A Hybrid Model with Mathematical Relations and Neural Network Relations for Optimal Concurrent Design

Dong Zhao[1], Wei Dong[2] and Deyi Xue[1*]

[1]University of Calgary, zdong@ucalgary.ca, dxue@ucalgary.ca
[2]Versa Power Systems, Wei.Dong@versa-power.com

ABSTRACT

A hybrid model is introduced in this research to describe both the mathematical relations and the neural network relations. Parameters of a design are associated by these two types of relations through a parameter relation network. An optimal concurrent design is achieved by changing values of variable parameters based on evaluation of different performance and cost parameters through optimization. A case study to design a 4-linkage mechanism considering both design and manufacturing requirements is conducted to demonstrate the effectiveness of this optimal concurrent design method.

Keywords: Concurrent design, optimization, performance, cost.

1. INTRODUCTION

Concurrent design is an approach to incorporate the considerations in down-stream product development life-cycle aspects, such as manufacturing and maintenance, into the early design stage for shortening product development lead time, reducing production cost, and improving the overall design quality [2,4]. Presently many methodologies and computer tools have been developed for solving concurrent design problems.

Among various concurrent design approaches, the optimization based concurrent design approach has demonstrated the effectiveness due to the advances in optimization techniques and product life-cycle database modeling. In this research area, Wong et al. developed an optimization based concurrent design method to minimize production cost considering functional requirements as constraints [7]. In this research, meta-modeling was employed for improving the efficiency and quality of optimization. Chang et al. introduced a multi-level product model to describe both the geometric parameters of mechanical systems and behaviors of these systems such as reliability and maintainability, and to achieve the optimal design parameters through tradeoff of these evaluation measures [1]. Wong and Wang developed a customized CAD system using a commercial CAD system, Pro/ENGINEER, for optimal design of industrial silencers considering performance and cost [6]. Many advanced computing techniques, such as genetic algorithm and neural networks, have also been used for optimal concurrent design [3].

In our previous research, a database scheme was introduced to model the different life-cycle aspects of a design and to associate these life-cycle aspects through their relations [8]. A hybrid optimization mechanism was developed to identify the optimal design configuration and its parameters through genetic programming and particle swarm optimization considering functional performance and production cost [10]. Modeling of the non-linear relations among functional performance measures and production cost measures was also studied [9].

Despite the progress, the relations between design variables and design evaluation measures in these optimal concurrent design methods are modeled by mathematical functions usually implemented by computer programs. Since the mathematical relations may not be achieved due to the uncertainty factors, new methods to model the relations considering uncertainties have to be developed. To address this problem, a hybrid model is introduced in this research to describe both the mathematical relations and the neural network relations among design parameters.

2. A HYBRID MODEL WITH MATHEMATICAL RELATIONS AND NEURAL NETWORK RELATIONS

The relation between a number of input parameters, x_1, x_2, ..., x_n, and an output parameter, y, can be defined by a function

$$y = F(x_1, x_2, ..., x_n) \tag{1}$$

* Corresponding author.

When the mechanism of mapping from the input parameters to the output parameter given in Eq. (1) is explicitly known, this relation can be modeled by a mathematical relation

$$y = M(x_1, x_2, \ldots, x_n) \tag{2}$$

A mathematical relation can be defined either by a simple mathematical expression, such as

$$y = x_1 + 4x_2 \tag{3}$$

or by a number of mathematical expressions, such as

$$\begin{aligned} y_1 &= x_1^2 + x_2^2 + x_3^2 + x_4^2 \\ y_2 &= x_1^4 + x_2^4 + x_3^4 + x_4^4 \\ y &= y_1 / y_2 \end{aligned} \tag{4}$$

When the mapping from the input parameters to the output parameter is not exactly known, approximation process is usually considered to achieve the relation. Among various approximation methods, the feed-forward neural network with the back-propagation learning algorithm has demonstrated the effectiveness to achieve such a relation when a large amount of training data is available [5]. In this research, a neural network relation is modeled by

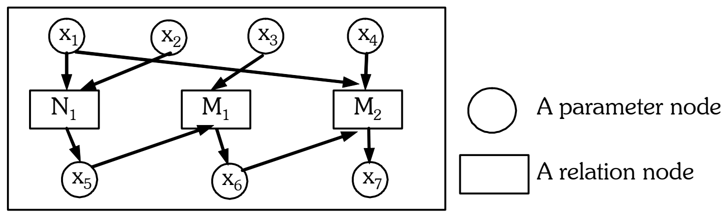
$$y = N(x_1, x_2, \ldots, x_n) \tag{5}$$

The various mathematical relations and neural network relations of a design form a parameter relation network. Fig. 1(b) shows a parameter relation network created from the 2 mathematical relations and 1 neural network relation given in Fig 1(a).

R1: $x_6 = M_1(x_5, x_3)$

R2: $x_5 = N_1(x_1, x_2)$

R3: $x_7 = M_2(x_1, x_4, x_6)$



(a). Two types of relations. (b). A parameter relation network.

Fig. 1: A hybrid model with mathematical and neural network relations.

