Quasi-Aesthetic Curves in Rational Cubic Bézier Forms

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ABSTRACT

Designing aesthetically appealing models is vital for the marketing success of industrial products. In this paper, we propose quasi-Aesthetic Curves that can be used in CAD systems for aesthetic shape design. Quasi-Aesthetic Curves represented in rational cubic Bézier Forms are curves whose logarithmic curvature histograms (LCHs) become nearly straight lines. The monotonicity of curvature of quasi-Aesthetic Curves is checked by the proposed method. We generate quasi-Aesthetic Curves by approximating the Aesthetic Curves whose LCHs are strictly represented by straight lines. We show that one Aesthetic Curve segment whose change of tangential angle is less than 90 deg. can be replaced by one quasi-Aesthetic Curve segment guaranteeing the monotonicity of the curvature in most of practical situations.

Keywords: Aesthetic Curves, logarithmic curvature histogram, rational cubic Bézier curves, monotone curvature

1. INTRODUCTION

CAD systems for designing aesthetically appealing models are considered to be the next generation CAD systems. In the design of aesthetic shapes such as automotive bodies, designers determine shapes with their great concern for the reflected images of the surroundings, shade lines and highlight lines [7]. In designing aesthetic surfaces, it is desirable to use curves that can control the variation of curvature, which dominates the distortion of reflected shapes on curved surfaces. The Aesthetic Curves [6,9,15,16] are curves that have such a property.

The Aesthetic Curves are curves whose logarithmic curvature histograms (LCHs – to be described in Section 3.1) are represented strictly by straight lines. We use capitals in the first letters (like the “Aesthetic Curve”) to mean curves with linear LCH. We use “aesthetic curves” to mean beautiful curves in artificial and the natural objects. The Aesthetic Curves have the following properties: (1) The Aesthetic Curves can represent many of aesthetic curves in artificial and the natural objects. (2) The Aesthetic Curves include the Clothoids, logarithmic spirals, circle involutes and circles as special cases. (3) The curvature is of monotone. (4) The curvature variation can be controlled by one parameter α. (5) Aesthetic Curve segments can be interactively generated by three control points and α. Although the Aesthetic Curves have many desirable properties, they are represented in quadrature forms. In CAD systems, it is de facto standard to use freeform curves and surfaces, such as B-spline or Bézier curves. To get the Aesthetic Curves incorporated into CAD systems, we need to show that they can be replaced in the form of freeform curves and present a method for doing this.

This paper proposes quasi-Aesthetic Curve segments in the form of rational cubic Bézier curves. Quasi-Aesthetic Curves are curves with approximate linear LCH. We show that the Aesthetic Curve segments can be replaced by quasi-Aesthetic Curve segments of monotone curvatures with the approximate linearity of LCH. Except when an aesthetic curve segment includes a nearby point of infinite curvature, our results show that the proposed method can replace very well the Aesthetic Curve segments by quasi-Aesthetic Curves preserving the monotonicity of curvature and the approximate linearity of LCH.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 briefly describes the LCH, the family of the Aesthetic Curves and Aesthetic Curve segments. Section 4 presents a method for representing quasi-Aesthetic Curves in rational cubic Bézier forms. Section 5 describes a method for checking the monotonicity of curvature of rational cubic Bézier curve segments. The final two sections present results and conclusions.

2. RELATED WORK

A lot of work has been done for designing aesthetically pleasing curves and surfaces. Higashi et al. proposed a method for controlling the curvature distribution of a curve by its evolute, which is the locus of the curvature center [7]. They also propose a method for generating smooth surfaces. Miura proposed unit quaternion integral curves for more direct control of their curvatures and variations than Bézier or B-spline curves [8]. Recently, Farin proposed a method for generating control points of a Bézier curve segment of arbitrary degree of monotone curvature and torsion [4]. The generated curves are called Class A curves. The book edited by Sapidis [11] includes a collection of papers focusing on aesthetic aspects of geometric modeling.

The Aesthetic Curves are curves whose logarithmic curvature histograms are represented by straight lines [6,16]. They include the Clothoids, logarithmic spirals, the circle involutes, and circles as special cases. In this paper, we represent the Aesthetic Curves in rational Bézier forms guaranteeing the monotonicity of curvature. Aesthetic Curves are briefly described in the next section.

Methods have been proposed for approximating logarithmic spirals or the Clothoid curves, which are included in the Aesthetic Curves. Baumgarten and Farin proposed a method for approximating logarithmic spirals by rational cubic Bézier curves [1]. In their method, the positions, tangents and curvatures at the two endpoints of the approximated curve are the same as those of the original spiral. We use a similar technique for the endpoints constraints. Wang et al. have described a method for approximating the Clothoid curve by polynomial Bézier curve segments of degree n using Taylor expositions [13].

Concerning the methods for checking the monotonicity of curvatures, Sapidis and Frey presented the necessary and sufficient condition of monotone curvature for polynomial Bézier curves of degree two [11]. Frey and Field presented the condition for rational quadratic curves [5]. Dietz and Piper proposed a method for controlling polynomial cubic Bézier curves such that they become spirals (thus become of monotone curvature) [2]. They used precomputed tables for the control of Bézier curves. Wang et al. presented sufficient monotone curvature conditions of polynomial Bézier and B-spline curves of degree n [14]. The condition of the monotonicity of the curvature of rational cubic Bézier curves has yet to be derived.

