Current Trends and Issues in Automatic Mesh Generation

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ABSTRACT

This paper presents current trends and issues in automatic mesh generation. Although automated mesh generation methods in two and three dimensions have been studied intensively, many analysis engineers still craft meshes manually for a certain class of analysis problems. In order to realize fully automated high-quality mesh generation, two technical issues need to be addressed: (1) automated mesh generators should be able to control the anisotropy and directionality of a mesh, and (2) geometric operations required prior to mesh generation should be made more robust and automated. This paper outlines recent development of the two technical issues in order to encourage further research and development of advanced mesh generation technology.

Keywords: Mesh Generation, FEM, Anisotropy, Directionality, Geometric Operations

1. INTRODUCTION

Rapid improvement of computer performance has enabled the simulation of complex physical phenomena using Finite Element Method (FEM). For example, the automotive industry has integrated FEM-based crash simulation as an integral part of the design process for evaluating the crashworthiness of a vehicle. The calculated impact force history and the computer-generated animation of a crash event help engineers improve passenger safety in a crash. In such simulation and rendering of physical phenomena, it is necessary to represent a geometric domain as a "mesh," or a discretized geometry consisting of a set of simple geometric elements such as triangles and tetrahedrons.

Because mesh generation, or meshing, is a critical task in FEM and computer graphics, many researchers and practitioners have studied extensively the theory and applications of meshing technologies over the past four decades. The technologies have matured and become available in many commercial packages. It is often claimed that mesh generation problems in two dimensions, surface, and three dimensions have been satisfactorily solved. In fact the current meshing technologies offer reasonably good solutions for basic linear FEM analysis and basic rendering tasks. More complex, non-linear analyses, however, often require high quality meshes that cannot be generated automatically by current commercial mesh generators. Analysis engineers still spend considerable time and manual labor to make ideal meshes for such analyses.

One of the missing capabilities of current commercial mesh generators is the versatile control of mesh anisotropy and directionality. They also cannot perform automatically the geometric operations required prior to the meshing process itself; such operations include, among others, feature identification and removal, medial-surface extraction from a thin-walled solid, and removing noise from laser-digitized data. Most of the time and labor an analysis engineer exerts in order to create an ideal mesh are caused by the lack of these two capabilities of current commercial packages.

In many industries—including automotive, aerospace, consumer electronics, medical, computer animation, movies, and video games—the computer simulation of physical and visual phenomena is critical in the design or production processes. Such physical and visual simulation requires a mesh as a fundamental geometric representation before the geometry can be analyzed by FEM and other numerical methods. Because the quality of the numerical solution depends on the quality of the mesh, and because current commercial packages cannot generate an ideal mesh automatically, there is a large demand for further research and development for advanced meshing technologies. This paper reviews the recent development of these technologies.

2. MESHING FOR SOLUTION ACCURACY AND COMPUTATIONAL EFFICIENCY

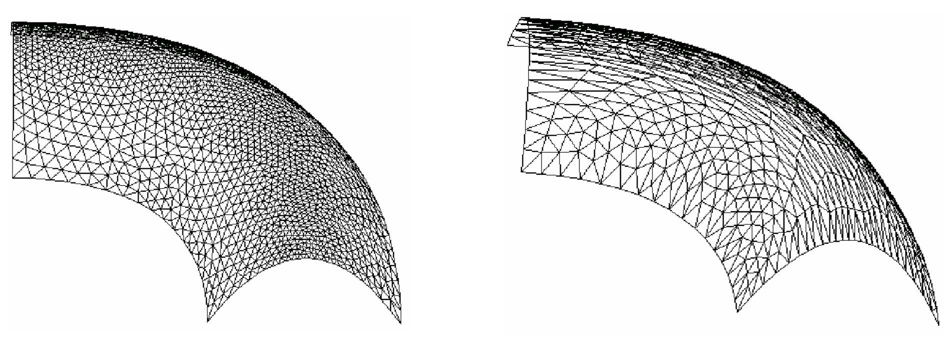
A mesh is a discretization of a two or three-dimensional geometry. Although there are some applications of a mesh of four- or higher dimensional geometry, we will not discuss them in this paper. A mesh consists of a set of nodes and elements. An element is either one-dimensional, two-dimensional, or three-dimensional. A two-dimensional element can be a triangle, quadrilateral, or general n-sided polygon. A three-dimensional element can be a tetrahedron, hexahedron, pyramid, prism, or general n-sided polyhedron. Different types of elements are often combined to form an entire mesh. In simulating a physical or visual phenomenon of a system by solving a governing differential equation with numerical methods, the problem is reduced to solving for a finite number of representative solution values at mesh nodes. The solution inside an element is interpolated from the solution values at its nodes.