A parameter relation network is modeled by two types of nodes: parameter nodes and relation nodes. When a parameter node is used only as an input node of one or several relation nodes, this parameter node is called an *independent parameter node*. When a parameter node serves as an output node of a relation node, this parameter node is called a *derived parameter node*. In the example given in Fig. 1, x_1, x_2, x_3 and x_4 are independent parameter nodes, while x_5, x_6 and x_7 are derived parameter nodes. Each parameter node can only be used as the output parameter node of one relation node. Relation nodes are classified into mathematical relation nodes and neural network relation nodes.

Each parameter node in the parameter relation network is associated with a parameter value. Value of an independent parameter can be modified. When value of a parameter is changed, the relations that use this parameter as an input parameter should be activated to update the values of their output parameters. This propagation process is continued until all the relevant parameters are updated. For example, when x_1 in Fig. 1 is changed, this change is then propagated to x_5 and x_7 through the relations N_1 and M_2, respectively. The change of x_5 is further propagated to x_6 through the relation M_1. Change of x_6 is subsequently propagated to x_7 through the relation M_2.

Propagation of parameter value changes through the hybrid parameter relation network is different from the traditional parametric design using the relations defined in commercial CAD systems in the following two aspects.

1. In the traditional parametric design, all the relations are used sequentially to get the value of the right side expression for each relation and assign this value to the left side of this relation. In the hybrid parameter relation network, only when the input parameter value of a relation is modified, this relation is then activated to get the output parameter value. This new output parameter value is compared with the current parameter value to see whether a modification is required.
2. In the traditional parametric design, each relation is only used once for parameter change propagation. Therefore different sequences of these relations could achieve different propagation results. In the hybrid parameter relation network, since a relation could be activated several times during the parameter change

propagation process, the same propagation result is obtained when these relations are organized in different sequences.

3. OPTIMAL CONCURRENT DESIGN USING THE HYBRID MODEL TO DESCRIBE THE RELATIONS AMONG PARAMETERS

In this research, the design modeling scheme introduced in [8] is employed to describe the relations among parameters. In this scheme, a design is modeled using primitives called *artifacts* including *components* and *assemblies*. Fig. 2 shows a gear-pair mechanism modeled by 1 assembly and 2 components. Each artifact is modeled by parameters and relations among parameters. Constraints are also described in artifacts. The parameter relations in different artifacts of a design form a parameter relation network.

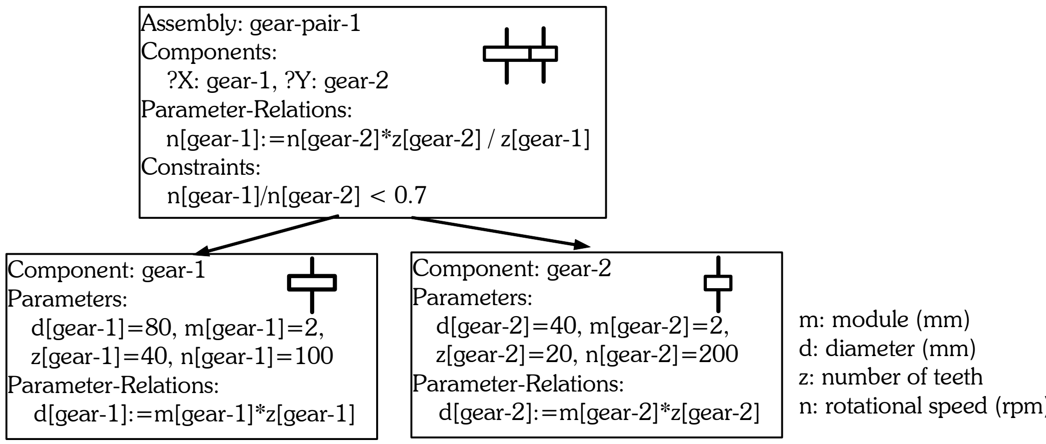


Fig. 2: Modeling of a gear-pair mechanism.

Among various parameters, the design engineer can select some independent parameters as design variables, as shown in Fig. 3. The value of each design variable can be changed between two boundary limits. The vector of design variables is defined as $\mathbf{X}$. In Fig. 3, x_1, x_6 and x_{11} are selected as design variables defined by