3. THE AESTHETIC CURVES

The Aesthetic Curves are curves based on the analysis of many aesthetic curves in artificial and the natural objects. Harada et al. have shown that many of aesthetic curves in artificial and the natural worlds are curves whose LCHs can be approximated by straight lines [6,16]. The slope of the straight line of LCH (also called the slope of LCH) is called α. Miura derived the general formula of the Aesthetic Curves [9]. Using the general formula, we have identified the family of the Aesthetic Curves and presented a method for interactively drawing a segment by specifying three control points and the slope of LCH α. This section introduces the LCH, the family of the Aesthetic Curves and Aesthetic Curve segments.

3.1 Logarithmic Curvature Histograms

A curve and its logarithmic curvature histogram are shown in Fig. 1. Let ρ and s be the radius of curvature and the arc length, respectively. When a curve is subdivided into infinitesimal segments such that $\Delta\rho/\rho$ being constant, the LCH represents the relationship between ρ and Δs in a double logarithmic graph [9]. Harada showed that many of aesthetic curves, such as the key lines of automobiles, the birds' eggs and the butterfly's wings, are curves whose LCH can be approximated by straight lines with slope α. Harada et al. insisted that the slope of LCH α is closely related to the impression of a curve [6,16].

Miura derived the general formula of the Aesthetic Curves whose LCHs are represented by straight lines:

$$\log\frac{\Delta s}{\Delta\rho/\rho} = \alpha\log\rho + c \tag{3.1}$$

where c is a constant.

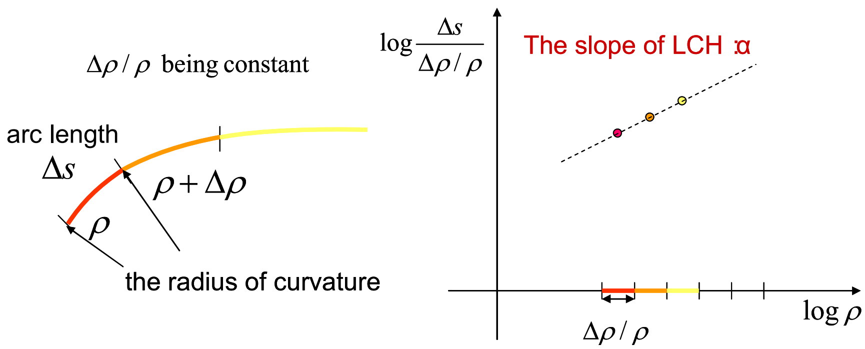


Fig. 1: A curve and its Logarithmic Curvature Histogram.

3.2 The Family of Aesthetic Curves

We have derived the equations of the Aesthetic Curves [15] by integrating the general formula derived by Miura using the constraints of the standard form. In the standard form, constraints of translation, rotation and scaling are placed at a certain point of the curve. We call the point of the curve at $\mathrm{d}\rho/\mathrm{d}s=\Lambda$ the reference point. At the reference point, the following constraints are placed to obtain the standard form: the curve goes through the origin (translational constraint), the tangential direction is the positive x axis (rotational constraint) and the radius of curvature is 1 (scaling constraint).

The point on an Aesthetic Curve whose tangential angle is θ with the slope of LCH α in the standard form is represented on the complex plane by

$$\mathbf{P}_{AE}(\theta)=\begin{cases}\int_0^\theta e^{(1+i)\Lambda\varphi}d\varphi & \text{if } \alpha=1\\ \int_0^\theta((\alpha-1)\Lambda\varphi+1)^{\frac{1}{\alpha-1}}e^{i\varphi}d\varphi & \text{otherwise}\end{cases} \tag{3.2}$$

where i is the imaginary unit, Λ is the parameter for specifying $\mathrm{d}\rho/\mathrm{d}s$ at the origin. Changing Λ means a similarity transformation of the curve when $\alpha\neq1$ or the change of the shape of the curve when $\alpha=1$. The point on an Aesthetic Curve can also be formulated using the arc length. θ and Λ may have either an upper bound or a lower bound. See [15] for the details. The curvature κ of the Aesthetic Curves are given by

$$\kappa=\begin{cases}e^{-s} & \text{if } \alpha=0\\ (\alpha s+1)^{-\frac{1}{\alpha}} & \text{otherwise}\end{cases} \tag{3.4}$$

Tangential angle θ and arc length s are related by

$$\theta=\begin{cases}1-e^{-s} & \text{if } \alpha=0\\ \log(s+1) & \text{if } \alpha=1\\ \dfrac{(\alpha s+1)^{\left(1-\frac{1}{\alpha}\right)}-1}{\alpha-1} & \text{ohterwise}\end{cases} \tag{3.5}$$

For understanding the family of the Aesthetic Curves and drawing curve segments, two kinds of standard forms are considered. Standard Form I is used for identifying the overall shapes of the Aesthetic Curves. Λ is set to 1 in Standard Form I. Standard Form II is used for interactively drawing curve segments and Λ can be an arbitrary value (Λ may have bounds depending on α).

Fig. 2 shows the family of the Aesthetic Curves in Standard Form I. When $\alpha=-1,1,2$ or $\pm\infty$, the Aesthetic Curve is the Clothoid curve, a logarithmic spiral, a circle involute or a circle, respectively. The characteristics of the Aesthetic Curves at $\rho=0$ or $\rho=\infty$ are analyzed in [15]. For example, when $\alpha<0$, the Aesthetic Curve includes the point of inflection. When $\alpha > 0$, the Aesthetic Curve includes the point at $\rho = 0$. If $0 < \alpha \leq 1$, the Aesthetic Curve spirally converges to the point at $\rho = 0$. If $\alpha > 1$, the tangential direction is determined at $\rho = 0$.