If an ideal mesh is a mesh that yields a high-accuracy solution with small computational expense (memory size and computational time), the ideal mesh should be different for each analysis problem. In FEM analyses of physical phenomena, such as structural, thermal, and fluid phenomena, analysis engineers rely on their experience and knowledge about the target physical phenomena to create an ideal mesh. For example, in a flow simulation around an airfoil, the analysis engineer creates a mesh around the airfoil boundary so that mesh elements are stretched in the boundary direction and compressed in the orthogonal direction. This helps capture the boundary layer of the flow more accurately. Another example is automotive crash simulation. The analysis engineer often adjusts the mesh manually so that the mesh is aligned in the direction of impact force transmission, and the mesh elements' shape and size fall within the ranges specified by an FEM solver. One of the main differentiating characteristics of those hand-crafted meshes created by analysis experts versus meshes created by commercial packages is the precise control of mesh anisotropy and directionality.

3. MESH ANISOTROPY AND DIRECTIONALITY

3.1 Mesh Anisotropy

In an anisotropic mesh, mesh elements are stretched in a specified direction with a specified aspect ratio. Usually the direction and aspect ratio need to be changed locally. Here we will use a simple meshing problem to illustrate the importance of controlling the anisotropy of a mesh: "Subdivide a torus-like piece of smooth surface into a set of triangles." The output triangular mesh should approximate the original smooth surface as closely as possible with a minimal number of triangles. To achieve this we should use smaller triangles for a high-curvature region and larger triangles for a low-curvature region. Figure 1 (a) shows an example of such a curvature-adaptive mesh. This mesh consists of near-equilateral triangles; the mesh is thus isotropic. In contrast, some triangular elements in Figure 1 (b) are stretched in a certain direction, the direction of principal curvature. The aspect ratio of the stretching is also changed according to the ratio of the two principal curvature values. The mesh shown in Figure 1 (b) is thus anisotropic, and its anisotropy is directly controlled by the surface curvature. The curvature of a surface is defined by a 2x2 tensor field, from which one can calculate the two principal curvature directions and principal curvature values. The anisotropic mesh shown in Figure 1(b) is superior to the isotropic mesh shown in Figure 1(a) if the goal is to subdivide this smooth surface into a set of triangles while minimizing the approximation error.



(a) Isotropic triangular mesh (b) Anisotropic triangular mesh

Fig. 1 Two graded triangular meshes that approximate a torus-like piece of smooth surface [25][39].

Many researchers have proposed different methods for generating an anisotropic triangular mesh [3-5], [8], [12], [25], [39]. The two triangular meshes shown in Figure 1 were created using the BubbleMesh method [22-24]. Inspired by the repeating mesh-like patterns found in nature, BubbleMesh mimics nature's shape-finding process for finite element mesh generation. The method packs "bubbles," or cells, tightly in a geometric domain, places finite element nodes at the center of each bubble, and connect them to form a high-quality mesh. Various types of cell shapes, including circles, ellipses, rectangles, spheres, ellipsoids, and rectangular solids, can be packed to generate different types of meshes. For example, packing circles on a surface while adjusting the diameters of the circles according to the surface curvature yields a graded isotropic triangular mesh as shown in Figure 1 (a). In contrast, packing ellipses whose principle axis directions and diameters are adjusted based on the surface curvature yields a graded anisotropic mesh as shown in Figure 1 (b). The anisotropic mesh is more efficient because it approximates the original smooth surface as well as the isotropic mesh but with a much smaller number of triangles.

