Nesting of Complex Sheet Metal Parts

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ABSTRACT

In the mass production of sheet metal parts, saving of materials is very important as material cost is the major portion of the overall production cost. By making use of the Minkowski sum evaluation, efficient nesting of part blanks is achieved. In the part layout formation, strip pitch and width are calculated for different blank-pair orientations. The optimum orientation of two nested pairs that results in the greatest material utilization is then obtained. These algorithms for nesting and part layout formation are implemented in SolidWorks and some case studies carried out on typical parts to demonstrate the method are discussed. It was found that for parts that have the Minkowski sum inner loop, a very high material utilization can be achieved.

Keywords: Minkowski sum, nesting, part layout, material utilization, optimization, SolidWorks

1. INTRODUCTION

Sheet metal parts are widely used in daily life and engineering field. In today's highly competitive industrial environment, it is very important to cut down the production cost. As the material cost is the major portion of the cost involved in mass producing sheet metal components, efficient nesting of parts will minimize the amount of scrap material and reduce the overall production cost significantly.

Traditionally, nesting layouts were carried out manually and it is a very time consuming process. Depending on the designer's skill and experience, the optimal layout is not always obtained. In recent years, computer-aided software tools are used to carry out the nesting of part blanks automatically. Some computer nesting systems are demonstrated by Choi et al. [3], Huang et al. [8] and Zhao and Peng [15]. However, most of the nesting algorithms are limited to regular blank shapes such as rectangles or simple polygon shapes. When the blank shapes are irregular, initial conversion to approximate manageable shapes are performed before the nesting process.

In our work, an automatic nesting system for relatively more complex parts is devised and implemented on the computer software tool SolidWorks, and Visual C++ 6.0 is used to create the SolidWorks application programming interface for algorithm demonstration. The nesting process is divided into two main stages: the nesting of two blanks and the part layout formation of two nested pairs. The basic idea of using the Minkowski sum in an algorithm to orient a part on the strip for maximizing the material utilization, which had been presented previously by Nye [12], was adopted and a modified Minkowski sum formation algorithm was developed. In order to deal with the nesting of more complex part shapes, our algorithm would start with the extraction of the blank profiles, which may consist of straight or circular edges and even concave features, and then use the Minkowski sum formation to determine the optimum orientation of the nested blank pair and the width of the metal strip.

In the following sections, previous work on the nesting problem and an introduction to the Minkowski sum are presented in Section 2. Section 3 introduces our method of applying the Minkowski sum to nesting of complex sheet metal parts. In Section 4, the nesting algorithm of a pair of convex and concave blanks is presented. Section 5 describes the algorithm of part layout formation together with the calculations of the nesting parameters. In Section 6, an example of a pair of convex and concave blanks is used to demonstrate the idea of nesting and part layout formation. Finally conclusions are drawn in Section 7.

2. PREVIOUS WORK

Since the 1960s, many researchers have been working on the two dimensional cutting stock problems. These problems appear in sheet metal stamping, leather production and cutting wood for furniture making. With the rapid growth of computer technology, it is fast and economical to perform nesting by computers. Different types of parts were considered in the cutting stock problems. In this section, some of the previous nesting algorithms are described. The idea of Minkowski sum and difference is introduced.

2.1 Nesting

The application of the mathematical theory in cutting problems was undertaken in the early 1960s by Gilmore and Gomory [7] and Adamowicz and Albano [1]. Regular shapes such as squares and rectangles were considered. In the study of Gilmore and Gomory [7], a mathematical programming for a two-dimensional cutting problem was developed. According to the algorithm, a large rectangular sheet was cut into groups of smaller squares and rectangles. Adamowicz and Albano [1] presented the algorithm to specify the exact number of small rectangles that should be cut from a large material sheet. These algorithms are applicable to general layout problems, but limited to regular shapes only. The two-stage approach of cutting non-rectangular shapes has been suggested in which irregular shapes are initially converted to approximate manageable shapes before being nested. The shapes can be rectangular enclosures or polygons. Dori and Ben-Bassat [5] presented an algorithm for converting a part to a two-dimensional convex polygon which is then used as pavers and arranged within a rectangular board. This approach is limited to parts with convex shapes only.

Ismail and Hon [9] proposed an algorithm for pairing the shapes before being nested in press tool design. The data representation is by means of a rectangular grid. By extracting the edge information of the shape in the form of two dimensional edge arrays, maximum amount of overlap or material utilization factors of each possible combination are compared. This approach considers only the edges of the shapes and neglects the internal geometry of the part shapes.

Zhao and Peng [15] investigated the basic principles of part layout optimization. The general structure of the system includes the algorithms of layout calculation and manufacturing requirements. The one-step translation algorithm for calculating the layout parameters is improved and the conflict between precision and efficiency in the layout optimization is eliminated.

In these approaches, the nesting algorithms are limited by the geometry of parts, such as regular or convex shapes. In this paper, the nesting of irregular shaped parts with circular profiles and concave features are focused. The pairing of two different parts and the part layout formation of a nested pair are also presented.