$$\boldsymbol{X} = (x_1, x_6, x_{11})^T \tag{6}$$

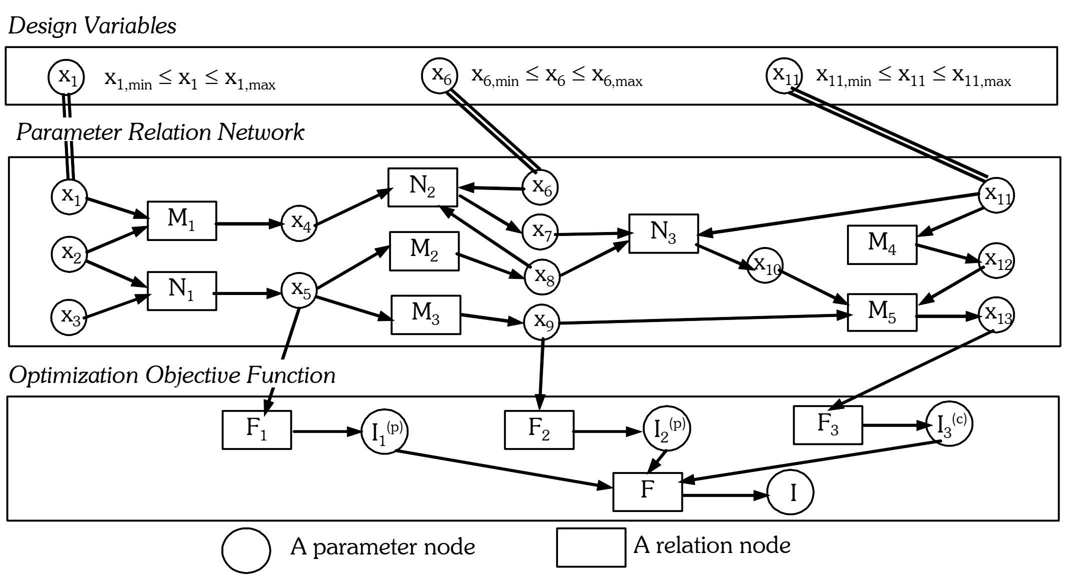


Fig. 3: Modeling of optimal concurrent design considering performance and costs.

Among the parameters in the parameter relation network, the design engineer can select a number of derived parameters as the evaluation measures. An evaluation measure can be achieved from design variables by $f(\mathbf{X})$. In Fig. 3, performance measures of x_5 and x_9 and cost measure of x_{13} are selected as the evaluation measures.

$$x_5 = f_5(\mathbf{X}) \qquad (7)$$
$$x_9 = f_9(\mathbf{X})$$
$$x_{13} = f_{13}(\mathbf{X})$$

Since the different evaluation measures are usually described in different units such as kilowatts, percentages and dollars, these measures need to be converted into comparable measures for evaluating the design considering all relevant evaluation aspects. In this research, these evaluation measures are converted into evaluation indices, I_i $(i=1,2,\ldots,m)$, representing the degrees of satisfaction in these evaluation aspects. The method to model the non-linear relations between evaluation measures and evaluation indices introduced in [9] is employed in this work to convert the evaluation measures into the evaluation indices. An evaluation index is defined by

$$I_i(\mathbf{X}) = F_i[f_i(\mathbf{X})], \quad i = 1, 2, \ldots, m \qquad (8)$$

Since the m evaluation indices are described by values between 0 and 1, the overall evaluation index, I, can then be defined by considering the importance of these evaluation aspects:

$$I(\mathbf{X}) = \frac{1}{W_1 + W_2 + \ldots + W_m}[(W_1 I_1(\mathbf{X}) + W_2 I_2(\mathbf{X}) + \ldots + W_m I_m(\mathbf{X})] \qquad (9)$$

where W_1, W_2, ..., W_m are m weighting factors for the m evaluation indices. These individual weighting factors are selected by design engineers according to their preference.

The optimal concurrent design problem is defined by

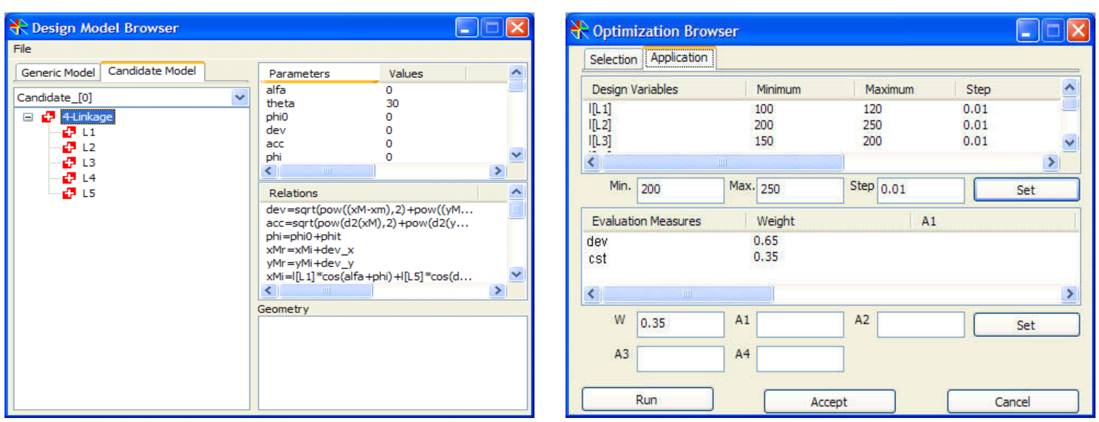
$$\underset{w.r.t.\ \mathbf{X}}{Max}\ I(\mathbf{X}) \qquad (10)$$

subject to:

$$\mathbf{X}_{\min} \le \mathbf{X} < \mathbf{X}_{\max}$$
$$h_j(\mathbf{X}) = 0, \quad j = 1, 2, \cdots, k$$
$$g_j(\mathbf{X}) \le 0, \quad j = k+1, k+2, \cdots, l$$

4. SYSTEM IMPLEMENTATION

The optimal concurrent design system was implemented using Visual C++. Two snapshots of this system are given in Fig. 4. In this system, the *Design Model Browser* is used to build the design using artifacts. The mathematical relations and neural network relations defined in artifacts are used to form the hybrid parameter relation network. The *Optimization Browser* is used to select design variables, evaluation measures, mapping from evaluation measures to evaluation indices, and weighting factors of these evaluation indices for modeling the optimal concurrent design problem. The optimization result is also achieved using this browser.