While the previous example highlights the advantage of a curvature-based anisotropic mesh for improving the accuracy of shape approximation, the same argument can be extended to general FEM analyses. Instead of controlling the anisotropy based on a surface curvature, in an FEM analysis we should control the anisotropy based on how much the solution value, such as temperature in thermal analyses and velocity in fluid analyses, changes in a certain direction. This will help improve the solution accuracy and reduce the required number of mesh elements. If a solution value changes drastically in one direction, the mesh elements should be made smaller in that direction to improve the solution accuracy. If a solution value does not change much in the other direction, mesh elements should be made larger in that direction to reduce the computational cost. Figure 2 illustrates an example of such an anisotropic mesh suitable for automotive air flow analysis. The mesh is stretched along the vehicle boundary in order to capture the physics of the boundary layer flow more accurately. Such desired anisotropy can be represented by using a 2x2 tensor field for a 2D problem and a 3x3 tensor field for a 3D problem. Figure 3 illustrates an example of a 3x3 tensor field and a resultant tetrahedral mesh obtained by tightly packing ellipsoidal cells and connecting the centers by Delaunay tetrahedrization [36].

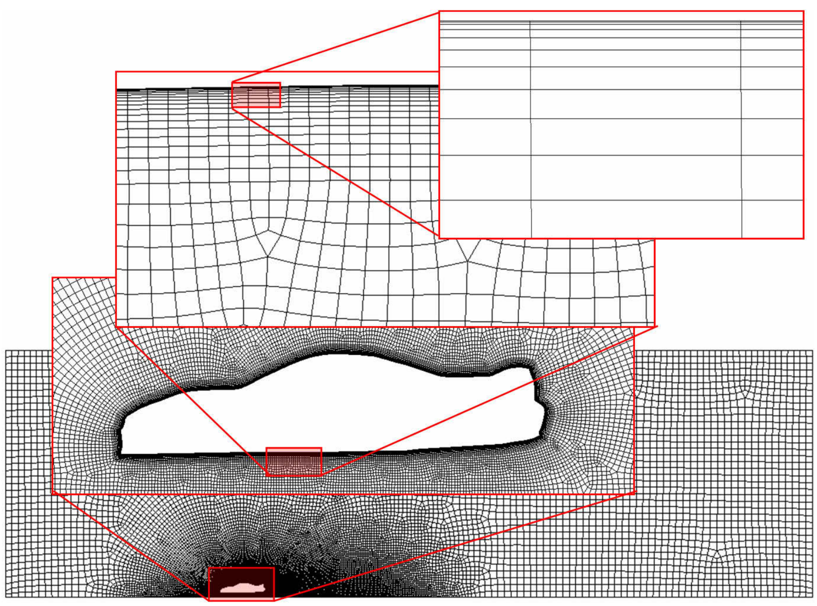


Fig. 2 An anisotropic quadrilateral mesh for an automotive air flow analysis. The mesh anisotropy and directionality are controlled so that the numerical solution accurately captures the boundary layer flow.

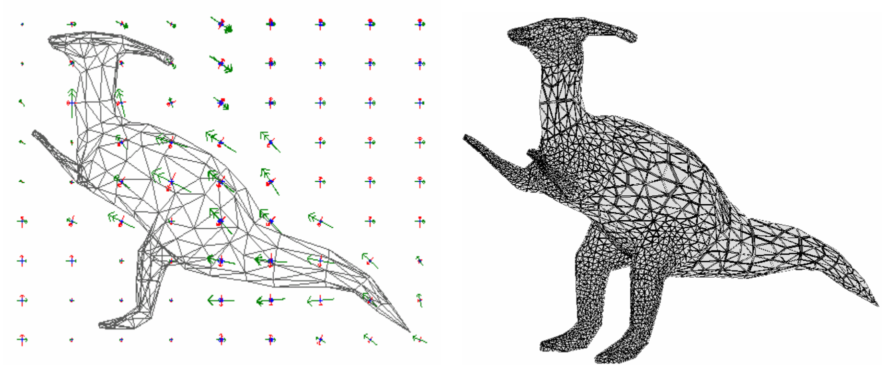


Fig. 3 Desired anisotropy is given as a 3x3 tensor field (left). The centers of tightly packed ellipsoids are connected to form an anisotropic tetrahedral mesh (right) [36].

