Optimum Matching of Geometric Features for Material Metamorphosis in Heterogeneous Object Modeling

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ABSTRACT

This paper presents a new methodology of material blending for heterogeneous object modeling by matching the material governing features for designing a heterogeneous object. The proposed geometric feature matching method is used to generate non-self-intersecting and non-twisted connecting surfaces. The proposed method establishes point-to-point correspondence represented by a set of connecting lines between two material directrices. To blend the material features between the directrices, a heuristic optimization method developed with the objective is to maximize the sum of the inner products of the unit normals at the end points of the connecting lines and minimize the sum of the lengths of connecting lines. By subdividing the connecting lines into equal number of segments, one can construct a series of intermediate piecewise curves that represent the material metamorphosis between the governing material features. Implementation and examples are also presented.

Keywords: Heterogeneous object modeling, curve matching, metamorphosis, ruled surface generation.

1. INTRODUCTION

Heterogeneous objects are made of different materials where each material contributes to a certain property. By proper control of the constituent material compositions, a heterogeneous object can be designed to exhibit different properties at different portions of the part. If designed properly, these objects can perform better than their homogeneous counterparts in many different engineering applications because of their ability to satisfy multiple property requirements [13]. In our earlier work [13], a feature-based heterogeneous object modeling method is presented. Object features that control material composition are identified as material governing features because they dictate the material variation inside the object. A lofting based method [13] was used to blend the material features. In [13], it was found that the property requirements at each material governing feature follow the normal direction as shown in Fig. 1. The outcome of the lofting process represents a smooth blending among its generators. In case of surface lofting from its generator curves, the resulting surface passes smoothly through each of the curves. In the same way lofting can be used to get a smooth transition from one governing feature to another. As shown in Fig. 1, the property requirements at each of the generators are blended along the normal direction from the generators. It is assumed that each iso-parametric entity in the blend direction t will represent constant property requirements. Fig. 1 shows how two different material property requirements are blended together using a lofting process. Two curves, $GF_1(u)$ and $GF_1(u)$ are exposed to high and low temperatures, respectively, and therefore exhibit different property requirements. It is a known fact that from a heated body, heat flows in the normal direction from every point of the body. Therefore, the iso-parametric curves on the loft surface will represent iso-conditions (iso-temperatures). In the figure, the thicknesses of the curves show the temperature variations and therefore different property requirements.

As shown in Fig. 1, a smooth transition between two given material governing features is provided to blend the material properties for heterogeneous object modeling. This can be also used as shape blending, morphing, or metamorphosis which is represented by a series of “in-between” curves generated by matching the generators curves’ normal vectors. In other words, a ruled surface is generated using the inputs as directrices and each iso-parametric curve on the surface constitute a stage of the metamorphosis.

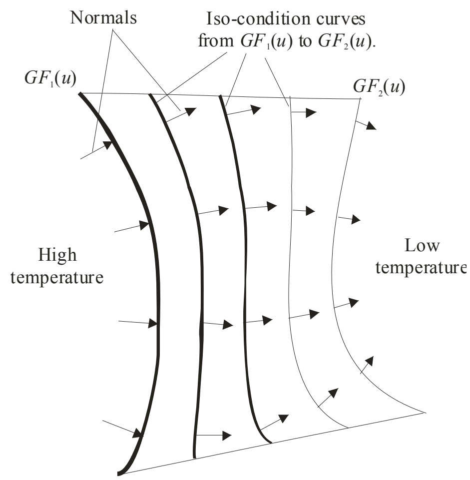

Fig. 1: Material (property) blending between two material governing features $GF_1(u)$ and $GF_2(u)$.

To be able to blend the material between two or more generators along their normal direction, the normal vectors of the material governing features must match. While matching the normal vectors, the following conditions must be met for smooth transition:

a) The connecting normal lines must not self-intersect.
b) The length of the ruling lines must be minimum possible.
c) The end points of each ruling line must be matching i.e. they must have some common or similar properties.

This problem can be generalized at generating ruling surface between two directices. A naïve way of constructing the ruling lines is by parametrically connecting the points on the two directrices. The rationale here is that both end points of each ruling line have the same parameter values. This does not guarantee a non-twisted ruled surface, particularly in case of directrices given as closed curves. The surface may also become self-intersecting and therefore unsuitable for metamorphosis, as also pointed out by Elber [3] and Surazhsky et al [18]. Therefore, other sophisticated methods are required to find the "best" set of ruled lines which satisfy all those conditions mentioned above.