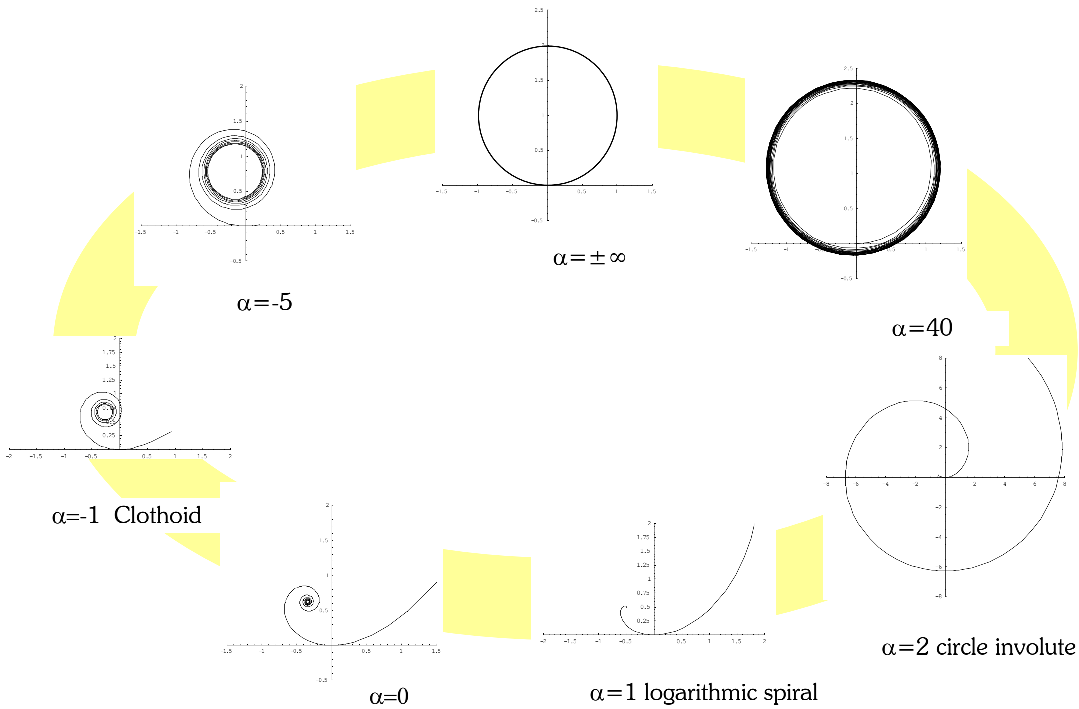


Fig. 2: The family of Aesthetic Curves in Standard Form I.

3.3 Aesthetic Curve Segments

An Aesthetic Curve segment can be interactively drawn by specifying three control points $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$ and α. We briefly describe how the curve segment is computed. From the control points given, the change of tangential angle θ_d can be computed as the angle formed by $\mathbf{p}_b - \mathbf{p}_a$ and $\mathbf{p}_c - \mathbf{p}_b$ (See Fig. 5 (a)). On the overall shape of the Aesthetic Curve with the slope of LCH α, we can decide the points $\mathbf{p}_0$ and $\mathbf{p}_2$ such that their tangential angles are 0 and θ_d, respectively(See Fig. 5 (b)). $\mathbf{p}_1$ is the intersection between the tangent lines at $\mathbf{p}_0$ and $\mathbf{p}_2$. If the two triangles $\mathbf{p}_a\mathbf{p}_b\mathbf{p}_c$ and $\mathbf{p}_0\mathbf{p}_1\mathbf{p}_2$ are similar, the Aesthetic Curve segment can be drawn by transforming the points on the Aesthetic Curve in Standard Form II such that $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$ are transformed to $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$, respectively. The similar triangle $\mathbf{p}_0\mathbf{p}_1\mathbf{p}_2$ can be found by changing Λ using the bisection method[15]. Note that the positions of control points and α dictate whether an Aesthetic Curve segment can be drawn. See [15] for the drawable regions depending on α. Fig. 3 shows an Aesthetic Curve segment of $\alpha = -1$ and its corresponding overall shape.

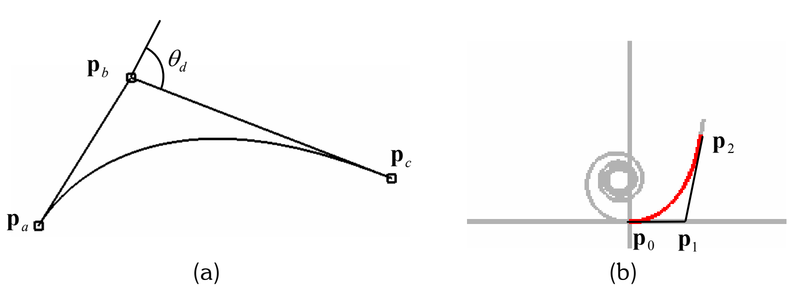


Fig. 3: (a) An Aesthetic Curve segment and (b) the corresponding overall shape.