3.2 Mesh Directionality

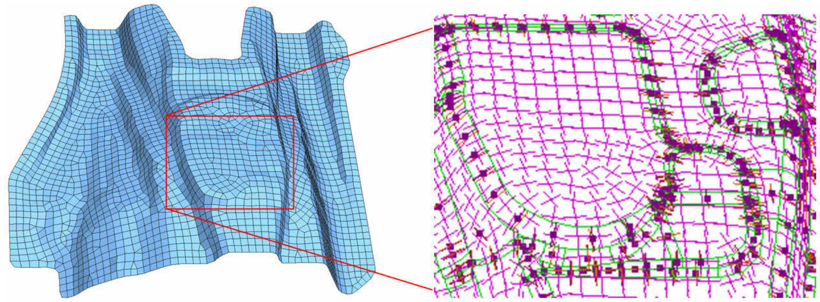
Although mesh directionality affects the solution accuracy of FEM analyses, this has not been addressed sufficiently in current commercial meshing packages. Meshes created by a commercial mesh generator often exhibit recognizable directionality, but such directionality is usually a byproduct of a particular meshing algorithm used in the package and thus is not controllable by a user. For example, many commercial surface mesh generators use a variation of the advancing front method, which places triangular or quadrilateral elements one by one starting from the boundary. The resulting mesh therefore has a nested offset pattern with the mesh directionality aligned in the boundary direction.

While such boundary-aligned mesh directionality is acceptable for many FEM analyses, in some highly non-linear analyses, such as automotive crash simulation and flow simulation, analysis engineers often need to adjust the mesh directionality manually in order to obtain an accurate solution. A typical automotive crash FEM analysis uses a quad-dominant mesh that consists of 70-90% quadrilateral elements and 30-10% triangular elements. In simulating a crash with FEM, the large deformation of a sheet-metal part can be captured by an angle change, or folding, on the boundary between two adjacent quadrilateral elements. The deformation can also be attained by the diagonal folding of a quadrilateral element. Because the former folding requires much less energy than the latter, the direction of the mesh-element boundary, or mesh directionality, influences strongly how a sheet-metal part is deformed and buckled. When a part is smashed into a folded pattern, similar to accordion pleats, the starting point and the shape of the folded pattern change depending on the mesh directionality. This affects the impact force history, the accuracy of which is critical in assessing the crashworthiness of a vehicle. It is also required that the directionality of a quad-dominant mesh should be aligned in the principal curvature direction for high-curvature regions such as rounds and fillets, common features of a sheet-metal part. This is to minimize the discrepancy between the original geometry and an FEM mesh.

Figure 4 shows a high-quality quad-dominant mesh with controlled directionality. The mesh was created using BubbleMesh. Given a 2x2 tensor field that specifies the desired mesh directionality, shown in Figure 4 (b), the method packs square cells tightly on the surface, creates mesh nodes at the center of each cell, and connects the nodes into a quad-dominant mesh as shown in Figure 4 (a). This method is later extended to a 3D meshing problem, a hex-dominant mesh generation with controlled mesh directionality. Figure 5 shows a 3x3 tensor field that specifies a desired mesh directionality. Rectangular solid cells are packed tightly in the volume while the orientation of the cells is controlled by the tensor field. The centers of the cells are then connected to form a hex-dominant mesh consisting of hexahedrons, pyramids, prisms, and tetrahedrons. The majority of the volume, 70-80%, is filled with hex elements.

