

Local and Global Accessibility Evaluation with Tool Geometry

Jinnan Wang¹, Chell A. Roberts² and Scott Danielson³

¹Arizona State University, wangjn@asu.edu

²Arizona State University, chell.roberts@asu.edu

³Arizona State University, scott.danielson@asu.edu

ABSTRACT

Accessibility analysis is an important step for the automatic generation of machining planning. One aim of accessibility analysis is to find a domain in which the tool can maneuver without colliding with the workpiece. Most approaches to accessibility analysis use rays or half-line projections that do not include tool geometry considerations. In this paper we present an approach for the global accessibility analysis that includes tool geometry. The paper demonstrates how local accessibility analysis forms an upper bound for the tool radius selection and how a global accessibility approach determines point and part accessibility. The results in this paper are limited to ball-endmill tool geometry. The paper also only considers tool – workpiece interference and collisions and does not consider tool holder geometry. Experiments on sample parts are shown to validate the approach.

Keywords: accessibility evaluation, manufacturing planning, tool geometry

1. INTRODUCTION

Automated machining planning includes machine selection, tool selection, part orientation, fixture planning, toolpath planning, and parameter selection operations towards optimizing objectives, such as cost, time and/or quality [4]. Much of the automated manufacturing planning research for machining processes is based on manufacturing features (*MFs*). Over the last decade, the concept of *MFs* has emerged as a powerful and widely adopted representation to assist in the conversion of design information into machining instructions. *MFs* have proved to be convenient because they characterize the capabilities of machining processes such as 3-axis milling and turning fairly well. However, the feature-based approach is not without its disadvantages [1]. First, any feature-based system is limited by the extent of its vocabulary. Secondly, *MFs* are not directly available from CAD representations. They must be extracted by a process referred to as feature extraction or they must be designed into the model. While there has been significant progress with feature-based approaches, there still remain problems to fully automated manufacturing planning due to recognition of interacting features and complex features within sculptured surfaces.

Decades of machining planning research has resulted in some limited successes of algorithmic planning, but has failed to produce a complete and consistent approach. This is due, in large part, to the complexity of analyzing a computer product model relative to a specific manufacturing process and a specific tool. Common problems include mating the geometry of a tool with the geometry of the product model, determining the set of possible tool orientations for the model geometry and tolerances, and fitting inspection data to the model geometry and tolerances. Key to the solution of these problems is the availability of mathematical models for tool/part accessibility. Both the feature-based approach and the primitive-based approach need to solve the problem of tool/part accessibility.

One aim of accessibility analysis is to find a domain in which the tool can maneuver without colliding with the workpiece [6]. There are two approaches to analyzing the accessibility of a workpiece: (a) entire feature accessibility, where the accessibility of the whole workpiece is evaluated, and (b) point accessibility, where the directions along which the tool can reach a specific point are examined. The entire feature accessibility domain can be derived from the point accessibility domains. The accessibility domains are independent of the part orientation, and optimal part orientations are determined after analyzing the accessibility domains of cutting location (CL) points.

Point *p* is *accessible* to a tool in NC machining if the tool does not penetrate the part when its cutting point is placed at *p* [5]. Fixturing devices and other obstacles are not considered in this paper.

In collision-free machining, both local and global gouging effects should be avoided. Local gouging means that in all arbitrary small neighborhoods of the current touching point, the milling tool cuts into the final surface of the workpiece, thereby destroying it. More complicated are global collisions occurring at some distance from the touching point [7]. Local accessibility means the tool is cutting without local gouging; so does global accessibility. Fig. 1 demonstrates the distinction between local and global accessibility. To perform automated machining planning, the accessibility information with tool geometry of the part should be evaluated. This can then be used for machine selection, tool selection, part orientation, fixture planning, tool path planning and parameter selection operations.