(a). Design Model Browser. (b). Optimization Browser.

Fig. 4: Two snapshots of the implemented optimal concurrent design system.

5. A CASE STUDY OF OPTIMAL 4-LINKAGE MECHANISM DESIGN

A case study of optimal mechanism design was conducted using the hybrid model with mathematical relations and neural network relations. In this case study, the geometric parameters of a 4-linkage mechanism (Fig. 5) are optimized to achieve a required motion path of point *M*. The mathematical relations are used to calculate the coordinates of the path, while the neural network relations are used to obtain the systematic errors of the path for error compensation. Accuracy and cost are selected to evaluate the designed mechanism in the optimization.

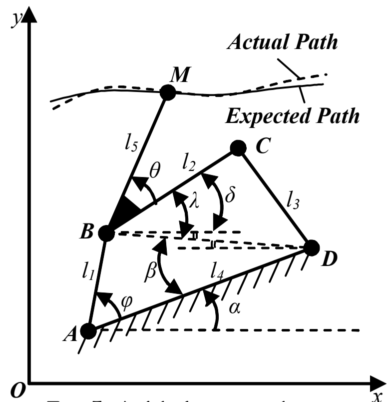


Fig. 5: A 4-linkage mechanism.

5.1 The 4-Linkage System

In the 4-linkage mechanism given in Fig. 5, l_1 is the length of the driving link *AB*, l_2 is the length of the connection link *BC*, l_3 is the length of the driven link *CD*, l_4 is the length of the fixed link *AD*, l_5 is the length of the arm *BM*, α is the angle between the horizontal line and the fixed link *AD*, φ is the driving angle between the fixed link *AD* and the driving link *AB*, β is the angle between the diagonal line *BD* and the fixed link *AD*, λ is the angle between the connection link *BC* and the diagonal line *BD*, δ is the angle between the connection link *BC* and the horizontal line, and θ is the angle between the connection link *BC* and the arm *BM*. *A*, *B*, *C* and *D* are rotational joints. The arm *BM* and the connection link *BC* have no relative motion. When the driving angle φ rotates from φ_0 to φ', the point *M* on the arm of the mechanism moves along a path.

5.2 Behaviors of the 4-Linkage System

When the geometric parameters of the 4-linkage system are perfectly manufactured, the coordinates of the point *M* can be calculated mathematically by:

$$x_M = x_A + l_1 \cos(\alpha + \varphi) + l_5 \cos(\delta + \theta) \tag{11}$$

$$y_M = y_A + l_1 \sin(\alpha + \varphi) + l_5 \sin(\delta + \theta) \tag{12}$$

where x_A and y_A are the *x* and *y* coordinates of the anchor point *A*. The driving angle φ in Eqs. (11) and (12) is defined by

$$\varphi = \varphi_0 + \varphi(t) \tag{13}$$

where φ_0 is the initial angle, $\varphi(t)$ is a function of the time *t* defined by

$$\varphi(t) = \varphi_s t \tag{14}$$

where φ_s is a constant. The driving angle φ is changed from φ_0 to φ'. In Eqs. (11) and (12), α and θ are design variables, and δ is calculated by:

$$\beta = \arctan\left(\frac{l_1 \sin\varphi}{l_4 - l_1 \cos\varphi}\right) \tag{15}$$

$$\lambda = \arccos\left(\frac{l_1^2 + l_2^2 - l_3^2 + l_4^2 - 2 l_1 l_4 \cos\varphi}{2 l_2 \sqrt{l_1^2 + l_4^2 - 2 l_1 l_4 \cos\varphi}}\right) \tag{16}$$