In the literature, several methods have been proposed, mostly catering to specific applications. For shape blending, attention is focused on establishing matching between the input curves based on common local properties such as position, edge/arc length, angle, parameter, tangent and curvature. Sederberg and Greenwood [15] established vertex-to-vertex matching based on locations and angles at the vertices of 2-D polygonal curves. The intermediate shapes are obtained by linear interpolation of the matched vertices. The method also tries to avoid local self intersections. A similar method has been used by the same authors in [16] where the inputs are given as piecewise polynomial curves.

When combinations of some of the common characteristics are found over a range of the input curves, they are referred to as "features." Although the exact definitions of "features" vary by methods, the common goal is to establish matching between the features to retain the characteristic features of the objects during the blending process. Hui and Li [6] used 2D shapes composed of curve segments and defined rules in order to identify features. They developed algorithms to match the features based on their positions and shapes.

The approach of Cohen et al [2] is to establish matching between two given C1 continuous parametric curves based on tangent maps. To construct a set of non-self-intersecting ruling lines, one curve is reparameterized with respect to the other by means of formulating an optimization problem. The objective is to maximize the sum of the dot products between the unit tangents to the curves at the matched points. Constraints are specified to avoid local self intersection. An approximate solution to the problem is obtained by first discretizing the curves into piecewise polygonal curves and subsequently solved using dynamic programming. However, the method is explained in detail for matching open curves only, where matching of the end points of the curves are constrained. For matching closed curves, the authors suggest using a k-shift of tangents approach.

Johan et al [7] also uses optimization to match equally spaced sampled points from two given input curves. The objective here is to minimize the sum of a cost function which is composed of a weighted sum of the difference of angles and difference of parameters at the matched vertices. Dynamic programming is used to solve the problem. The overall approach, as the authors mentioned, is an extension of [15] and [2].

It is argued that features identification and matching is important so that the matched features transform into each other smoothly while ensuring no deformation or distortion of the in-between curves [13, 14]. In most cases of shape blending, feature identification is easy because the input shapes already contain some similarities between themselves. Although this helps in achieving visually pleasing animations, the resulting ruled surface may not always be twist or stretch free. Moreover, in cases of input curves that exhibit very little or no similarity, it may be very difficult to identify common features. Therefore, in such applications, other local properties are used for establishing the matching.

In [18], the authors presented optimal boundary triangulations of interpolating ruled surfaces. This paper presents an algorithm for constructing an optimal triangulated ruled surface that interpolates two discrete directrices. The developed algorithms are used to establish equivalence between the optimal triangulation and the single-source shortest path problem on the graph.

In the use of ruled surfaces in CAD/CAM applications, such as adaptive ruled layers approximation for multi-axis machining in rapid prototyping, the directrices are given as two consecutive piecewise linear curves (or, polygonal contours). In Koc and Lee [8], the directrices are obtained from slicing a stereolithography (STL) file. The authors find the matching points so as to minimize the twisting of the resulting ruled surface. Both curves are reparameterized by means of inserting points so that one-to-one matching can be achieved.

This paper proposes a new methodology for matching between two material governing features [13] in closed freeform B-spline curves. The conditions for matching of two points, each on one of the given curves, are specified as follows: (i) the sum of the angular distances between the normals to the curves at the matching points must be minimized, and (ii) the sum of the Euclidean distances between the matching points must be minimized. The normal vector matching criterion has been selected because the matching is not dependent on any feature identification. The problem is formulated as a continuous optimization problem. A Greedy Ruling Line Construction (GRLC) method is proposed to find the global optimum discrete matching that establishes point-to-point correspondence between all vertices. The proposed method guarantees that the total number of ruling lines generated in the process is also minimized. A vertex insertion method is also proposed to ensure one-to-one correspondence.

2. MATERIAL GOVERNING FEATURES

In this paper, the directrices used for generating the ruled line set are given freeform B-spline curves [12]. B-spline curves are chosen because of their superiority over Bezier curves. The proposed method utilizes the normals at the sampled points for matching. Therefore it is necessary that both the curves be at least $\mathbf{C}^1$ continuous so that well-defined normals exist at all points on the curves. In cases where the inputs are given as point sets or polygonal contours, one can fit a closed B-spline curve that satisfies at least $\mathbf{C}^1$ continuity.