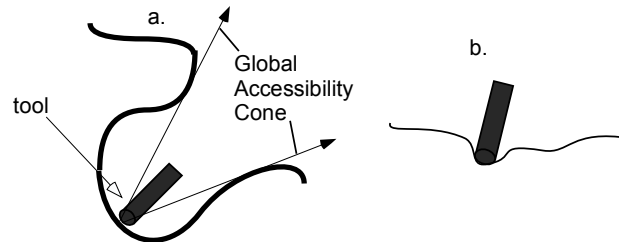


Fig.1: a. Global accessibility: tool relative to entire part, and, b. Local accessibility: tool relative to part surface.

In this paper, we present an approach for the global accessibility analysis with tool geometry. In section 2, we show how local accessibility analysis forms an upper bound for the tool radius selection. Section 3 presents an approach to analyze global accessibility with tool geometry. Section 4 presents the experiments performed on the sample parts to verify and validate the approaches. Finally, in section 5, we draw our conclusions.

2. LOCAL ACCESSIBILITY ANALYSIS

To find a feasible tool radius for a given surface, both local and global accessibility are analyzed. In this section we consider the tool radius constricted by the local accessibility sets an upper bound of the minimum size tool (which means that this tool can cut entire part). In the next section, local accessibility is used as an initial minimum size tool for global accessibility analysis. In multi-axis (4-axis and 5-axis) NC machining, using different types of endmills (e.g., flat-endmills or torus-endmills) for sculptured surface machining is generally far more efficient than using traditional spherical endmills on 3-axis machines. This is due mainly to the number of paths needed to obtain small cusps and to the cutting inefficiency of a spherical endmill, which has a low speed cutting zone at its bottom [3]. But in this work, we only consider spherical endmills for sculptured machining to simplify the problem. The local accessibility of a point on a free form surface for a spherical endmill is equivalent to the maximum curvature of that point. We will use both NURBS models and triangular mesh models to demonstrate our approach.

We tessellate NURBS models by partitioning them into small facets. Each facet is a triangular patch that is small enough to approximate the curved surface by a plane triangle within a prescribed tolerance. We set the centroids of the triangular facets as access points (the point we will consider for accessibility). The local accessibility of the entire feature can be derived from the local accessibility of these access points. On the NURBS model, the two principle curvatures of the point can be found by its surface parameters. The maximum principle curvature is the maximum curvature at the point. On the triangular mesh model, an approximated quadratic surface of a triangular facet is created from a group of neighborhood points. The maximum curvature is then found by calculating the principle curvatures at the centroid according to the approximated quadratic surface. The maximum curvature of the feature is derived from the maximum curvature of the maximum curvatures of the access points. The upper bound of the tool radius is the reciprocal of the maximum curvature.

A free form surface usually has some high curvature regions. In Fig. 2 the male face NURBS model and the female face triangular mesh model is shown. Each patch can also be coded with different radii to analyze the local accessibility. This provides a way to find all the regions of the surface that can be cut out by a given tool. The upper bound of the minimum tool radius for the highest curvature regions is also calculated. Figure 2 also shows four spherical endmill tools of different radii and the high curvature regions of each face model. The tool with the smallest radius is the minimum size tool needed to cover the part, the other three tools are 0.25 inch, 0.50 inch and 1.00 inch. The figure is the result of using all four tools where we first cover all the facets of the model possible with the largest tool, we then cover the portion of the model not covered by the largest tool with the next largest tool and so on.

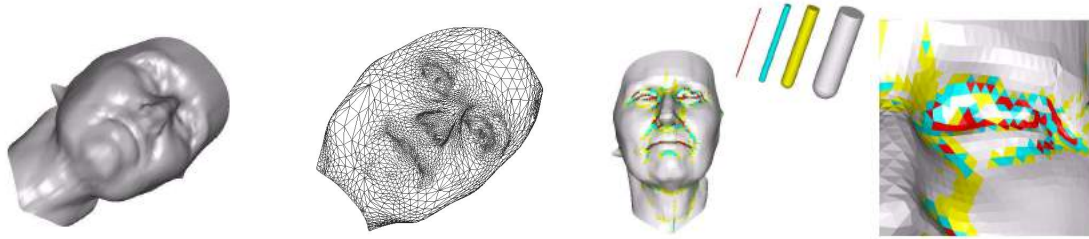


Fig. 2: Models: NURBS, tessellated, tool mappings, and tessellation mappings.