$$\delta = \lambda - (\beta - \alpha) = \alpha + \lambda - \beta \tag{17}$$

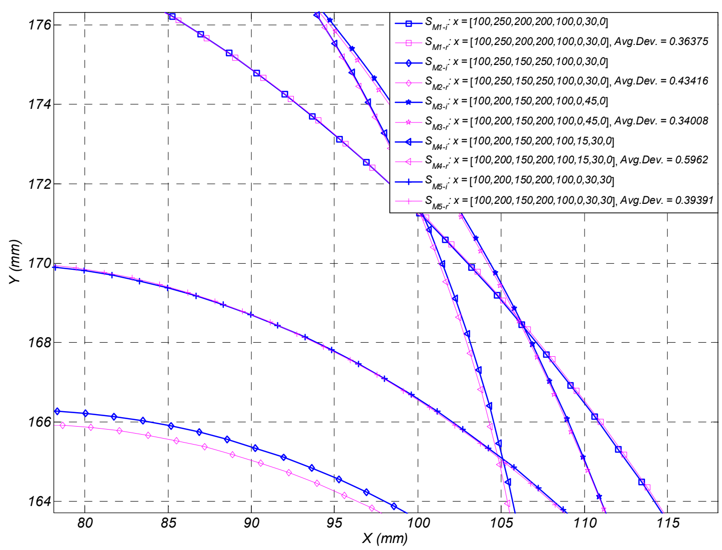
From Eqs. (11)-(17), we can see the coordinates of the point *M* in the path are determined by the parameters of [x_A, y_A, l_1, l_2, l_3, l_4, l_5, α, θ, φ_0]. To simplify the design problem, x_A and y_A are selected as 0 in this case study. Therefore the 8 design parameters for the 4-linkage system are selected as [l_1, l_2, l_3, l_4, l_5, α, θ, φ_0]. The coordinates of the point *M* in the path can be calculated by the following two mathematical functions.

$$x_M = x_M(l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0) = l_1 \cos(\alpha + \varphi) + l_5 \cos(\delta + \theta) \tag{18}$$

$$y_M = y_M(l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0) = l_1 \sin(\alpha + \varphi) + l_5 \sin(\delta + \theta) \tag{19}$$

5.3 Discussion on the Behaviors of the 4-Linkage System

Although the path of the point *M* for the 4-linkage system can be calculated using Eqs. (18) and (19), the actual path is usually different from the calculated path due to errors in manufacturing, measurement, etc. Fig. 6 shows the partial calculated paths and the actual paths when the 8 design parameters are assigned with different values. The different types of errors can be classified into 2 categories: statistical errors and systematic errors. Statistical errors are caused by random factors such as unpredictable changes of temperatures. The systematic errors, on the other hand, are caused by non-random factors such as the errors of the manufacturing machines. If the cause of the systematic errors can be identified, the systematic errors can be eliminated by compensation. The errors of the paths for the 4-linkage system are primarily caused by the length errors of the five linkages, l_1, l_2, l_3, l_4, and l_5, which are produced by a certain manufacturing machine. Therefore only the systematic errors are considered in this case study.



S_M: path of the point *M*. *i*: ideal path. *r*: real path.
x= [l_1, l_2, l_3, l_4, l_5, α, θ, φ_0]. *Avg. Dev.*: average deviation.

Fig. 6: Ideal and real paths of the point *M*.

Identification of the causes of the systematic errors, however, is a nontrivial task. In this case study, the neural network is employed to predict the systematic errors of the path when the 8 design parameters are assigned with certain values. By observing the calculated paths and the actual paths of the point *M* when the driving angle change, $\varphi(t)$ in Eq. (13), is selected with a series of values from 0° to 180° as shown in Fig. 7, the deviations of an actual path from a calculated path in the *x* and *y* directions, D_x and D_y, can be approximated using the following oscillation functions:

$$D_x = a_0 + a_1 \cos\varphi + a_2 \sin\varphi + a_3 \cos^2\varphi + a_4 \sin^2\varphi \tag{20}$$

$$D_y = b_0 + b_1 \cos\varphi + b_2 \sin\varphi + b_3 \cos^2\varphi + b_4 \sin^2\varphi \tag{21}$$

where a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, and b_4 are 10 coefficients to obtain the errors in the *x* and *y* directions for a manufactured 4-linkage mechanism. These 10 coefficients can be achieved from the data points corresponding to different φ angles in the calculated path and the data points in the actual path using the least square method. Since a calculated path and an actual path can be obtained for each design with the 8 design parameters, the 10 coefficients can be achieved for each of the design modeled by the 8 design parameters to obtain the errors of *x* and *y* coordinates in the path. The *x* and *y* coordinates in the actual path, x_M' and y_M', can then be calculated by

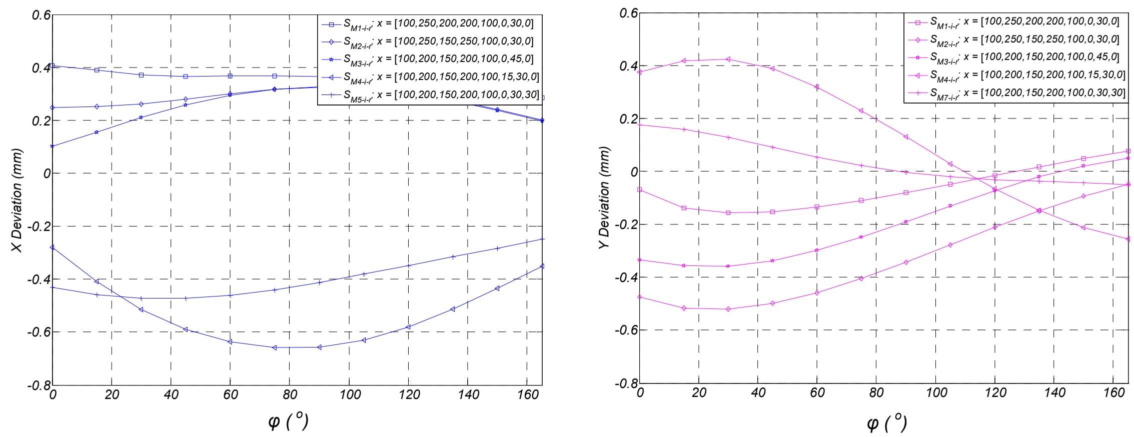


Fig. 7: Coordinate deviations between actual paths and calculated paths.