2.2 The Minkowski Sum

Minkowski sum is a useful tool in the computational geometry field. It is widely used in robot motion planning and image analysis. One of the examples is the computation of the collision-free robot paths [13]. By using the idea of Minkowski sum, a polygon-polygon intersection query can be simplified into a point-in-polygon location query. The definition of the Minkowski sum and the intersection detection among two polygons are presented below graphically. Given two polygons, A and B, their Minkowski sum $A \oplus B$ is the set of all vector sums generated by all pairs of points in A and B as defined in Eqn. (1)

$$A \oplus B = \{a + b | a \in A, b \in B\} \tag{1}$$

where a and b are arbitrary points in A and B respectively and $a+b$ is the point representing the vector sum of these two points. Fig. 1(a) shows the Minkowski sum of two polygons graphically. The light area of each polygon represents the interior part of the polygon. In the vector addition, all the points in the interiors as well as the boundaries of the two polygons are considered. The set of vector sums forms the Minkowski sum polygon and its interior is represented by the dark area.

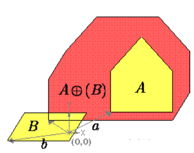


Fig. 1(a): Minkowski sum of polygons

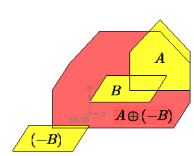


Fig. 1(b): Intersection detection of polygons

The Minkowski difference $A \oplus (-B)$ is used in detecting intersections among polygons A and B , where $(-B) = \{-b | b \in B\}$ is the polygon B rotated 180 ˚ around the origin (0,0). As shown in Fig. 1(b), the polygon $(-B)$ is located at the left-bottom quadrant of the coordinate system. The dark polygon is the Minkowski difference $A \oplus (-B)$ of polygon A and B , which includes the origin. When there exists two points from polygons A and B , in which $a = b$, the vector sum $a + (-b)$ is equal to zero. It is observed that polygons A and B intersect if and only if $A \oplus (-B)$ contains the origin.

With the use of the Minkowski difference, intersection of polygons is easily detected. In this paper, two parts are initially overlapped and the corresponding Minkowski difference is computed. The translational distance among the two parts are computed according to the geometry of the Minkowski sum. A part is then translated with respect to another part so that the overlapping among them is eliminated.

In the computational geometry field, there are many approaches in the Minkowski sum formation of two polygons. These include the formation of the Minkowski sum boundary and formation of the Minkowski sum with polygon decomposition. O' Rourke [13] proposed the Minkowski sum construction method by determining and arranging all edge vectors of polygon shapes. This method is simple and efficient, but is limited to polygons with straight line edges only. The construction complexity will be much higher when dealing with circular or curved edges. Lee and Kim [11] presented the approximation of the Minkowski sum boundary curves formation by the conventional techniques of offsetting curves of the shapes.

The algorithm for computing the Minkowski sum of two polygons is performed by initially decomposing the polygon into convex sub polygons. Agarwal et al. [2] proposed different methods of decomposing polygons and examined the suitability for efficient construction of Minkowski sum. With the decomposition of the polygons, the details of the inner part of the Minkowski sum are computed. Fernández et al. [6] described a procedure for the decomposition of a polygon into convex polygons without adding new vertices. The algorithm is limited to the polygon shapes and blanks with circular shapes are not considered.

In some recent approaches, the idea of 'Minkowski sum' is introduced to help solve the nesting problems [12]. Minkowski sum is the area generated from the addition of two blank areas and its basic concept is computationally similar to the 'Obstacle space' [10] whose boundary is defined as the 'no-fit polygon' [4]. The geometry of the Minkowski sum is useful in detecting the blank intersections. By using the no-fit polygon, a blank can be translated with respect to another blank in which they just touch one another and without overlapping.

Joshi and Sudit [10] and Nye [12] presented the algorithms for finding the optimal blank orientations by using the idea of obstacle space or Minkowski sum. The width and pitch of the strip are computed directly from the geometry of the no-fit polygon. With the use of Minkowski sum, a part is oriented on the strip for maximizing the material utilization. Vamanu and Nye [14] proposed the algorithm for creating optimal strip layouts for pairs of parts nested together. In these approaches, the computation time for calculating the nesting parameters is reduced when compared with the traditional nesting algorithms. However, it is limited to the use of the no-fit polygon in nesting of polygons with straight line edges only. The inner geometry of the Minkowski sum is not considered in the nesting calculation.

3. THE APPLICATION OF MINKOWSKI SUM TO NESTING OF COMPLEX SHEET METAL PARTS

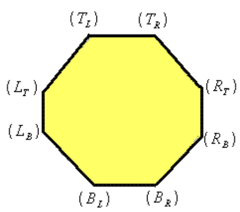
In the die layout design for sheet metal parts, the nesting problem may be split into two main stages. They are the nesting of two part blanks and the layout formation of two nested pairs. In this paper, the nesting of relatively complex parts for die layout design is presented. The basic idea of using the Minkowski sum of sheet metal parts as presented by Nye [12] was adopted and the Minkowski sum formation algorithm is modified. Typical cases of sheet metal parts considered include irregular parts with circular profiles and concave features. For parts with concave features, the decomposition of parts is performed first. Minkowski sub-sum hull is then computed by using the modified edge copying method. Union of all sub-sum hulls gives the overall Minkowski sum that includes the inner geometry. In cases where the overall Minkowski sum of two blanks consists of an outer envelope and an inner loop, the optimum nesting arrangement of the two blanks may make use of the inner loop to achieve a very high material utilization.