3. GLOBAL ACCESSIBILITY ANALYSIS

The global accessibility analysis is easy to start as using a pencil of rays of zero radius emanating from a selected point. The accessibility can be evaluated by checking to see if any rays intersect other portions of the part and building up a point visibility cone (PVC) or binary spherical map (BSM) to represent the global accessibility (Fig. 3). Kang & Suh [2] created an approach to tessellate the product model into 'n' patches, with a unit sphere placed in the centroid of each patch. Each unit sphere is also tessellated into 'm' patches. Each patch of the unit sphere represents a normal or a direction from that patch. All patches of the product model are then mapped to each unit sphere. If there are 'n' patches on the surface, 'n' projections need to be mapped onto the binary sphere, which would entail the sampling of 'm' normals or directions within sphere. If 'n' such points exist on the surface under evaluation, then the order of the algorithm that can determine the surface visibility would be $m \cdot n^2$. Generating the BSMs by patch projection is computationally expensive. Kang & Suh's approach is a typical one to generate the global accessibility information without a known tool radius.

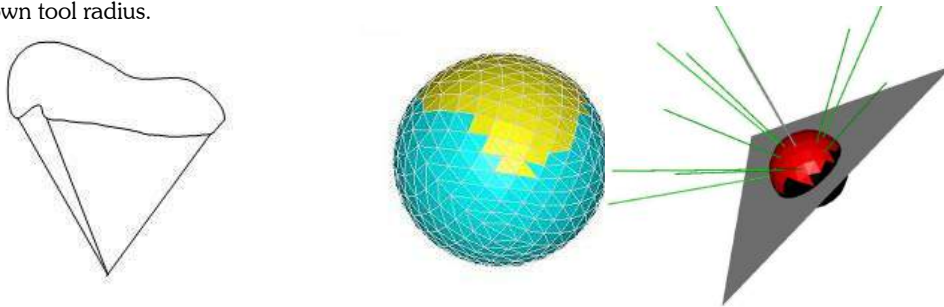


Fig. 3: Point visibility cone (PVC), binary spherical map (BSM), and a BSM embedded in a facet.

Spitz & Requicha [5] presented an approach to include tool radius into the global accessibility analysis. The approach can verify that a tool radius r penetrates an obstacle X iff the nongrown half-line penetrates $X \uparrow r$, where $X \uparrow r$ denotes X grown by r . In this approach, an object is grown by the current tool radius r , including all the points that are at a distance no greater than r from another point in the object, and then the global accessibility analyzed for the grown object with rays. The result of the analysis includes the current tool radius r . Unfortunately, computing the solid model of a grown object is an expensive and nontrivial task prone to precision errors and it produces curved objects even when the input is polyhedral. Woo et al [8] presented the concept of spherical and visibility maps where surface normals were enhanced with cutting tool geometries. Woo et al. recognize the concept of partial visibility which they presents as an equivalent to the global accessibility notion of Spyridi. Woo categorizes a wide range of manufacturing processes by their point, line and surface visibilities besides considering the various degrees of freedom in the mechanism of the machines that give a computational structure in terms of a point great circle and a spherical rectangle. Our approach is to develop an augmented surface model where each access point on the surface is associated with a BSM, a reflection of the point accessibility of the entire model. First we create a set of BSMs with rays. We then develop an approach to modify all of the BSMs with the tool radius.