4. QUASI-AESTHETIC CURVES IN RATIONAL CUBIC BÉZIER FORMS

We are given an Aesthetic Curve segment s_{AE} defined by three control points $\mathbf{b}_a, \mathbf{b}_b, \mathbf{b}_c$ and the slope of LCH α (Fig. 4 (a)). We try to find a quasi-Aesthetic Curve segment represented by a rational cubic Bézier curve segment $s_{\mathbf{B}}$ that replaces the given Aesthetic Curve segment $\mathbf{s}_{AE}(\theta) = \mathbf{P}_{AE}(\theta)$ preserving the approximate linearity of LCH. Thus, we can draw a quasi-Aesthetic Curve segment similarly as an Aesthetic Curve segment by specifying three control points and α. Finding a curve with monotone curvature and prescribed positions, unit tangents and curvatures at the end points is a sensitive problem. As will be shown in Fig. 7, however, Aesthetic Curves are well-approximated by Quasi-Aesthetic Curves except when the curves include the point at $\rho = 0$ or ∞.

Let a rational cubic Bézier curve segment $s_{\mathbf{B}}$ be defined by

$$\mathbf{s}_B(t) = \frac{\mathbf{p}(t)}{w(t)} \quad t \in [0,1] \tag{4.1}$$

where

$$\mathbf{p}(t) = \sum_{i=0}^{3} B_i^3(t)\mathbf{P}_i, \; w(t) = \sum_{i=0}^{3} B_i^3(t) w_i, \tag{4.2}$$

$B_i^3(t)$ is the Bernstein polynomial, $\mathbf{P}_i = w_i \mathbf{b}_i$, $\mathbf{b}_i (\in \mathbf{E}^2)$ is the i-th control point vector and $w_i \in \mathbf{R}^+$ is its weight. Although the rational cubic Bézier curve has 12 parameters, its degree of freedom (DOF) is actually 10. Thus 2 of 12 parameters may be chosen arbitrarily. This is because either scaling all $\mathbf{P}_i$ by a scalar $s_0 (\neq 0)$ or scaling w_0, w_1, w_2, w_3 by $s_1^{\,0}, s_1^1, s_1^2, s_1^3 (s_1 \neq 0)$ respectively does not change the shape of the curve. For approximating the given Aesthetic Curve, we need to fix 10 DOFs.

To determine the rational cubic Bézier curve segment $s_{\mathbf{B}}$, we need to determine the control point vectors $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ and their weights w_0, w_1, w_2, w_3. We choose the weights w_1 and w_2 for arbitrarily chosen parameters. We define:

$$w_1 = w_2 = (2\cos(\theta_D / 2) + 1)/3. \tag{4.3}$$

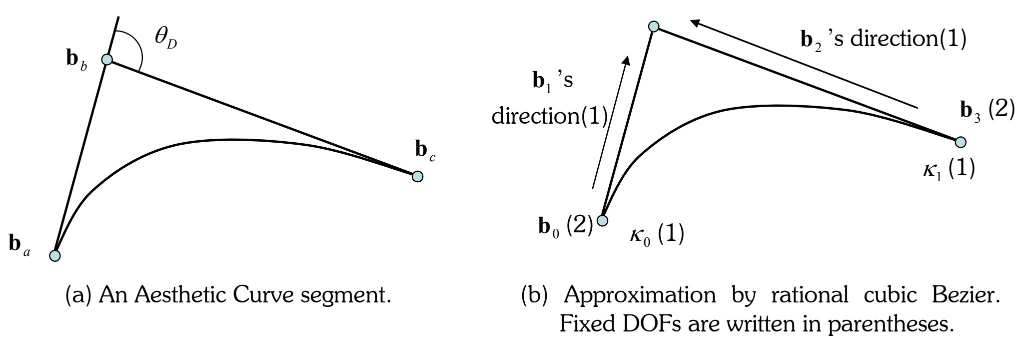
w_1 and w_2 are derived by performing degree-elevation to a rational quadratic Bézier curve that represents a circular arc with the change of tangential angle θ_D assuming $w_0 = w_3 = 1$. θ_D is the angle formed by $\mathbf{b}_b - \mathbf{b}_a$ and $\mathbf{b}_c - \mathbf{b}_b$ (Fig. 4 (a)). Positional and tangential constraints at the start and end points of $\mathbf{s}_{AE}$ yields:

$$\mathbf{b}_0 = \mathbf{b}_a, \; \mathbf{b}_1 = \mathbf{b}_a + b_1(\mathbf{b}_b - \mathbf{b}_a), \; \mathbf{b}_2 = \mathbf{b}_c + b_2(\mathbf{b}_b - \mathbf{b}_c), \; \mathbf{b}_3 = \mathbf{b}_c,$$

where $b_0, b_1 (0 \le b_0, b_1 \le 1)$ are unknowns. The curvatures at the start point (κ_0) and at the end point (κ_1) are [1,3]

$$\kappa_0 = \frac{2w_0 w_2}{3w_1^2} \frac{\det(\mathbf{b}_1 - \mathbf{b}_0, \mathbf{b}_2 - \mathbf{b}_1)}{|\mathbf{b}_1 - \mathbf{b}_0|^3}, \tag{4.4}$$

$$\kappa_1 = \frac{2w_1 w_3}{3w_2^2} \frac{\det(\mathbf{b}_2 - \mathbf{b}_1, \mathbf{b}_3 - \mathbf{b}_2)}{|\mathbf{b}_3 - \mathbf{b}_2|^3}. \tag{4.5}$$



(a) An Aesthetic Curve segment.