4. GEOMETRIC OPERATIONS FOR MESHING

In order to generate an ideal mesh that yields a high-accuracy solution with a small computational expense, certain geometric operations are required prior to the meshing process. Such geometric operations include converting a thin-walled solid geometry to a medial surface, feature detection and removal, and polygonal mesh processing. Current commercial meshing packages do not support fully automated tools for these geometric operations, requiring a significant investment in manual labor prior to the mesh generation. These pre-meshing operations have become more arduous than the meshing process itself.



(a) Quad-dominant mesh with controlled mesh directionality (b) The desired anisotropy is given as a 2x2 tensor field.

Fig. 4 Mesh directionality control is critical in automotive crash simulation.

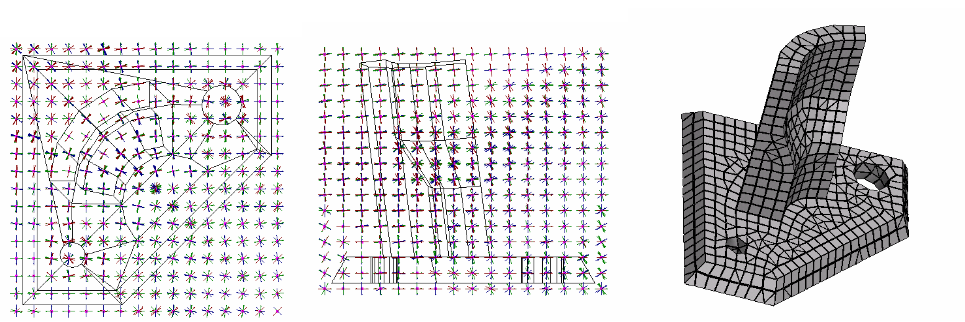


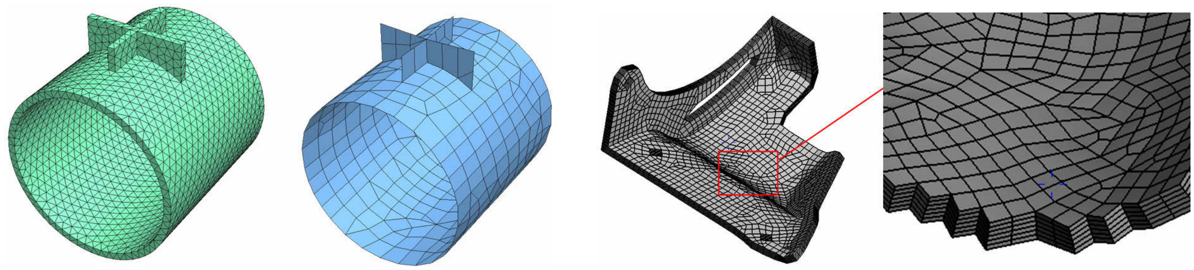
Fig. 5 The desired anisotropy is given as a 3x3 tensor field (left). The centers of tightly packed rectangular solids are connected to form a hex-dominant mesh (right) [37].

4.1 Medial-Surface Generation from a Thin-Walled Solid and Layered Mesh Generation

As the usage of plastic parts has been increasing in many industries, including automotive and consumer electronics, the demand for FEM simulations of injection-molded plastic parts has been growing. For example, in the manufacturing of injection molded plastic parts, FEM can be used for estimating cooling time, the flow pattern of the material, and the amount of warpage in a final product. Another example is an impact analysis of a crash between a human head and the plastic dashboard of a car. FEM analyses have successfully replaced some of the physical experiments of such an impact analysis for passenger safety assurance.