3.1 Global Accessibility Analysis

The first step is to evaluate the global accessibility with rays by creating BSMs for all access points and generate the augmented surface model. A BSM is a unit sphere created with its surface decomposed into 'm' triangular facets according to the global coordinate system. The 'm' is then calculated by the resolution of the accessibility

representation that is the grid number of the unit sphere, $m = 12 \times grid^2$ (as an example, if $grid = 8$, $m = 12 \times 8^2 = 768$). Each BSM is translated to each access point by placing the center of the BSM on the access point, which allows the accessibility of the facets can be represented in a unified algebraic fashion. However, the spherical triangular facets of a BSM under, or intersecting with the tangent plane of the access point, would be marked as inaccessible before the patch projection. The patch projection only performs on the facets of the BSM above the tangent plane. Fig. 3 depicted a tessellated model of a BSM and a BSMs on a single facet with its accessible orientation vectors. The darker shaded areas on the BSM represent *inaccessible* facets where a projection vector would intersect the part model. The following algorithm detects and marks each of these facets as being inaccessible if a ray emanating from the center of the facet sphere intersects any facet of the part. The procedure is as follows.

- Step 1. Create tessellations on the surface, F_i , $i \in [1, n]$. The access points are the centroids of the facets, P_i , $i \in [1, n]$.
- Step 2. For point P_i , $i \in [1, n]$, calculate the coordinates of P_i , and calculate the surface normal \vec{n}_i , at P_i .
- Step 3. Create a unit sphere with its surface decomposed into 'm' triangular facets according to the global coordinate system, $A = \{a_t\}$, $t \in [1, m]$. a_t represents a spherical triangular facet. \vec{a}_t represents a direction from center of the unit sphere to the centroid of the spherical triangular facet a_t .
- Step 4. Translate the unit sphere to each P_i as A_i . $A_i = \{a_{it}\}$, $i \in [1, n]$, $t \in [1, m]$ and initialize $a_{it} = 0$ (accessible). The center of the unit sphere is coincident with P_i . Check the spherical triangular facet a_{it} , $t \in [1, m]$. If a_{it} is under the tangent plane of access point P_i (which means if one of the three vertices of a_{it} is under the tangent plane, calculated by dot product with the patch normal), then mark it as inaccessible (set $a_{it} = 1$).
- Step 5. Project all the surface triangular facets F_j , $j \neq i$, $j \in [1, n]$ that lie above the tangent plane of the access point P_i to the unit sphere (which means if one of the three vertices of F_j lies above the tangent plane, calculated by dot product with the patch normal). F_j^p is the projection of F_j on the unit sphere A_i .
- Step 6. Check the spherical triangular facet a_{it} , $t \in [1, m]$, if $a_{it} = 0$ and \vec{a}_t fall into F_j^p , $j \neq i$, $j \in [1, n]$, then mark it as inaccessible (set $a_{it} = 1$). If only one F_j^p causes a_{it} to become inaccessible, then the distance between P_i and P_j is calculated and recorded. If more than one F_j^p causes a_{it} to become inaccessible, then all the distances between P_i and P_j are calculated but only the shortest one is recorded.

In this procedure, we tessellate the surface into 'n' facets. A unit sphere also tessellated into 'm' facets is then placed on the facet such that the centroid of the sphere and the centroid of the facet are calculated. Finally all the facets of the part surface are mapped to each of the unit spheres to create the binary sphere map (BSM) on each facet of the surface. During the facet mapping, we add a distance calculation to Kang's approach. We record the shortest distance of the inaccessible orientation in Step 6. The distance data will be used for modifying the BSM with the tool radius. The mapping of one facet onto a BSM is depicted in Fig. 4.

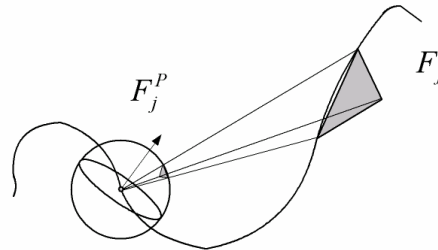


Fig. 4: The mapping of one facet onto the BSM.