(b) Approximation by rational cubic Bezier. Fixed DOFs are written in parentheses.

Fig. 4: (a) An Aesthetic Curve segment and (b) its approximation.

Solving these equations for w_0, w_3, respectively, using Eqn.(4.3), we obtain

$$w_0 = \kappa_0 \frac{3w_1}{2} \frac{|\mathbf{b}_1 - \mathbf{b}_0|^3}{\det(\mathbf{b}_1 - \mathbf{b}_0, \mathbf{b}_2 - \mathbf{b}_1)}, \tag{4.6}$$

$$w_3 = \kappa_1 \frac{3w_2}{2} \frac{|\mathbf{b}_3 - \mathbf{b}_2|^3}{\det(\mathbf{b}_2 - \mathbf{b}_1, \mathbf{b}_3 - \mathbf{b}_2)}. \tag{4.7}$$

Thus, w_0, w_3 can be computed if two unknowns b_0, b_1 are determined. In Fig. 4(b), fixed DOFs are shown in parentheses.

We use the optimization approach to determine b_0, b_1 such that the sum of squared errors between points on the rational cubic Bézier curve segment $\mathbf{s}_B$ and the corresponding points on the Aesthetic Curve $\mathbf{s}_{AE}$. The correspondence between the points on $\mathbf{s}_B$ (Eqn.(4.1)) and $\mathbf{s}_{AE}$ (Eqn.(3.2)) is established as follows. The change of tangential angle θ at $\mathbf{s}_B(t)$ is computed as the angle formed by $\dot{\mathbf{s}}_B(0)$ and $\dot{\mathbf{s}}_B(t)$. Then the corresponding point on the Aesthetic Curve is computed by $\mathbf{s}_{AE}(\theta)$. See Fig. 5. We minimize the following function $f(b_0, b_1)$:

$$f(b_0, b_1) = \sum (\mathbf{s}_B(t) - \mathbf{s}_{AE}(\theta)). \tag{4.8}$$

If b_0, b_1 are found such that $f(b_0, b_1)$ is a minimum, all the parameters of a rational cubic Bézier curve are determined. Thus we can represent quasi-Aesthetic Curves in rational cubic Bézier forms.

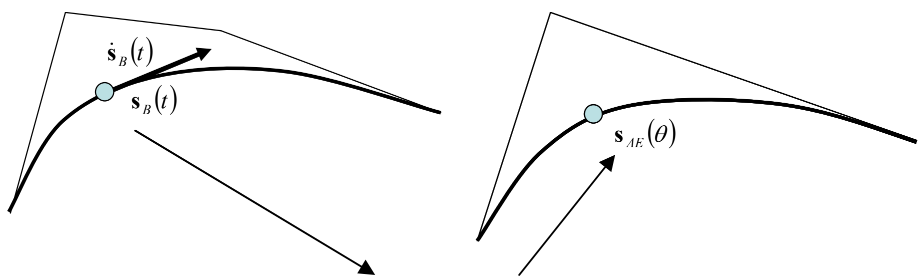


Fig. 5: A point (left) on the rational cubic Bézier curve segment $\mathbf{s}_B$ and the corresponding point (right) on the Aesthetic Curve $\mathbf{s}_{AE}$.

5. THE MONOTONICITY OF THE CURVATURE OF RATIONAL CUBIC BÉZIER CUREVES

To confirm that the approximated curve $\mathbf{s}_B$ is of monotone curvature, we need a method for checking the monotonicity of the curvature of rational cubic Bézier curves. We briefly describe our method for checking the monotonicity of the curvature based on Wang et al.'s approach [14].

The derivative of curvature κ of a curve $\mathbf{x}(t)$ with respect to arc length s is [3,10]

$$\frac{d\kappa}{ds} = \frac{\det(\dot{\mathbf{x}}, \mathbf{x}^{(3)})\dot{\mathbf{x}} \cdot \dot{\mathbf{x}} - 3\det(\dot{\mathbf{x}}, \ddot{\mathbf{x}})\dot{\mathbf{x}} \cdot \ddot{\mathbf{x}}}{|\dot{\mathbf{x}}|^6} \tag{5.1}$$

where $\dot{\mathbf{x}}, \ddot{\mathbf{x}}, \mathbf{x}^{(3)}$ are the first, second and third derivatives of $\mathbf{x}$ with respect to t. If $\frac{d\kappa}{ds} \geq 0$ or $\frac{d\kappa}{ds} \leq 0$ within $t \in [0,1]$, then the curve is of monotone curvature. Note that this definition of monotone curvature includes circular arcs where $\frac{d\kappa}{ds} = 0$ without depending on t.

We check the monotonicity of curvature of a rational cubic Bézier curve segment $\mathbf{s}_B(t)$ using Eqn.(5.1). If the positions of all the control points are different, the denominator of the right-hand side of Eqn.(5.1) is always positive.

Thus, we only need to consider the numerator of the right-hand side of Eqn.(5.1). Let K_n be the numerator of the right-hand side of Eqn.(5.1). Computing K_n replacing $\mathbf{x}$ by $\mathbf{s}_B$, we obtain

$$K_n(t) = \frac{k_n(t)}{k_d(t)} \tag{5.2}$$

where $k_n(t)$ is a polynomial of degree 12 and $k_d(t)$ is $w(t)^8$ after simplification. Since we assume all the weights are positive, $k_d \geq 0$. Thus we need to consider $k_n(t)$ only.