$$x'_M = x_M + D_x = l_1 \cos(\alpha + \varphi) + l_5 \cos(\delta + \theta) + a_0 + a_1 \cos\varphi + a_2 \sin\varphi + a_3 \cos^2\varphi + a_4 \sin^2\varphi \tag{22}$$

$$y'_M = y_M + D_y = l_1 \sin(\alpha + \varphi) + l_5 \sin(\delta + \theta) + b_0 + b_1 \cos\varphi + b_2 \sin\varphi + b_3 \cos^2\varphi + b_4 \sin^2\varphi \tag{23}$$

A neural network is used to predict the 10 coefficients of a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4 from the 8 design parameters of l_1, l_2, l_3, l_4, l_5, α, θ, φ_0, as shown in Fig. 8. First for each design modeled by the 8 design parameters, the coordinates in the calculated path and the actual path can be achieved. The errors are used to obtain the 10 coefficients using Eqs. (20) and (21) through the least square method. The 8 design parameters of l_1, l_2, l_3, l_4, l_5, α, θ, φ_0 and the 10 coefficients of a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4 are then selected as the input data and the output data in a data set to train the neural network. Since the 10 coefficients can be calculated from the 8 design parameters using the trained neural network, the errors of D_x and D_y in Eqs. (20) and (21) can then be achieved using the following two neural network relations.

$$D_x = D_x(l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0) = a_0 + a_1 \cos\varphi + a_2 \sin\varphi + a_3 \cos^2\varphi + a_4 \sin^2\varphi \tag{24}$$

$$D_y = D_y(l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0) = b_0 + b_1 \cos\varphi + b_2 \sin\varphi + b_3 \cos^2\varphi + b_4 \sin^2\varphi \tag{25}$$

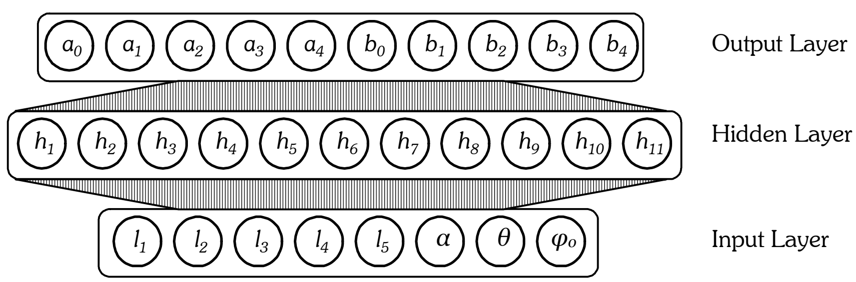


Fig. 8: A neural network.

5.4 Evaluation Measures

Two measures are selected in this case study to evaluate the design modeled by the 8 design parameters. These two measures are: (1) the average deviation between the required path and the designed path of the point M, and (2) the cost of a device used at the point M.

5.4.1 Average Deviation between the Required Path and the Designed Path at Point M

The average deviation between the required path and the designed path of the point M in the mechanism can be calculated by:

$$Dev_{avg,M} = \frac{\sum_{i=0}^{n} \sqrt{\left(x_R - x_M^{'}\right)^2 + \left(y_R - y_M^{'}\right)^2}}{n} \tag{26}$$

where n is the number of sampling points along the path, x_R and y_R are the required x and y locations of the point M, and $x_M^{'}$ and $y_M^{'}$ are the coordinates in the designed path calculated from the 8 design parameters using Eqs. (22) and (23).

5.4.2 Cost of the Device Used at Point M

The velocities of the point M in the x and y directions are calculated by:

$$v_x = \dot{x}_M^{'} = \frac{dx_M^{'}}{dt} = \lim_{\Delta t \to 0} \frac{x_M^{'}(t_{i+1}) - x_M^{'}(t_i)}{\Delta t} \approx \frac{x_M^{'}(t_{i+1}) - x_M^{'}(t_i)}{\Delta t_s} \tag{27}$$

$$v_y = \dot{y}_M^{'} = \frac{dy_M^{'}}{dt} = \lim_{\Delta t \to 0} \frac{y_M^{'}(t_{i+1}) - y_M^{'}(t_i)}{\Delta t} \approx \frac{y_M^{'}(t_{i+1}) - y_M^{'}(t_i)}{\Delta t_s} \tag{28}$$

where Δt_s is the different between t_{i+1} and t_i. The accelerations in the x and y directions are calculated by:

$$a_x = \ddot{x}_M^{'} = \frac{d\dot{x}_M^{'}}{dt} = \lim_{\Delta t \to 0} \frac{\dot{x}_M^{'}(t_{i+1}) - \dot{x}_M^{'}(t_i)}{\Delta t} \approx \frac{\dot{x}_M^{'}(t_{i+1}) - \dot{x}_M^{'}(t_i)}{\Delta t_s} \tag{29}$$

$$a_y = \ddot{y}_M^{'} = \frac{d\dot{y}_M^{'}}{dt} = \lim_{\Delta t \to 0} \frac{\dot{y}_M^{'}(t_{i+1}) - \dot{y}_M^{'}(t_i)}{\Delta t} \approx \frac{\dot{y}_M^{'}(t_{i+1}) - \dot{y}_M^{'}(t_i)}{\Delta t_s} \tag{30}$$