3.2 Mathematical Model of Global Accessibility Analysis with Tool Geometry

In this section, we demonstrate how a BSM can be used to capture global accessibility with a tool radius. In Kang's approach, BSMs are created with rays (no tool size). A BSM (tessellated unit sphere) is composed of triangles with each triangle representing a potential tool orientation where the tool orientation is a projection of the vector from the centroid of the BSM through the centroid of the facet. The process of mapping surface facets to the BSMs creates a map of all inaccessible tool orientations for each of the BSMs. The projection of a surface facet onto the BSM is equivalent to saying that a tool cannot be oriented in that same projection orientation to access the part because it would have to pass through the projected surface in order to reach the BSM. When the tool geometry is included in the mapping, the global accessibility will either remain unchanged or the accessibility map (BSM) will decrease. That means some or all of the accessible orientations evaluated with rays might become inaccessible orientations when evaluated using the tool radius. Figure 5 shows BSMs with and without tool geometry included.

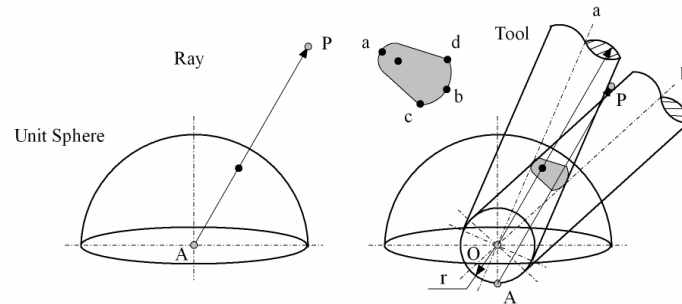


Fig. 5: Global accessibility changed by including tool geometry.

The first step of the BSM modification approach is to pick one BSM of the model that has been created using ray intersections. The second step is to pick an inaccessible orientation of the upper hemisphere of the BSM (which is above the tangent plane of the access point). Figure 5 shows an orientation evaluated with a ray that we will consider to be inaccessible (AP). Point A is an access point. Point P represents a triangular surface facet of the other portion of the part (the facet is small enough to be approximated as a point). The inclusion of tool radius will create a small inaccessible region for a tool radius ($r > 0$). The unit hemisphere on the left represents the upper hemisphere of the original BSM. Its center is on point A. Point P is on a line passing through the centroid of a triangular facet of the surface of the BSM (sphere facets are not shown). The unit hemisphere on the right represents the BSM after the modification with a spherical endmill. Its center is on point O (the center of the spherical end of the tool). To avoid a collision of the tool shaft with the other portion of the part, point P, an inaccessible region, was marked on the unit sphere. The boundary of the inaccessible region is formed by the orientations of the tool when its shaft just touches point P. Since an inaccessible orientation from the ray approach becomes an inaccessible region with the tool radius, this change will affect the other orientations that are ray accessible.

All inaccessible orientations of the upper hemisphere of the original BSM would go through the same process to complete one BSM modification with the tool radius. And all the BSMs would be modified by sequence. The small gray region shown in Fig. 5 is the general case. It could approximately be defined by four critical orientations, a , b , c and d . Orientation a is the orientation of the tool that is closest to the normal vector of Point A. Orientation b is the orientation of the tool that is furthest from the normal vector of Point A. Orientations c and d are two orientations in the same plane with the inaccessible orientation, and this plane is perpendicular with the plane that consists of normal vector of point A and orientation a . To demonstrate the change of BSM with tool radius, the problem can be stated as: that given a tool radius and an inaccessible orientation with a distance recorded to find the four critical orientations, a , b , c and d . After a small gray region is defined by these four orientations, we change the status of all the orientations that are ray accessible within the small gray region to be tool inaccessible. We adopt the following model notations:

A - An access point, centroid of a triangle facet of the surface.

O - Center of the sphere of the spherical endmill.

P - A point representing a facet of the other portion of the part, the reason causes the orientation to be inaccessible.

L - Distance between point A and point P.

L' - Distance between point O and point P. $L' = \sqrt{L^2 + r^2 - 2Lr \cos \theta}$

θ - Angle between the access point's normal and the original inaccessible orientation.

θ' - Angle between the access point's normal and the tool's orientation.

R - Distance between point P and point A 's normal vector. $R = L \sin \theta$

H - Distance between point P and point A 's tangent plane. $H = L \cos \theta$

r - Radius of the spherical endmill tool.