We change the basis of $k_n(t)$ to Bernstein basis and represent $k_n(t)$ by

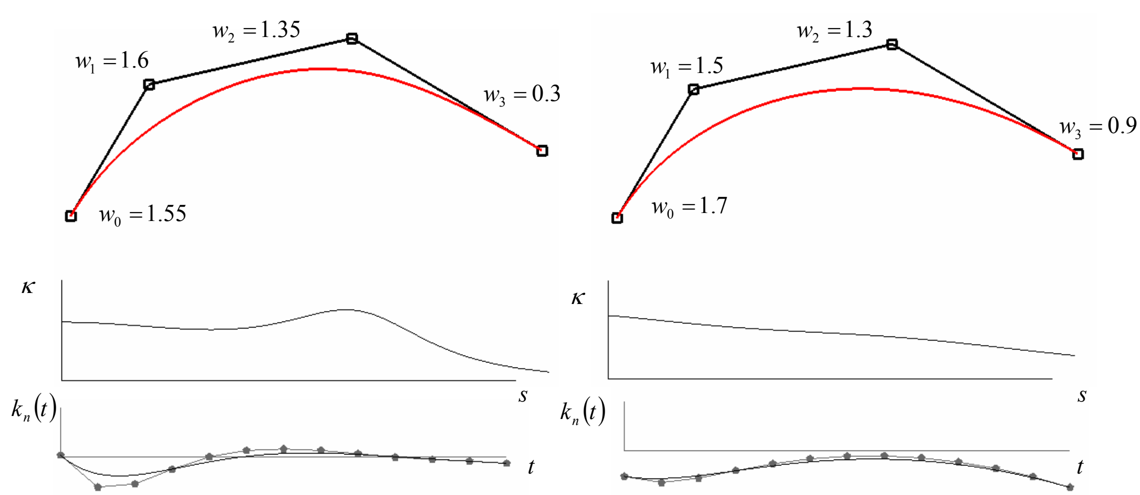
$$k_n(t) = \sum_{i=0}^{12} B_i^{12}(t) b_i \ . \tag{5.3}$$

Now $k_n(t)$ is a Bézier curve of degree 12. To see if $k_n(t)$ changes its sign within $t \in [0,1]$, we consider the following two conditions.

(a) $b_i \geq 0\,(0 \leq \mathrm{i} \leq 12)$ or $b_i \leq 0\,(0 \leq \mathrm{i} \leq 12)$
(b) $b_0 \cdot b_{12} < 0$

If the condition (a) holds, we can immediately conclude that $k_n(t) \geq 0$ or $k_n(t) \leq 0$ within $t \in [0,1]$ from the convex hull property. Thus the curve is of monotone curvature. If the condition (b) holds, $k_n(t)$ changes its sign within $t \in [0,1]$. Thus the curve is not of monotone curvature. If neither (a) nor (b) holds, we recursively subdivide the Bézier curve of Eqn.(5.3) using the de Casteljau algorithm until the condition (a) holds in one of the subdivided curves or (b) holds in all of the subdivided curves. If the condition (a) holds in one of the subdivided curves, the curve is not of monotone curvature. If the condition (b) holds in all of the subdivided curves, the curve is of monotone curvature.

Fig. 6 shows the rational cubic Bézier curve segments with weights, the curvature plots (the graphs of κ with respect to s), and the graphs of $k_n(t)$ with respect to t. Note that the vertical scales of κ and $k_n(t)$ are arbitrarily scaled. The curve segment of Fig.6(a) is not of monotone curvature, whereas that of Fig.6(b) is. The described method correctly checks the monotonicity of the curvature.



(a) Rational cubic Bézier of not monotone curvature (b) Rational cubic Bézier of monotone curvature

Fig. 6: Monotonicity of the curvature of rational cubic Bézier curve segments.

6. RESULTS

Fig. 7 and 8 show quasi-Aesthetic Curves in rational cubic Bézier forms with various α s. All of these curve segments are confirmed to be of monotone curvature by the method described in Section 5. If a curve should find not to be of monotone curvature, the aesthetic curve may be subdivided into segments until the monotone curvature condition is satisfied. However, in our experiments, the monotonicity of curvature was confirmed in all the situations where the optimization process was successful. In Fig. 7, two kinds of errors (rms and errMax) between the original Aesthetic Curve segment and the created quasi-Aesthetic curve segment are shown. "rms" means the root mean square, and "errMax" means the maximum distance between the corresponding points (See Fig. 5). These errors are normalized so that the arc length becomes 1. As the change of tangential angle of the original Aesthetic Curve segment gets larger, both rms and errMax get worse. The changes of tangential angles of all the curve segments shown in Fig. 7 and 8 are set to be 90 deg, which can be considered as the largest change of the tangential angle in practical situations.

If the control points of the original Aesthetic Curve segment form an isosceles triangle, the Aesthetic Curve segment becomes a circular arc without depending on α . In such a situation both rms and errMax are around 1×10^{-16} because a rational cubic Bézier curve can exactly represent a circular arc. As the control points gets away from an isosceles triangle, the approximation error gets worse as shown in Fig. 7. Aesthetic curve segments of $\alpha > 1$ may include a point at infinite curvature. Because a rational cubic Bézier curve cannot represent such a point, the optimization process may get unstable (trapped in local minima). However, it is rare to use curves that include points at infinite curvature. Thus, quasi-Aesthetic Curves are practical in most situations.