3.1 Computing the Minkowski Sum Hull

Initially, two blanks are imported to the SolidWorks system and the faces that points to the positive z direction are assigned as the blank shapes. The bottommost-left point of each blank is assigned to be its reference point. All blanks are translated from its blank reference point to the origin and are overlapped. Blank(A) and Blank(B) are the stationary and the orbiting blanks respectively. Blank(-B) is then formed by rotating Blank (B) 180 ° around the origin. The Minkowski sum hull is defined as the boundary of the Minkowski sum.

3.1.1 Detection of 8 Extreme Points

Four extreme positions in the coordinate system are defined as the topmost, bottommost, leftmost, and rightmost. Eight extreme points are defined by using the four extreme directions twice. Fig. 2(a) illustrates the idea of the eight extreme points of an octagon. Eight extreme points are selected among the set of the points on the octagon profile. Given the profiles of Blank(A) and Blank(-B), the end points of each edge segment are computed and are defined as the blank points. The eight extreme points are selected among the list of blank points. Fig. 2(b) shows the eight labeled extreme points of the Blank(A) and Blank(-B) respectively. However, in the construction of Minkowski sum hull, those non-extreme blank points are still considered during the edge copying process.



(T_L) Topmost-Left ; (T_R) Topmost-Right, ;

(R_T) Rightmost-Top ; (R_B) Rightmost-Bottom

(B_L) Bottommost-Left ; (B_R) Bottommost-Right ;

(L_T) Leftmost-Top; (L_B) Leftmost-Bottom

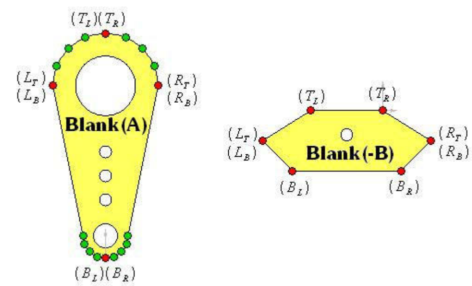


Fig.2(a): The 8 extreme points of an octagon

Fig.2(b): The 8 extreme points of Blank(A) and Blank(-A)

3.1.2 Formation of Initial Sketch of Minkowski Sum Hull

In the Minkowski addition, a vector sum point is generated by adding a point on a blank profile to a point from another blank profile. After performing the vector addition of all combinations of profile points, a set of vector sum points is resulted. The order of connecting all the vector sums is not known. As a result, it is not possible to compute the Minkowski sum hull by connecting the vector sums points directly.

In the formation of the Minkowski sum, the geometry of the blank profiles is extracted. By copying and translating the edges of the blank profiles, an initial sketch of Minkowski sum hull is generated. A translation point of edges is the vector sum of a point on a blank profile with an extreme point of another blank profile. By considering each of the extreme point, a set of edges is copied and translated to the corresponding translation point. Fig. 3(a)-(c) illustrate an example of copying and translating edges of blank profile graphically. The initial sketch of Minkowski sum hull is generated and is shown in Fig. 3(d). It consists of a number of sketch segments.

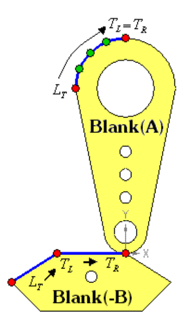


Fig. 3(a): Edges of blank profiles from L_T - T_L - T_R

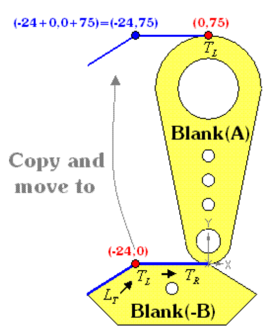


Fig. 3(b): Copy and translate edges of blank profile(-B)

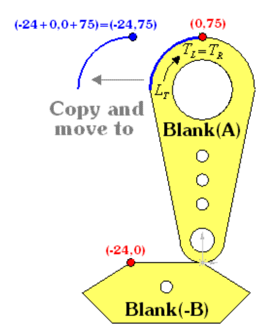


Fig. 3(c): Copy and translate edges of blank profile(A)

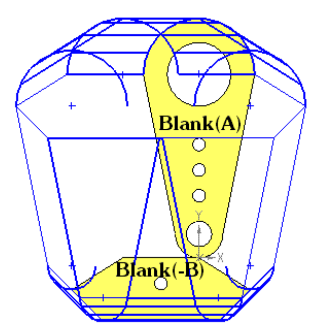


Fig. 3(d): Generated initial sketch of Minkowski sum hull

The detailed algorithm is as follows.
Step1: Determine the set of extreme points $EP_1, EP_2, ..., EP_8$, and arrange them in clockwise direction of the sketch.