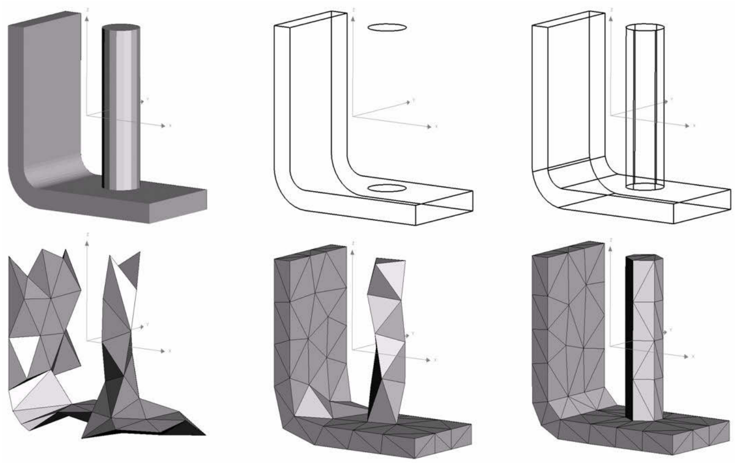
In these FEM analyses of injection-molded plastic parts, one geometric operation that is commonly performed with considerable manual labor is the conversion of a thin-walled solid geometry to a medial surface. Analysis engineers often prefer to model the shape using shell finite elements. Figure 6 (a) shows an example of converting a thin-walled solid geometry to a medial surface. Although some commercial packages offer some capability for the automatic generation of a medial surface, none of them works robustly for a complicated part with many overlapping ribbing structures.

One of the common goals of the FEM analysis of an injection molding process is to estimate the amount of warpage of a part after the part is removed from a die and cooled. The warpage happens due to the residual thermal stress, which is governed by the temperature distribution inside the die. The temperature distribution is in turn governed by the flow pattern. The accurate warpage estimation with an FEM analysis is a challenge because the phenomenon is governed by three types of physical phenomena, fluid, thermal, and structural. There has been a study on how to adjust the material property and boundary conditions in FEM in order to calibrate analysis results and match them with experimental results [30]. Another recent study [20][40] confirmed that the characteristics of an FEM mesh influence greatly the solution accuracy and convergence of a numerical solution. The study also showed that layered anisotropic meshes that are stretched in a thin-wall's tangential direction are more effective for such an FEM analysis than unstructured isotropic meshes [40].



(a) Medial surface generation from a thin-walled solid (b) Layered hexahedral mesh generation [20][40]

Fig. 6 Automated medial surface generation from a thin-walled solid and layered mesh generation.



(a) No constraint edge (b) Fewer constraint edges than ideal (c) Ideal set of constraint edges [38]

Fig. 7 A high-quality mesh can be obtained only with an appropriate set of constraint edges.

4.2 Geometry Simplification and Constraint Edge Detection

Because geometric data created in the design process usually contain unnecessarily detailed geometric features, they are not suitable as an input for a mesh generator. Such detailed features need to be simplified or removed prior to meshing. This task is often performed manually by an analysis engineer. This section discusses two examples of such geometric operations: automated generation of constraint edges, and decimation of small CAD patches.

Figure 7 shows three examples of tetrahedral mesh generation from an STL file of a mechanical part. Such tetrahedral meshing is trivial if the specified mesh element size is much smaller than the thickness of the geometry. In that case, any commercial mesh generator should be able to create a satisfactory mesh. It is, however, quite challenging if the target mesh size is comparable to or larger than the thickness of the geometry. This is because mesh generators cannot create a quality mesh without a proper set of constraint edges. The constraint edges are a subset of all the edges included in the original STL model that need to be included in the final mesh. Figure 7 (a) shows a resultant mesh with no constraint edges. This leads to an unusable tetrahedral mesh. Figure 7 (b) shows a set of constraint edges extracted by using a simple angle criterion: an edge is a constraint edge if the two triangles that share the edge form an angle less than 135 degrees. This yields a better tetrahedral mesh, but it is still not ideal. Simply loosening the angle criterion to include more edges does not yield an ideal tetrahedral mesh either. To address this issue, a previous work proposed a method for detecting a near-optimal set of constraint edges automatically [38]. Figure 7 (c) illustrates the extracted constraint edges and the resultant tetrahedral mesh. This method is designed for a polygonal model generated by a CAD package. For the other type of polygonal geometry, created from a 3D laser digitizer, many different approaches have been proposed in the research communities of computer graphics and reverse engineering [1][13][14][17][18][27].