Logarithmic curvature histograms (shown as LCH) of quasi-Aesthetic curve segments (approximated rational cubic Bézier curve segments) are also shown in Fig. 7 and 8. To draw logarithmic curvature histograms, we need to compute $\log\left(\dfrac{\mathrm{d}s}{\mathrm{d}\rho/\rho}\right)$. This can be computed by

$$\frac{\mathrm{d}\rho}{\mathrm{d}s} = -\frac{1}{\kappa^2}\frac{\mathrm{d}\kappa}{\mathrm{d}s} \tag{6.1}$$

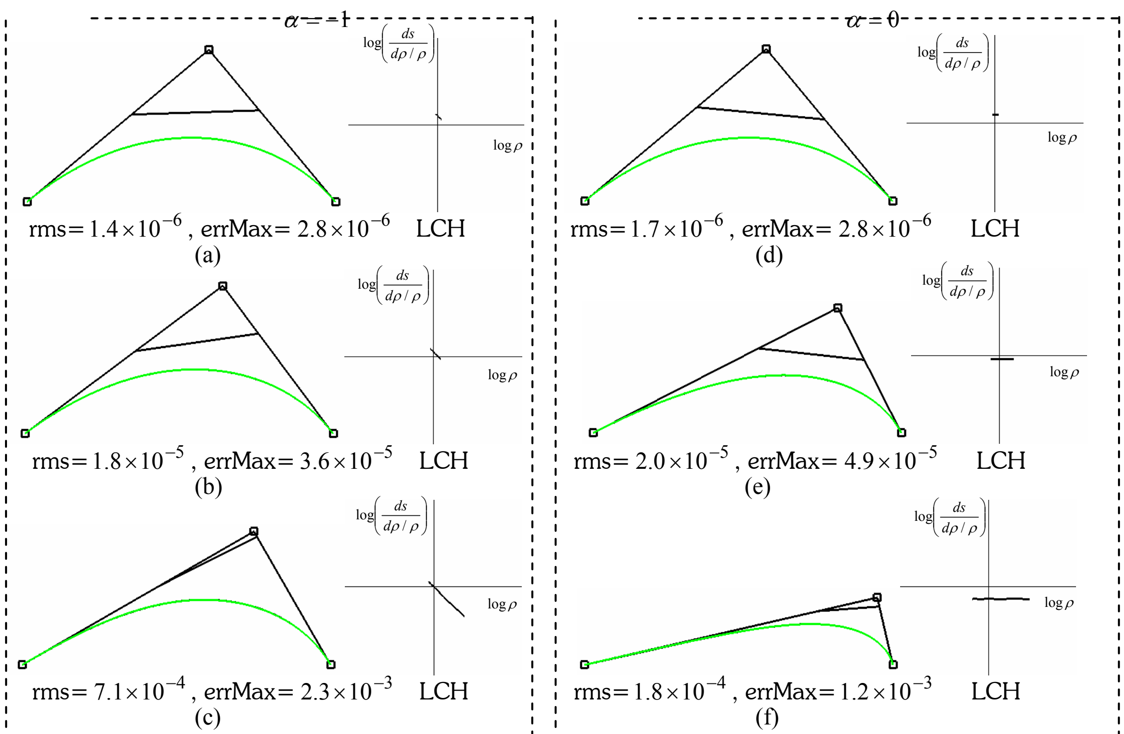


Fig. 7: Quasi-Aesthetic Curves of $\alpha = -1,0$ and their LCHs.