Step2: For each combination of three adjacent extreme points $EP_{i-1}, EP_i, EP_{i+1} \,|\, i \in [1..8], EP_0 = EP_8 = EP_9$,

(i) Copy the edges of a blank profile between these extreme points.
(ii) Compute the translation point as
Translation point = an extreme point of one blank profile + a point along the edges of another blank profile
(iii) Translate the copied edges from the extreme point to the translation point.

3.1.3 Splitting and Selecting the Contour of the Minkowski Sum Hull

Sketch segments in the initial sketch of Minkowski sum hull are connected together. All intersection points between sketch segments are determined. Fig. 4(a) shows the set of intersection points in dark colour. Based on the list of intersection points, splitting are performed among the sketch segments. When there is an intersection point lying on a sketch segment, this sketch segment is splitted at the intersection point. A set of mini sketch segments is resulted in the initial sketch of Minkowski sum hull. In order to reduce the computation time, all redundant sketch segments are automatically removed from the sketch.

The Minkowski sum hull is generated by selecting the contour segments in the initial sketch. Among the mini sketch segments, the topmost-left point in the sketch is defined as the starting point of the selection. By selecting the points along the clockwise direction, the direction vectors of each segment at its starting and ending points are determined. The direction vector is represented by means of an angle with respect to the positive x-axis. Starting from the topmost-left point, a set of sketch segments forming the contour is selected one by one. In each step of the segment selection, the direction vectors of segments are compared and the required contour segment is selected. Fig. 4(b) illustrates the start of the Minkowski sum hull selection. Fig. 4(c) shows a complete loop of the Minkowski sum hull in solid lines and curves. The area filled with light colour is the Minkowski sum of Blank(A) and Blank(-B). All the redundant sketch segments in dotted colour will be removed.

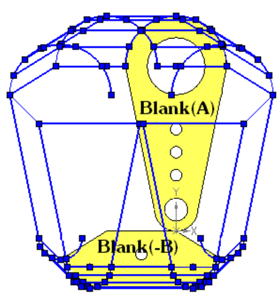


Fig. 4(a): The set of intersection points among the sketch

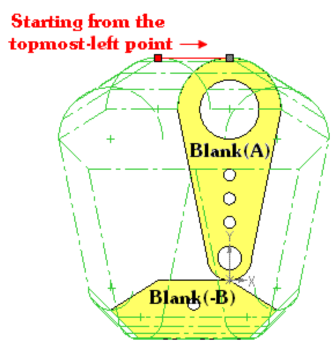


Fig. 4(b): Selection of the contour of the Minkowski sum hull

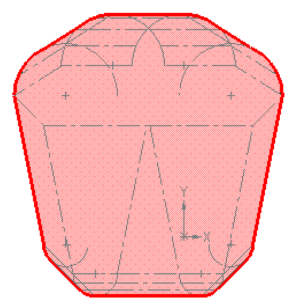


Fig. 4(c) The Minkowski sum (filled area) and its boundary

3.2 Formation of Overall Minkowski Sum with Decomposition of Concave Parts

In the previous section, the Minkowski sum hull of any two blanks is generated by selecting the contour of the mini sketch segments in the sketch. Only the boundary of the Minkowski sum is resulted and the inner geometry of the Minkowski sum is ignored. When the imported blanks are concave, holes may be generated in the Minkowski sum. The decomposition of blanks to convex parts is therefore performed before the construction of the Minkowski sum.

3.2.1 Algorithm of Decomposition of Blanks

When the inputted blank is concave, the decomposition of blanks to convex parts is initially performed. By considering the interior angles of points along the blank profile, splitting lines are drawn on the blank and used for the decomposition. Fig. 5 shows the Blank(A) which is used for the illustration. The detailed algorithm is as follows.
Step1: Determine the concavity of the blank points along the blank profile. The interior angle at the concave vertex is always greater than 180°. Fig. 5(a) shows two labeled concave vertices.

Step2: Choose one of the concave vertices to be the starting point of a splitting line. In Fig. 5(b) and 5(c), point 1 is selected as the starting point of the splitting line.

Step3: Select the ending point of the splitting line from the list of consecutive vertices one by one and compute the corresponding interior angles and concavity at the starting and ending points of the splitting line.

Step4: Repeat Step3 when both of the interior angles at the starting and ending points are smaller than 180˚. Fig. 5(b) shows a convex part, a rectangle, generated on the left hand side when the splitting line is “1–6”. Move on to Step5 when any concave part is resulted. Fig. 5(c) shows that a concave vertex is resulted at the starting point 1.

Step5: Decompose the convex part from the blank and repeat the algorithm again on the remaining part of the blank. Fig. 5(d)-(e) show the dark rectangle decomposed and the algorithm is repeated on the remaining part. The concave Blank(A) is finally decomposed into three convex parts and is shown in Fig. 5(f).