The average acceleration of the point M is calculated by:

$$a_{avg,M} = \frac{\sum_{i=0}^{n} \sqrt{a_x^2 + a_y^2}}{n} \tag{31}$$

When a device is used at the point M, due to different degrees of forces, different designs of this device are required. Suppose the cost of the device is proportional to the average applied force at the point M, and the force at point M is proportional to the acceleration, the cost of the device can then be calculated by

$$C_M = C\, a_{avg,M} \tag{32}$$

where C is a constant selected as 1,800 $\$\cdot s^2/m$.

5.5 Constraints

Based on the mechanism design theory, the following geometric constraints need to be satisfied when l_1 is selected as the driving link.

$$l_1 + l_2 \le l_3 + l_4 \tag{33}$$

$$l_1 + l_3 \le l_2 + l_4 \tag{34}$$

$$l_1 + l_4 \le l_2 + l_3 \tag{35}$$

In addition, l_1 is selected as the shortest link among the 5 links in this case study. The lower and upper boundaries of the 8 design parameters are defined as:

$$100 \le l_1 \le 120 \tag{36}$$

$$200 \le l_2 \le 250 \tag{37}$$

$$150 \le l_3 \le 200 \tag{38}$$

$$200 \le l_4 \le 250 \tag{39}$$

$$100 \le l_5 \le 120 \tag{40}$$

$$0^o \le \alpha \le 15^o \tag{41}$$

$$30^o \le \theta \le 45^o \tag{42}$$

$$0^o \le \varphi_0 \le 30^o \tag{43}$$

5.6 Optimal Mechanism Design

The required path is defined by 12 data points given in Table 1. The 8 design parameters are defined by a vector:

$$\boldsymbol{X} = (l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0)^{\mathrm{T}} \tag{44}$$

P	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
$\varphi(t)$	0	15	30	45	60	75	90	105	120	135	150	165
x_R	153.0	159.8	151.3	132.6	107.5	78.7	48.6	19.5	-7.0	-29.4	-46.8	-58.9
y_R	139.0	157.8	175.1	189.3	198.5	201.6	197.9	187.7	171.7	151.1	127.5	102.7

Tab. 1: Coordinates in the required path.

Two evaluation measures, the average deviation between the required path and the designed path of the point *M*, $Dev_{avg,M}$, and the cost of the device used at the point *M*, C_M, are selected in this case study. These two evaluation measures are calculated from the 8 design parameters using the mathematical relations and the neural network relations defined in the hybrid relation network. Since these two evaluation measures are modeled by different units, these evaluation measures are first converted into evaluation indices, I_1 and I_2, representing degrees of satisfaction in these evaluation aspects using the method introduced in [9]:

$$I_1(\boldsymbol{X}) = F_1(Dev_{avg,M}(\boldsymbol{X})) \tag{45}$$

$$I_2(\boldsymbol{X}) = F_2(C_M(\boldsymbol{X})) \tag{46}$$

Fig. 9 shows the non-linear relations between the evaluation measures and the evaluation indices.

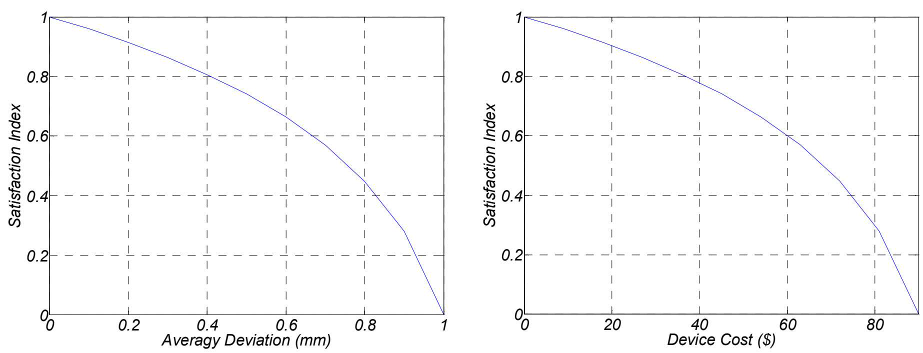


Fig. 9: Evaluation measures and evaluation indices.