Another meshing problem that illustrates the importance of geometry simplification is the quad-dominant meshing of a sheet metal part for automotive crash analysis. Such a CAD geometry consists of many small surface patches, in the order of 1-2mm, while a target mesh element size is typically 5-20mm. If we include all the CAD-patch boundaries in the final mesh, the quality of the mesh will be poor. It is thus critical to suppress excess CAD-patch boundaries prior to mesh generation as shown in Figure 8 [10].

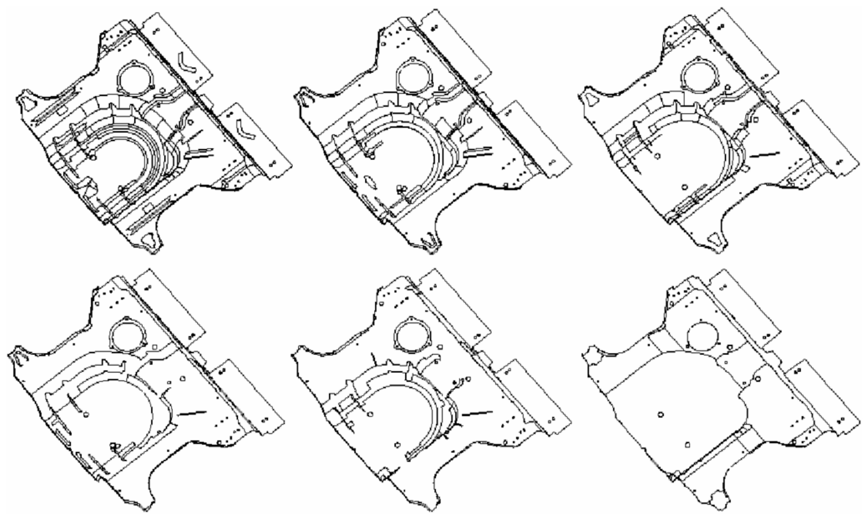


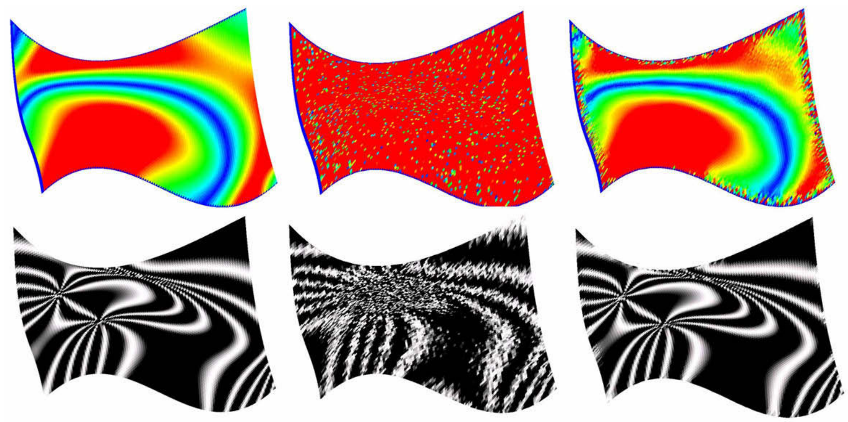
Fig. 8 Automated suppression of excess CAD-patch boundaries [10].

4.3 Noise Removal and Segmentation of Measured Geometry

One of the recent trends in mesh generation is that the input geometry is often created by using a 3D laser digitizer or other imaging device such as a CT scanner or ultrasound. Such a measured geometry is typically represented as a polygonal mesh or a cloud of points. In the manufacturing industry it is important to detect and evaluate the discrepancy between the shape of a manufactured part and the shape of the original design. In order to confirm, by using FEM, that such a shape discrepancy does not degrade the performance of a product, it is critical to be able to create a high-quality mesh directly from the measured geometry. Another task where the mesh generation of a measured geometry plays a key role is when evaluating a competitor's product – there is no CAD data available. Here we discuss two geometric operations that need to be performed prior to mesh generation: noise removal, and surface segmentation.