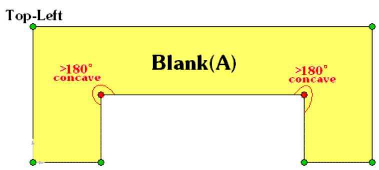


Fig. 5(a): Convex vertices and concave vertices (labeled)

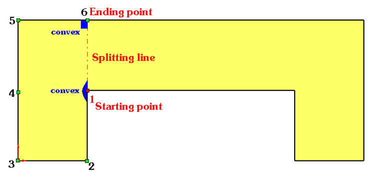


Fig. 5(b): Splitting line “1-6” (Convex part resulted)

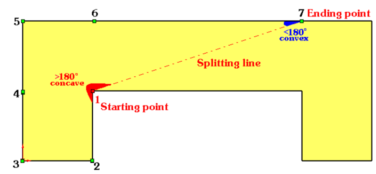


Fig. 5(c): Splitting line “1-7” (Concave part resulted)

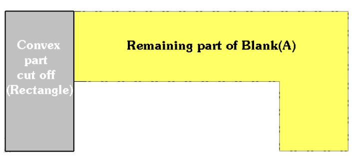


Fig. 5(d): Convex part (dark area) is decomposed

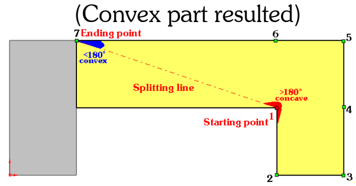


Fig. 5(e): Decomposition on the remaining part(Splitting line “1-7”)

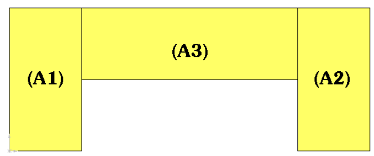


Fig. 5(f): Three convex parts (rectangles) are decomposed

3.2.2 Example of Minkowski Sum Formation with Decomposition

The proposed decomposition of the blanks to convex parts can also be applied to blanks with arc edges. Fig. 6(a) shows two blanks with arc edges, i.e. Blank(A) and Blank(-B). Each of them is decomposed into two convex parts. It is observed that only the blank profile is used in the decomposition of blanks, thus the internal geometry of the original blanks is not included in the decomposed parts. Minkowski sub-sum hull is generated by performing the Minkowski addition on two convex parts, in which one of the parts is from Blank(A) and the other one is from Blank(-B). The Minkowski sub-sum hulls generated are also convex. For all the combinations of a convex part of Blank(A) with another convex part from Blank(-B), a set of Minkowski sub-sum hulls is computed. Fig. 6(b) shows the formation of the Minkowski sub-sum hulls. The overall Minkowski sum is the union of all the Minkowski sub-sum hulls. Fig. 6(c) shows the union of all the Minkowski sub-sums.

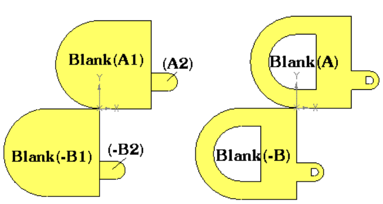


Fig. 6(a): Decomposition of blanks

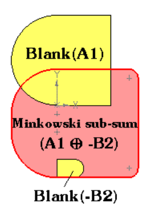


Fig. 6(b): Minkowski sub-sum

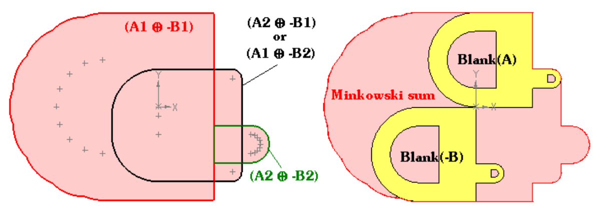


Fig. 6(c): Overall Minkowski sum

The summarized algorithm is as follows.

Step1: Decompose Blank(A) and Blank(-B) into a set of convex parts Blank(A_1), Blank(A_2), …., Blank(A_m) and Blank(- B_1), Blank(- B_2), …, Blank(- B_n) respectively.

Step2: For each $i \in [1..m]$ and for each $j \in [1..m]$, compute the Minkowski sub-sum $MSS_{ij} = A_i \oplus -B_j$.

Step3: The overall Minkowski sum $A \oplus -B$ is the union of all sub-sums $\left\{ MSS_{ij} \middle| i \in [1..m], j \in [1..n] \right\}$

4. NESTING OF TWO BLANKS

Nesting of two blanks means to pack the blanks as close as possible. If part of a blank can be included in another blank, a pair of nested blanks is resulted. The nesting algorithm consists of a series of steps, which are the decomposition of blanks to convex parts and the formation of the overall Minkowski sum. The geometry of the overall Minkowski sum is used for computing the nesting results. In this section, the detailed algorithm is proposed and the illustration of a pair of blanks is presented.