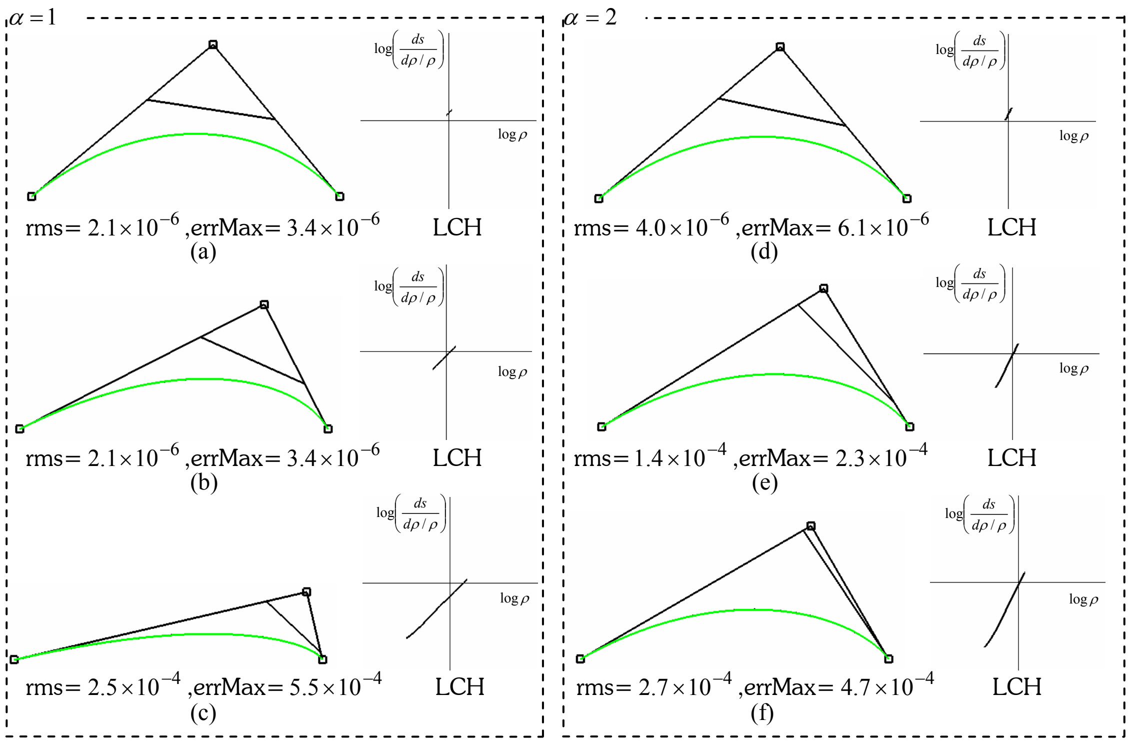


Fig. 8: Quasi-Aesthetic Curves of $\alpha = 1,2$ and their LCHs.

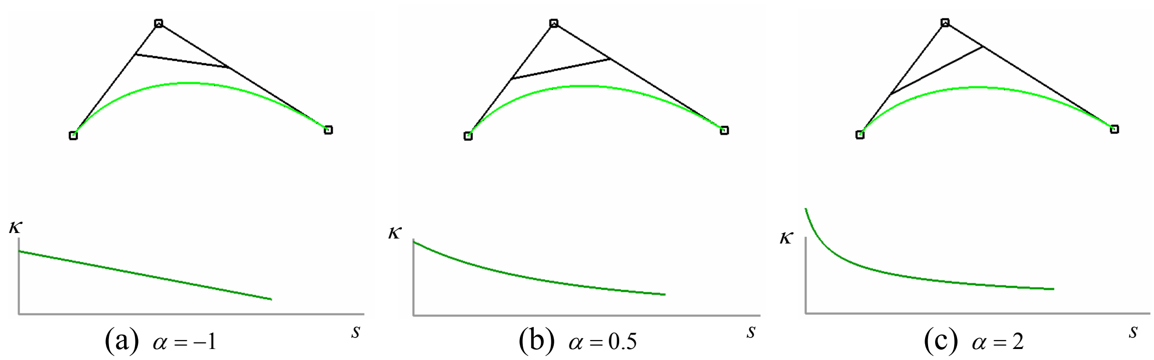


Fig. 9: Quasi-Aesthetic Curves and their curvature plots (1).

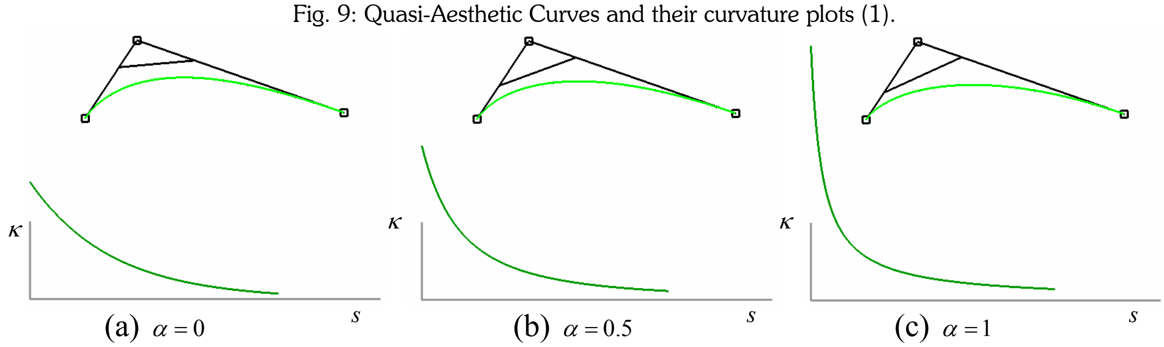


Fig. 10: Quasi-Aesthetic Curves and their curvature plots (2).

using Eqn.(5.1). From the LCHs shown in Fig. 7 and 8, we can conclude that the linearity of LCH is well-preserved comparing the results based on the analysis of many Aesthetic Curves [6,16]. Therefore, we can replace the Aesthetic Curves by quasi-Aesthetic Curves guaranteeing the monotonicity of the curvature and preserving the approximate linearity of LCH.

Fig. 9 and 10 show quasi-Aesthetic Curve segments in rational cubic Bezier forms and its curvature plots with various α s. In each of these figures, the positions of the control points are the same but α s are different. The figures show how the variation of curvature changes with different α s.

7. CONCLUSIONS

We have proposed quasi-Aesthetic Curves in rational cubic Bézier forms. Quasi-Aesthetic curves are curves with the approximate linear LCH. Comparing with the approximate linearity of LCH [6,17] of the aesthetic curves in artificial and the natural objects, we conclude that quasi-Aesthetic Curves preserve the approximate linearity of LCH very well. Thus quasi-Aesthetic Curves have a potential to be used in many aspects of aesthetic shape design. We showed that in most cases an Aesthetic Curve segment whose change of tangential angle is less than 90 deg. can be replaced by one rational cubic Bézier curve segment.

Future areas of research include a more efficient and more stable method for representing quasi-Aesthetic Curve segments, the connection of segments and the creation of surfaces.

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