And the initial ranges of some notations are given as follows.

$L > 0$ Distance should be greater than zero.

$0 \leq \theta < \pi/2$ Inaccessible orientation is upon the tangent plane of the access point.

$0 \leq \theta' < \pi/2$ Tool orientation is upon the tangent plane of the access point.

In order to create the mathematical model of BSM modification and solve it completely, the model has been considered in many cases according to the conditions set by L' , R , H and r . In table 1, the first column shows the math relationship, the second column presents the figure (the plane shown in each figure is the tangent plane of point A), and the third column is the unit sphere with gray region signified as the inaccessible region caused by the tool.

Math relationship	Figure	Inaccessible region	Math relationship	Figure	Inaccessible region
Case 1: $L' < r$			Case 4: $L' > r$ and $0 < H \leq r$		
Case 2: $L' = r$ and $H < r$			Case 5: $L' > r$ and $r < H < 2r$		
Case 3: $L' = r$ and $H > r$			Case 6: $L' > r$ and $H \geq 2r$		

Tab. 1: Six cases for modification of BSM's when considering tool geometry.

For illustration, we show the modification for case 6. Details for the other cases can be found in [6]. Fig. 6 (Case 6) shows that when $L' > r$ & $L \cos \theta \geq 2r$, the inaccessible region of the unit sphere is defined by four critical orientations, a , b , c and d . In Fig. 6, the third drawing shows how to find orientations a and b . The last two drawings show how to locate orientations c & d . The drawing on the right is the A-A view of the drawing in the middle.

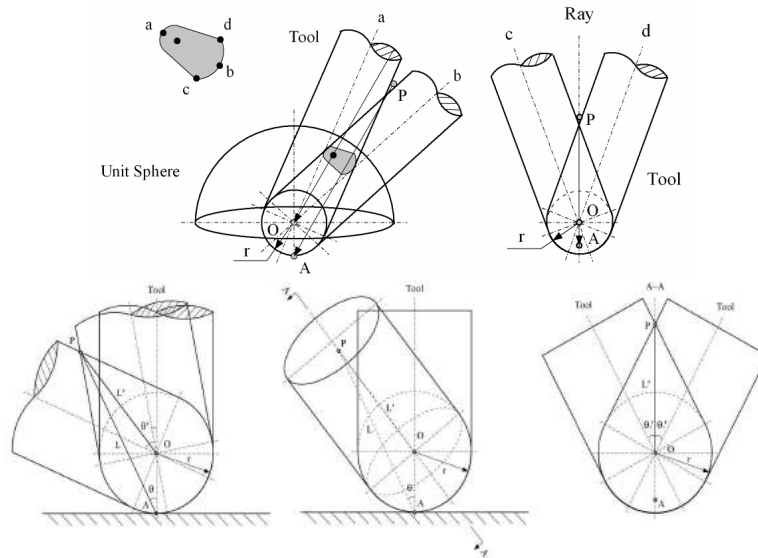


Fig. 6: Case 6 of the BSM modification with tool geometry.

3.3 BSM Modification Procedure

The procedure to modify the BSM's follows.






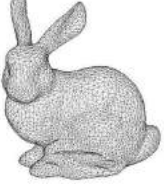
- Step 1.* Retrieve an access point P_i , $i \in [1, n]$ with its BSM $A_i = \{a_{it}\}$, $t \in [1, m]$ of the global accessibility and surface normal \vec{n}_i .
- Step 2.* Pick all a_{it} that are accessible and all a_{it} that are inaccessible and falling above the tangent plane to form two lists $A'_i = \{a_{it}\}$, $A''_i = \{a_{it}\}$, $t \in [1, m]$.
- Step 3.* By sequence take an inaccessible orientation a_{it} in inaccessible list A''_i , and name it the inaccessible orientation \vec{n}_i .
- Step 4.* Calculate the inaccessible angle θ between \vec{n}_i and \vec{n}_i .
- Step 5.* Use the formulations to calculate the inaccessible region with the tool radius.
- Step 6.* Check if any a_{it} which is in the accessible orientation list $A'_i = \{a_{it}\}$ falls into the inaccessible region. If yes, change the status of a_{it} to inaccessible and take it out of the list. If the accessible orientation list $A'_i = \{a_{it}\}$ is empty (which means the whole BSM is inaccessible), stop and go to the next access point.
- Step 7.* Repeat 3 through 6 until all inaccessible orientations a_{it} in A''_i have been checked.
- Step 8.* By sequence, retrieve the next access point P_i and go to Step 2 until all the BSMs on the access points have been modified.