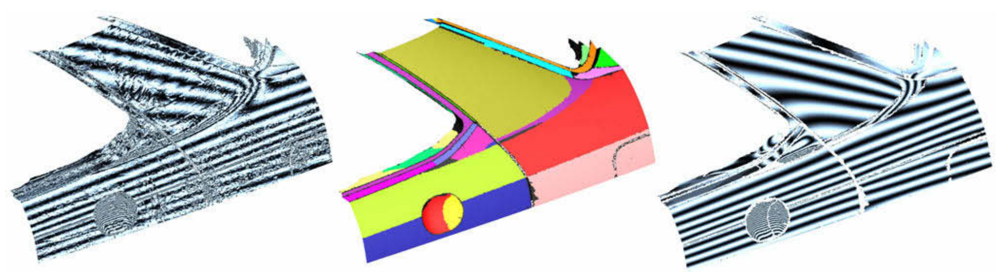
Measured geometries always contain noise, which must be removed prior to mesh generation. Two technical challenges in removing noise are: (1) to distinguish an intended geometric feature from noise; and (2) to smooth out noise without altering the overall shape of the measured geometry. Because noise removal is one of the central issues in reverse engineering and digital metrology, many researchers have studied it and proposed various solutions [6][15][19][28][29][33][35]. Figure 9 shows the result of noise removal using polynomial filtering and neighborhood erosion [33]. Such a noise removal technique is essential when creating a high quality FEM mesh whose mesh size and directionality are controlled based on the surface curvature; noise must be removed before calculating a curvature properly. One common current practice in industry is to convert a measured geometry to a CAD model with manual labor before creating a high quality FEM mesh. This lengthy manual labor could be eliminated by using proper noise removal and meshing techniques.

Another useful geometric operation in generating an FEM mesh from a measured geometry is segmentation. This operation identifies meaningful sub-regions of a surface, such as a plane, cylinder, torus, and fillet surface. A successful segmentation will enable an automated generation of constraint edges for mesh generation. Because segmentation is another fundamental process in reverse engineering and digital metrology, many researchers have studied this problem [2][7][9][16][21][31][32][41]. Figure 10 illustrates the result of a recent study on segmentation [31][32]. As can be seen in Figure 10 (a), the measured geometry contains noise that makes the visual inspection of reflection patterns difficult. The proposed segmentation method first divides the surface roughly using the estimation of curvature distribution, places a seed point in each of the sub-regions, adds neighbor points, and fits a polynomial surface. This process is repeated until most of the surface is covered by a set of sub-regions as shown in Figure 10 (b). The measured data points are then projected onto a polynomial surface. This removes noise and enables a visual inspection of reflection patterns without converting the measured geometry into a CAD model by manual labor.



(a) Original polynomial surface (b) Surface with noise (c) Surface recovered by noise removal

Fig.9 Noise removal from measured polygonal mesh, curvature (upper) and reflection pattern (lower) [10].



(a) Measured data with noise (b) Regions identified by segmentation (c) Improved reflection patterns

Fig. 10 Automated segmentation of a measured geometry [31][32].

5. CONCLUSIONS

Although automatic mesh generators are available in commercial CAD and CAE packages, many analysis engineers still craft meshes manually for advanced numerical simulations of physical and visual phenomena to achieve high solution accuracy. In this paper, we pointed out that there are two technical issues that need to be addressed to realize truly automated mesh generation: (1) anisotropy and directionality control, and (2) pre-meshing geometric operations. The recent technology development related to the two issues was reviewed and discussed along with related industry applications.

As more computer simulations are performed in the earlier stage of product design, the user base of mesh generators is expanding to general design engineers in addition to specialized analysis engineers. The primary goal for design engineers using FEM is to gain rapid insights and feedback on a new design. They are not interested in or capable of manually crafting an ideal mesh for computer simulation. This new user base will benefit significantly from a mesh generator that can control anisotropy and directionality and can perform necessary pre-meshing geometric operations automatically.

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