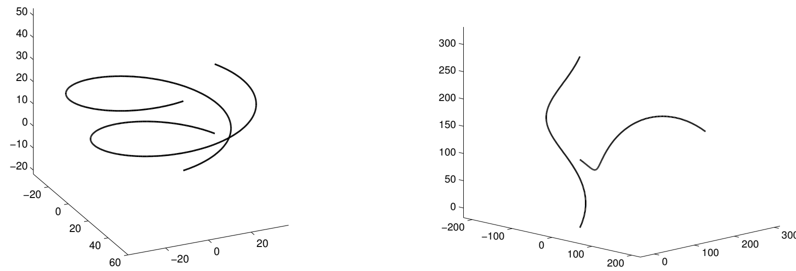


Fig. 3. Shape Matching of Spatial Curves.

Computational experiments were carried out to investigate the feasibility of using proposed method of energy based curve matching for part inspection. The traditional method of part inspection consists of localizing one shape with respect to other and calculating the maximum deviation of one curve with respect to other in a localized position. This maximum deviation in a localized position also known as *form error* is used as basis for part acceptance/rejection. Figure 4 shows four different cases of two curves being matched with increasing form error between them.

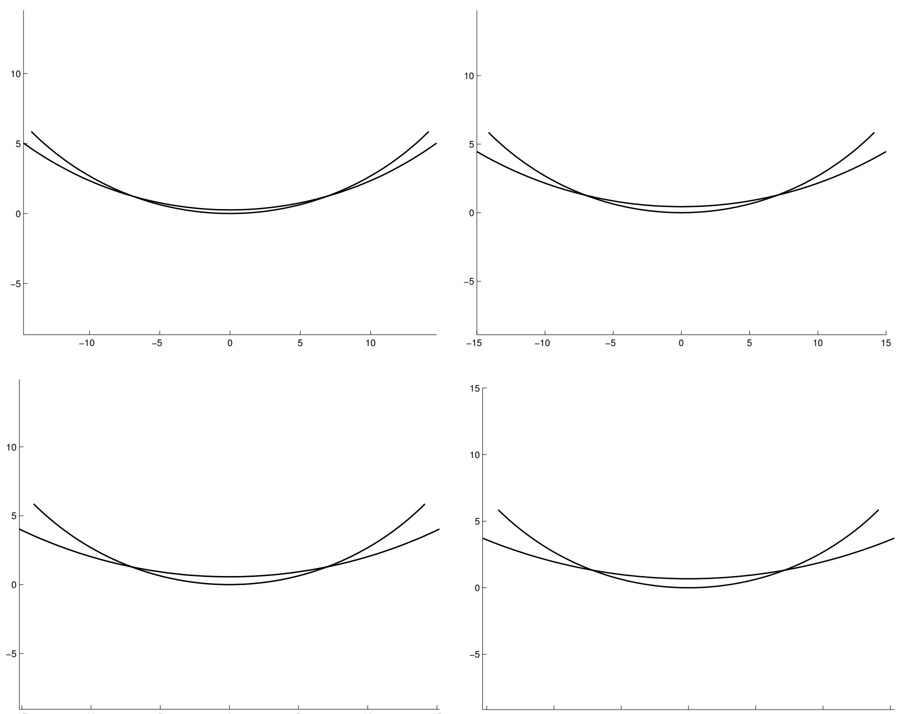


Fig. 4. Comparison of Traditional and Energy Based Shape Matching for Part Inspection.

For these cases, a comparative study of both traditional and proposed inspection was carried out. The results for four above cases are shown in Table 2. It can be seen that as the form error between two curves being compared increases by traditional method, strain energy too follows the same trend.

| Case | Form Error | Strain Energy |
|---|---|---|
| 1 | 0.8936 | 0.030 |
| 2 | 1.4726 | 0.050 |
| 3 | 1.8779 | 0.064 |
| 4 | 2.1774 | 0.075 |

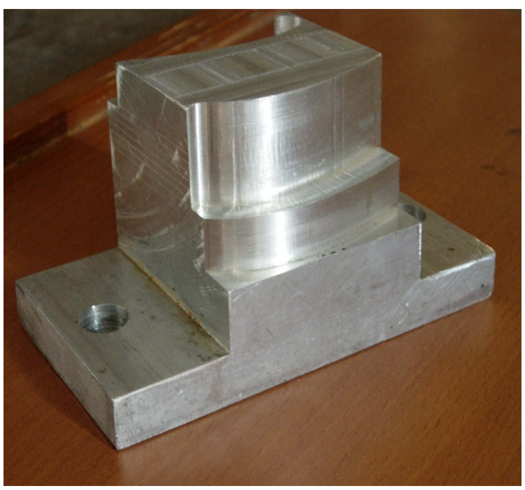
Tab. 2. Comparison of Traditional and Energy Based Shape Matching for Part Inspection.