To find a feasible tool radius for the high curvature regions of the surface, we use the tool radius found in the local accessibility analysis as the initial tool radius to modify the set of BSMs created by the global accessibility analysis with rays. If any of BSMs become totally inaccessible, we would shrink the tool radius to redo the modification until we find a tool radius with all the BSMs which have some accessible orientations.





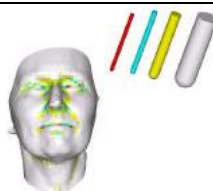







4. EXPERIMENTAL RESULTS

We demonstrate the accessibility approach on three models. The models include a male face model and a bunny model at one facet resolution (Tab 2). We use only one resolution of the bunny model because of the complexity of the model. The experimental set was considered adequate for demonstrating the validity of the approach. We use different resolutions to demonstrate accessibility computation and minimum tool size for varying granularities of model approximation. A tessellated model is, after all, an approximation of the NURBS model. The purpose of the experiments are to show that the approach for computing tool accessibility (local and global) can be accomplished for any size tool and for varying resolution of tessellated models. We also demonstrate that the approach can be used to examine different tool combinations on a model. The approach was implemented in C++ using Standard Template Libraries and the ACIS solid modeling kernel. All experiments were conducted using a Pentium® IV machine with a 2.2 GHz processing speed and 1 Gb of RAM.

For each experiment, three different tool sizes (0.25 inch, 0.50 inch, and 1.00 inch) were investigated to find the minimum size tool which can fit onto the highest curvature regions (which means that this tool can cut the entire part). Tab. 3-4 shows the results of the local accessibility analyses. Tab. 3 shows the results of the male face model and Tab. 4 shows the results of the bunny model. Local accessibility analysis is used to set the upper bound of the feasible tool radius (the minimum size tool). The experimentation shows that there is a difference in computing the minimum tool size based on the resolutions of the models. The higher the resolution, the more accurate the model. In practice, the resolution of the tessellated model should be determined by the tolerance on the manufacture drawing. We found for these complex sculpture models that the tool that would cover the entire part was too small for machining practice. From a machining time perspective, it is better to use larger tools to machine parts. So in the local accessibility analysis we also look at three different tool sizes (0.25 inch, 0.50 inch, and 1.00 inch). The results of the experiments show the percentage of area covered by each tool.

Model #	Model name	Resolution 1	Resolution 2	Resolution 3
1	 Male face	 2347	 5890	 9047
2	 Bunny	 10000		

Tab. 2: Three models used for experimentation tessellated at different resolutions (facets).

Resolution No. (facets)	Tool number Tool size (inch)	Tool 1 0.25	Tool 2 0.50	Tool 3 1.00
1 2347				
	(Accessible area percentage)	99.02	94.47	85.81
2 5890				
	(Accessible area percentage)	98.61	94.54	85.89
3 9047				
	(Accessible area percentage)	98.58	94.47	86.22

Tab. 3: Local accessibility analysis of the male face model with three resolutions.

Resolution No. (facets)	Tool number Tool size (inch)	Tool 1 0.25	Tool 2 0.50	Tool 3 1.00
1 10000	(Accessible area percentage)	99.99	99.33	95.75

Tab. 4: Local accessibility analysis of the bunny model (graphics not shown).