The optimization objective function is defined as:

$$I(\boldsymbol{X}) = \frac{1}{W_1 + W_2}\left(W_1 I_1(\boldsymbol{X}) + W_2 I_2(\boldsymbol{X})\right) \tag{47}$$

where W_1 and W_2 are the weighting factors selected as 65% and 35%, respectively. The optimal mechanism design problem is thus defined as:

$$\underset{w.r.t.\ l_1,l_2,l_3,l_4,l_5,\alpha,\theta,\varphi_0}{Max} \quad I(l_1,l_2,l_3,l_4,l_5,\alpha,\theta,\varphi_0) \tag{48}$$

The optimal design parameters are identified as:

$$\boldsymbol{X} = (l_1, l_2, l_3, l_4, l_5, \alpha, \theta, \varphi_0)^T = (115\ mm,\ 223\ mm,\ 183\ mm,\ 217\ mm,\ 117\ mm,\ 0^o,\ 30^o,\ 15^o)^T \tag{49}$$

Fig. 10 shows partial required path and designed path. The average deviation between the required path and the designed path of the point *M*, $Dev_{avg,M}$, and the cost of the device used at the point *M*, C_M, for this optimal design are calculated as

$$(Dev_{avg,M},\ C_M)^T = (0.0976\ mm,\ \$25.818)^T \tag{50}$$

From this case study, we can see the average deviation calculated using the hybrid relation network model is much smaller than the average systematic errors given in Fig. 6.

6. CONCLUSIONS

Characteristics of this newly introduced hybrid model with mathematical relations and neural network relations are summarized as follows.

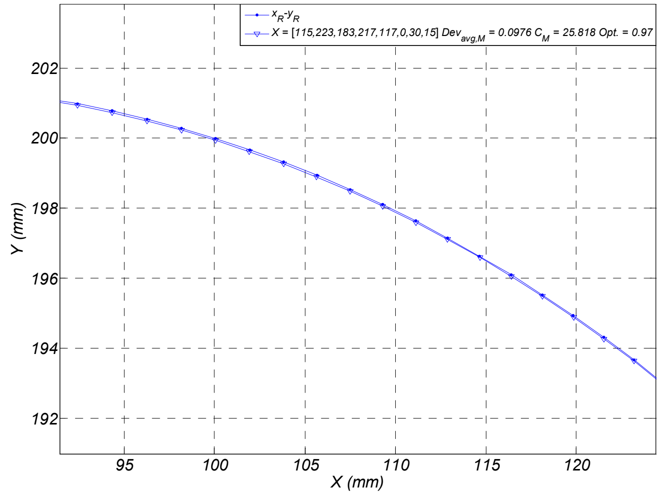


Fig. 10: Deviation of coordinates after the compensation of the systematic errors.

1. The hybrid model is effective for modeling both the explicit relations and the relations with uncertainties.
2. The hybrid model provides a new parametric design approach where the same result can be achieved when the relations are organized in different sequences.
3. The design engineer can select parameters of the hybrid model as design variables and evaluation measures for conducting the optimal concurrent design.

7. ACKNOWLEDGEMENTS

This research was initiated from a fuel cell design project supported by Alberta Energy Research Institute (AERI) and Western Economic Diversification (WED) through its research program. Research collaboration with Versa Power Systems, Calgary, Canada is also acknowledged.

8. REFERENCES

[1] Chang, K. H.; Choi, K. K.; Wang, J.; Tsai, C. S.; Hardee, E.: A Multilevel Product Model for Simulation-based Design of Mechanical Systems, Concurrent Engineering - Research and Applications, 6(2), 1998, 131-144.
[2] Kusiak, A. (ed.): Concurrent Engineering: Automation, Tools, and Techniques, John Wiley & Sons, New York, NY, 1993.
[3] Ozturk, N.; Yildiz, A. R.; Kaya, N.; Ozturk, F.: Neuro-Genetic Design Optimization Framework to Support the Integrated Robust Design Optimization Process in CE, Concurrent Engineering - Research and Applications, 14(1), 2006, 5-16.
[4] Prasad, B.: Concurrent Engineering Fundamentals: Volume I, Prentice Hall, Englewood Cliffs, NJ, 1996.
[5] Simpson, P. K.: Artificial Neural Systems: Foundations, Paradigms, Applications, and Implementations, Pergamon Press, Oxford, UK, 1990.
[6] Wong, L. M.; Wang, G. G.: Development of an Automatic Design and Optimization System for Industrial Silencers, Journal of Manufacturing Systems, 22(4), 2003, 327-339.
[7] Wong, L. M.; Wang, G. G.; Strong, D.: A New Design for Production (DFP) Methodology with Two Case Studies, Concurrent Engineering - Research and Applications, 12(4), 2004, 263-273.
[8] Xue, D.; Yang, H.: A Concurrent Engineering-Oriented Design Database Representation Model, Computer-aided Design, 36(10), 2004, 947-965.
[9] Yang, H.; Xue, D.; Tu, Y. L.: Modeling of the Non-linear Relations among Different Design and Manufacturing Evaluation Measures for Multi-objective Optimal Concurrent Design, Concurrent Engineering - Research and Applications, 14(1), 2006, 43-53.
[10] Zhang, F.; Xue, D.: Optimal Concurrent Design Based upon Distributed Product Development Life-cycle Modeling, Robotics and Computer Integrated Manufacturing, 17(6), 2001, 469-486.