## 5. MACHINING EXPERIMENTS

Machining experiments involving peripheral milling of free-form machined parts and inspection of the same were carried out to assess the potential of this method. An energy tolerance based on previous computational experiments was found and used for part acceptance/rejection. The parts were first machined on a CNC machine as per design geometry. Machined components were measured on a coordinate measuring machine (CMM) to capture manufactured geometry. The two representative parts used for these machining experiments are shown in Figures 5 and 6. The two parts machined are extruded surfaces and they can be expressed in parametric form as shown below:

Logarithmic Spiral: $X(t) = 5 + 2.75 \ \sin(t)e^{2t} \quad Y(t) = 20 - 2.75 \ \cos(t)e^{2t}$ (8)

Bezier Curve: $X(t) = 10t^3 - 30t^2 + 90t - 10 \quad Y(t) = 180t^3 - 270t^2 + 90t$ (9)



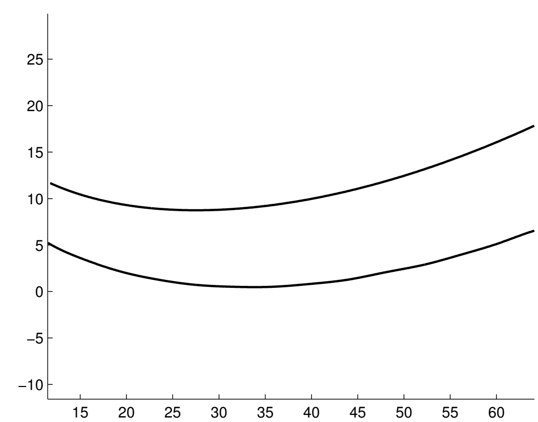



Fig. 5. Machined Parts for Shape Matching (A Logarithmic Spiral).

The manufactured geometries as captured from a measurement based on CMM were compared with design geometry using proposed energy based matching. As both the machined components are extruded shapes, curve matching as discussed for plane curves was sufficient. Strain energy evaluations in both these cases of inspection and comparison with traditional approach to inspection indicate that energy based shape matching can be a potential tool for part inspection.

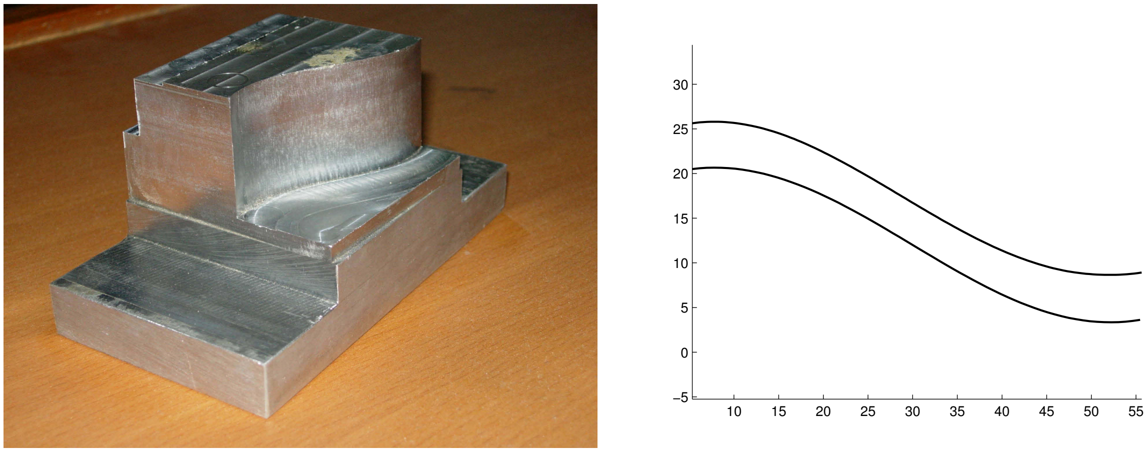



Fig. 6. Machined Parts for Shape Matching (A Bezier Curve).

## 6. ACKNOWLEDGEMENTS

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