The red area on each model represents the high curvature regions that cannot be covered by the tool. The results in the tables are presented in groups. The pictures for each resolution of the model form a group that occupies a row. In each group, the first picture is the result of using all four tools where we first cover all the facets of the model possible with the largest tool, we then cover the portion of the model not covered by the largest tool with the next largest tool and so on. The other pictures show the region that would be covered by using one tool size and the high curvature region uncovered is marked in red color. We also compute the percentage of accessible area covered by each tool.




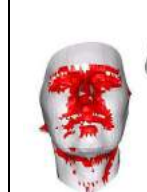
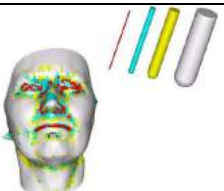



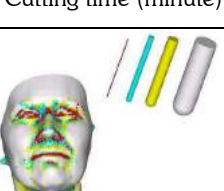



Tab. 5-6 shows the results of the experimentation using the global accessibility analysis. The minimum tool size calculated in the local accessibility analysis is used as the initial feasible tool radius to find the minimum tool size in the global accessibility analysis. The results show the minimum tool size found in the global accessibility analysis may be smaller than that found in the local accessibility analysis. The minimum tool size found in the local accessibility analysis might not have an accessible orientation when we perform the global accessibility with this tool size. In this case, we will shrink the tool size until we find a tool size that has at least one accessible orientation when we perform the global accessibility process. In the global accessibility analysis, we looked at four tool sizes. The accessible area percentage

and the machine time were calculated. By comparing the machine time, we demonstrate that using a combination of tools is better machine planning.

Tab 7 shows the computing times for the BSM modification approach for all model resolutions and tool sizes. The results indicate that the BSM modification procedure takes considerably less processing time than the patch projection procedure (half line projection). Also, the larger the size of the tool, the lower the processing time.

Resolution No. (facets)	Tool number	Tool 1	Tool 2	Tool 3	
	Tool size (inch)	0.25	0.50	1.00	
1	10000	(Accessible area percentage)	98.73	92.09	76.09
		(Cutting time in minutes)	1190.33	777.17	451.77

Tab. 5: Global accessibility analysis of the bunny model (graphics not shown)

Resolution No. (facets)	Tool number	Tool 1	Tool 2	Tool 3	
	Tool size (inch)	0.25	0.50	1.00	
1	2347				
		(Accessible area percentage)	95.97	86.19	68.05
		Cutting time (minute)	350.02	220.04	122.24
2	5890				
		(Accessible area percentage)	95.67	85.67	69.77
		Cutting time (minute)	351.25	220.20	126.17
3	9047				
		(Accessible area percentage)	95.65	85.99	71.37
		Cutting time (minute)	351.71	221.33	129.25

Tab. 6: Global accessibility analysis of the male face model with three resolutions.

No.	Model name	Tool size (inch) & computing time (second)						
		Number of facets	Min tool size	Tool size				
0*	Min			0.25	0.50	1.00		
1	Male face	2347	0.060	304	159	77	63	47
		5890	0.055	1771	210	183	154	118
		9047	0.045	4387	624	281	236	182
		10000	0.050	6219	617	273	227	161

* half line projection

Tab. 7: Computing times of the global accessibility analysis.

5. CONCLUSIONS

An approach for global accessibility using triangular tessellated solid models that considers tool size (spherical endmills) was demonstrated. It includes using local accessibility analysis to set an upper bound of the feasible tool radius, global accessibility analysis with rays (no tool size) to create a set of the BSMs, and modifying the BSMs with different tool radii to analyze the global accessibility with tool geometry. Kang's approach was used to create the initial set of the BSMs by patches projection. The approach is computationally expensive. Three models at different resolutions were used to demonstrate the approach. The experiments included finding the smallest tool that would cover the part for the high curvature regions of the surface and also a global accessibility analysis with different tool radii. The results demonstrate that the approach can be used to analyze the global accessibility of different tools and to compute the machining time with the various tools combinations. The superior computation efficiency of the new approach was demonstrated and compared with a previous approach. Future work will address other tool geometries and tool holder interactions with the workpiece. Optimization of tool selection is another area of future research.

6. REFERENCES

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