4.1 Algorithm

The following is a summary of the nesting algorithm of two blanks:

Step1: Import two blanks to the SolidWorks, named as Blank(A) and Blank(B). Initially, translate the blanks from its bottommost-left point to the origin (0,0). Compute Blank(-B) by rotating Blank(B) 180° around the origin.

Step2: By using the algorithms in section 3 of this paper, decompose Blank(A) and Blank(-B) into a set of convex parts.

Step3: Compute the set of Minkowski sub-sums and the overall Minkowski sum $A \oplus -B$.

Step4: Generate the geometry of the Minkowski sum, which includes the outer envelope and inner loop.

Step5: Determine the center position of the inner loop.

Step6: Compute the nested pair of Blank(A) and Blank(B) by translating the Blank(B) from the origin to the center position of the inner loop.

4.2 Illustration of Nested Pair Formation

Two blanks, a concave blank Blank(A) and a convex blank Blank(B) are used for the illustration and are shown in Fig. 7(a). The decomposition of the blanks and the formation of the overall Minkowski sum are performed. The geometry of the overall Minkowski sum is computed and a nested pair is resulted.

4.2.1 Decomposition of Blank(A) and Blank(-B)

Blank(A) and Blank(B) are initially imported to the SolidWorks and Blank(B) is rotated 180° around the origin to generate Blank(-B). The blank profile of Blank(A) and Blank(-B) are extracted and the blanks are decomposed to a numbers of convex parts. Blank(A) and Blank(-B) are decomposed to 5 and 2 convex parts respectively. Fig 7(a)-(c) illustrate the decomposition results of blanks.

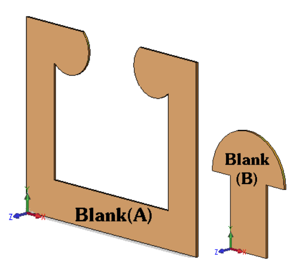


Fig. 7(a): Blank(A) and Blank(B) In isometric view

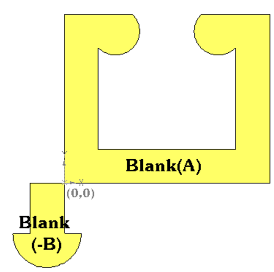


Fig. 7(b): Blank(A) and Blank(-B) In front view

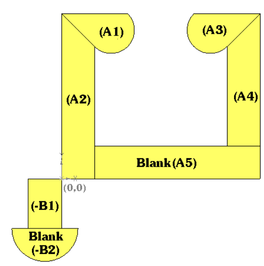


Fig. 7(c): Decomposition of Blank(A) and Blank(-B) into convex parts

4.2.2 Formation of the Overall Minkowski Sum and the Nested Pair

A set of Minkowski sub-sums is computed by performing the Minkowski addition on all the combinations of two convex parts, in which one of the convex part is from Blank(A) and another one is from Blank(-B). Fig. 8(a)-(b) show all the Minkowski sub-sums generated and the union of the all sub-sums respectively. The overall Minkowski sum consists of an outer envelope and inner loop. The inner loop of the Minkowski sum is the locus of the points traced by the reference point of Blank(B) in which part of Blank(B) is included in the Blank(A). All the points within the inner loop are the possible reference positions for placing the part of Blank(B) into Blank(A) to achieve a higher material utilization. The center of the inner loop is selected to be the optimal positioning for Blank(B) and a nested pair of two blanks is resulted. In the nested pair, one of the blanks is included into another and there is spacing between them. Fig. 8(c) shows the nesting results of Blank(A) and Blank(B) by using the center of the inner loop.

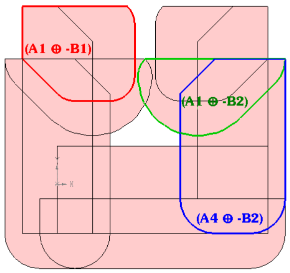


Fig.8(a): Minkowski sub-sums (filled area)

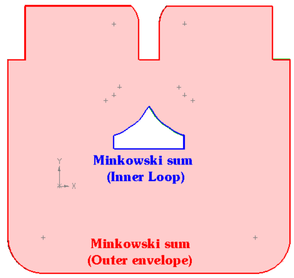


Fig. 8(b): Overall Minkowski sum (filled area)

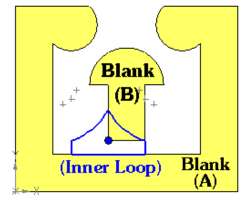


Fig. 8(c): Nesting results of Blank(A) and Blank(B) by assigning the center of the inner loop as the optimal positioning

5. PART LAYOUT FORMATION

In the die layout design for sheet metal parts, a large number of identical blanks are produced in the same metal strip. The material utilization depends on the orientation of the blanks. The optimal part layout is the arrangement of blanks in which the material utilization is the greatest. The geometry of the Minkowski sum of two identical blanks is used in computing the optimal part layout. In this section, the detailed algorithm is proposed and the illustration of a typical blank is presented.

5.1 Algorithm

The following is a summary of the nesting algorithm of two blanks:

Step1: Import a blank to the SolidWorks, named as Blank(A). Initially, translate the blanks from its bottommost-left point to the origin (0,0). Compute Blank(-A) by rotating Blank(A) 180° around the origin.

Step2: Compute the area A_B of the Blank(A) by using the SolidWorks function.

Step3: Compute the Minkowski sum $A \oplus -A$ and generate the outer envelope of the Minkowski sum.

Step4: According to the outer envelope of Minkowski sum, determine a set of points $MSP_1, MSP_2, \ldots, MSP_m$.

Step5: For each Minkowski sum point $(x_i, y_i) \,|\, i \in [1..m]$, calculate the following parameters:

(i) Sweepline vector V_i is the vector connected from the origin to the Minkowski sum point MSP_i (x_i, y_i).

(ii) Strip pitch p_i = distance between the origin and the Minkowski sum point MSP_i (x_i, y_i) = $\sqrt{x_i^2 + y_i^2}$.

(iii) Strip width w_i = maximum perpendicular distance from points on outer envelope to the sweepline vector V_i.

(iv) Strip area $A_{S_i} = p_i \times w_i$.

(v) Material utilization $\rho_i = \dfrac{A_B}{A_{S_i}}$.

Step6: Compute the optimal Minkowski sum point $(x_t, y_t) \,|\, t \in [1..m]$ in which the material utilization ρ_t is the greatest.

Step7: Translate the blank in the direction of sweepline vector V_t $|t \in [1..m]$.

5.2 Illustration of Part Layout Formation

Fig. 9(a) shows a sheet-metal blank, Blank(A), which is used for the illustration. By using the algorithms in section 3 of this paper, the geometry of Minkowski sum $A \oplus -A$ is computed and is shown in Fig. 9(b). For each for the Minkowski sum point on the outer envelope, the nesting parameters including the strip pitch, width and material utilization are calculated. The optimal material utilization of 68.4% is obtained when the sweepline angle is 63.08° as shown in the following calculations:

$$A_B = 1650.5mm^2$$

$$A_{S_t} = p_t \times w_t = 47.49 \times 51.41 = 2441.46mm^2$$

$$\rho_t = \frac{A_B}{A_{S_t}} = \frac{1650.5}{2411.46} = 68.4\%$$

Fig. 9(c) show the optimal part layout of three Blank(A) parts.

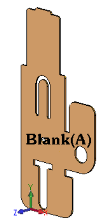


Fig.9(a): Blank(A) in isometric view

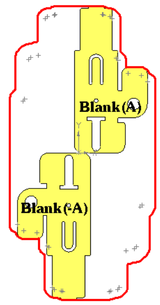


Fig.9(b): Minkowski sum (dark outer envelope)

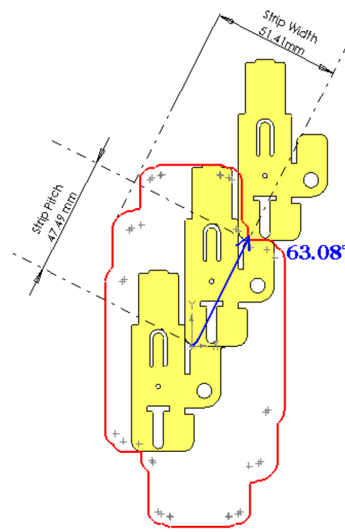


Fig.9(c): The optimal layout of 3 Blank(A), at the sweepline angle 63.08 °

6. AN EXAMPLE OF NESTING AND PART LAYOUT FORMATION

Fig. 10(a) shows a pair of convex and concave shapes, Blank(A) and Blank(B), which is used for the illustration. Blank(A) is the stationary blank and its bottommost-left vertex is fixed at the origin. Blank(B) is the orbiting blank and its bottommost-left vertex is the reference point. There is a semi-open space in Blank(A) which is large enough to include part of Blank(B). In this section, the nesting of Blank(A) and Blank(B) is analyzed. The nested pair of Blank(A) and Blank(B) is computed by finding the optimal position for placing Blank(B). The optimal part layout of two identical nested pairs is then computed by finding the optimal orientation.

6.1 Nesting of a Pair of Convex and Concave Blanks

After importing Blank(A) and Blank(B) to the SolidWorks, the rotated Blank(-B) is generated. Each of the Blank(A) and Blank(-B) are then decomposed to 5 convex parts. Fig. 10(a)-(c) show the formation of the set of convex parts. By using the convex parts, a set of Minkowski sub-sums is computed. Each of the Minkowski sub-sum is generated by performing the Minkowski addition on each combination of a convex part from Blank(A) with another convex part from Blank(-B). The union of all the sub-sums is the overall Minkowski sum, i.e. the filled area in Fig. 10(d). The overall Minkowski sum consists of an outer envelope and an inner loop. By treating the center of the inner loop as the optimal positioning for Blank(B), the pair of Blank(A) and Blank(B) is nested and is shown in Fig. 10(e). It is observed that Blank(A) and Blank(B) are not overlapped and part of Blank(B) is included in the inner space of Blank(A). Also, there are some free space between Blank(A) and Blank(B).

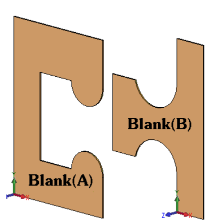


Fig. 10(a):Blank(A) and Blank(B) in isometric view

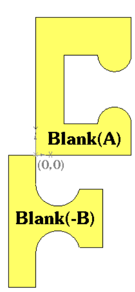


Fig. 10(b): Blank(A) and Blank(-B) in front view

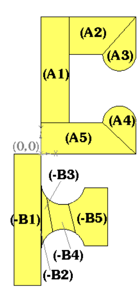


Fig. 10(c): Decomposition of blanks

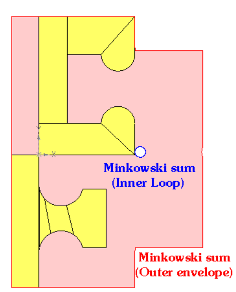


Fig. 10(d): Overall Minkowski sum of Blank(A) and Blank(-B) in pink area

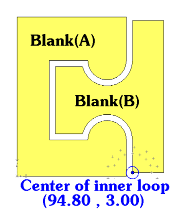


Fig. 10(e): Nesting of Blanks at center of inner loop

6.2 Part Layout Formation of two Identical Nested Pairs

In order to carry out the part layout on the metal strip, the nested pair of Blank(A) and Blank(B), named Blank(P), is imported to the SolidWorks. The idea of forming the overall Minkowski sum $P \oplus -P$ is repeated. Fig. 11(a) shows the geometry of the overall Minkowski sum. A set of nesting parameters is calculated at different Minkowski sum points on the outer envelope. The graph of material utilization against the sweepline angle is given in Fig. 11(b). The material utilization is the greatest, i.e. 89.8%, when the sweepline angles is -90˚, 0˚, 90˚ or 180˚. Fig.11(c) shows the optimal part layout of three pairs of Blank(A) and Blank(B) at 0˚ and 180˚. The following is the corresponding calculations:

$$A_B = 14151mm^2 \,;\, A_{S_t} = p_t \times w_t = 120.54 \times 130.68 = 15752mm^2$$

$$\rho_t = \frac{A_B}{A_{S_t}} = \frac{14151}{15752} = 89.8\%$$

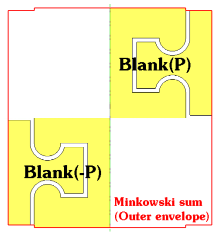


Fig. 11(a): Minkowski sum of Blank(P) and Blank(-P)

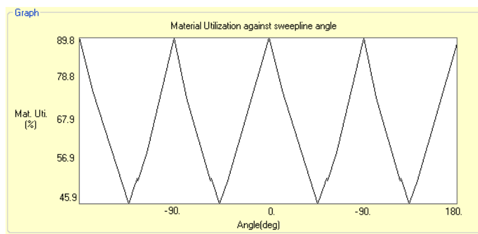


Fig. 11(b): Material utilization against sweepline angle

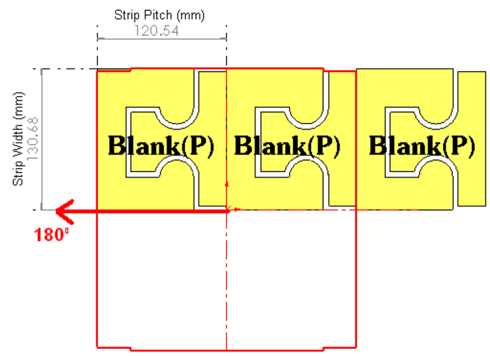


Fig. 11(c): The optimal layout of 3 nested pairs, Blank(P), at sweepline angle 0˚,180˚

7. CONCLUSIONS

A series of algorithms has been presented in this paper. These include the formation of the Minkowski sum hull by edge copying method; decomposition of blanks to convex parts; nesting of a pair of convex and concave blanks; and part layout formation of nested pairs. Illustrations of nesting and part layout formation of sheet-metal parts with circular profiles and concave features are presented. All algorithms are successfully implemented in the computer-aided software tool SolidWorks. By using the SolidWorks interface, the profiles of blanks are extracted and the Minkowski sum is computed by sketching. In the case of a pair of convex and concave blanks with a Minkowski sum inner loop, the convex blank is placed inside the concave blank to achieve a higher material utilization. With the use of the outer envelope of Minkowski sum, the nesting parameters of strip pitch and width are easily calculated. The optimal part layout is computed by finding the orientation that achieving the greatest material utilization. Calculations on the nesting parameters are performed and the optimal part layout is computed.

8. ACKNOWLEDGEMENTS

The authors would like to thank the Department of Mechanical Engineering, The University of Hong Kong, and 3D QuickTools Limited for supporting this project. The support from the Research Grant Council of the HKSAR government and the Croucher Senior Research Fellowship to the third author is also gratefully acknowledged.

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