A B-spline curve of degree *p* is defined as follows [12]:

$$\mathbf{C}(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i \tag{1}$$

where, the $\{\mathbf{P}_i\}_{i=0,\ldots,n}$ are the control points and $\{N_{i,p}(u)\}$ are the *p*-th degree B-spline basis functions defined on the knot vector $U = \{u_0, \ldots, u_h\}$, where, $h = n + p + 1$;

The input to the problem consists of two non-self intersecting, closed, and at least $\mathbf{C}^1$ continuous, planar, B-spline curves, $C_1(u)$ and $C_2(v)$ with degrees p_1 and p_2 , respectively. The degrees p_1 and p_2 need not be the same. Both curves are non-self intersecting (except at the joining point) and they do not intersect each other, i.e. the following conditions are satisfied:

$$
\begin{array}{ll}
C_1(u_i) \neq C_1(u_j) & u_i \in (u_{p_1}, u_{h_1-p_1}); u_j \in (u_{p_1}, u_{h_1-p_1}); u_i \neq u_j \\
C_2(v_i) \neq C_2(v_j) & v_i \in (v_{p_2}, v_{h_2-p_2}); v_j \in (v_{p_2}, v_{h_2-p_2}); v_i \neq v_j \\
C_1(u) \neq C_2(v) & u \in [u_{p_1}, u_{h_1-p_1}]; v \in [v_{p_2}, v_{h_2-p_2}]
\end{array} \tag{2}
$$

As have been mentioned before, the ruled surface constructed by parametrically connecting the points on the curves $C_1(u)$ and $C_2(v)$ is not guaranteed to be twist-free. Therefore, it is required that $C_2(v)$ be re-parameterized with respect to $C_1(u)$ before generating the surface and the ruling lines. The re-parameterization domain is same as that of the curve $C_1(u)$, i.e. $u \in [u_{p_1}, u_{h_1-p_1}]$. Let this domain be renamed as $u \in [u_{low}, u_{high}]$. If the re-parameterized version of $C_2(v)$ is denoted as $C_2(v(u))$, then a ruling line $RL(t)$ with end points $p = C_1(u_k)$ and $q = C_2(v(u_k))$ is given as:

$$RL(t) = tp + (1-t)q; \qquad t \in [0,1] \tag{3}$$

The ruling line $RL(t)$ in Eqn. (3) is represents a match between its end points. Therefore, it is required that the curve $C_2(v)$ is re-parameterized with respect to $C_1(u)$ such that the end points of all the ruling lines are matched. A ruling line $RL(t)$ is introduced only when the following two conditions are satisfied:

(a) The inner product of the unit normal vectors to the curves $C_1(u)$ and $C_2(v(u))$ at p and q respectively is maximized. The maximum value of the inner product is equal to one when the both unit normals become collinear with the ruling line, rendering p and q perfectly matched.

(b) The Euclidean distance between the points p and q, $\left|\overrightarrow{p-q}\right|^2$ is minimized. This condition is required to prevent twisting of the ruled surface.

To mathematically express these two conditions, a function *f* can be defined that assigns a value to each ruling line as follows:

$$f(p,q) = \frac{\langle N(p), N(q) \rangle}{\left|\overrightarrow{p-q}\right|^2} \tag{4}$$

Without loss of generality, it can be assumed that the curves $C_1(u)$ and $C_2(v)$ lie on the *xy*-plane and $C_1(u)$ is totally contained inside $C_2(v)$. The normals are calculated as follows:

$$N(p) = \frac{C_1'(u_k)}{\left|C_1'(u_k)\right|} \times k \text{ and } N(q) = \frac{C_2'(v(u_k))}{\left|C_2'(v(u_k))\right|} \times (-k) \tag{5}$$

where $\mathbf{k}$ is the unit vector in the positive z-direction.

Now the global curve matching problem can be formulated as a continuous optimization problem where the objective is to maximize the sum of the function *f* over the entire parameter domain of the curve $C_1(u)$, i.e. $u \in [u_{low}, u_{high}]$.

$$Maximize \int_{u_{low}}^{u_{high}} \frac{\left\langle N\left(C_1(u)\right), N\left(C_2(v(u))\right)\right\rangle}{\left|\overrightarrow{C_1(u) - C_2(v(u))}\right|^2} \tag{6}$$

Subject to the following constraints:

1. So that $C_2(v(u))$ is a valid re-parameterization, two consecutive ruling lines $\overline{C_1(u_i)C_2(v(u_i))}$ and $\overline{C_1(u_{i+1})C_2(v(u_{i+1}))}$ should not intersect each other, i.e. $v(u_i) < v(u_{i+1})$
2. No ruling line $\overline{C_1(u_i)C_2(v(u_i))}$ should intersect the directrices $C_1(u)$ and $C_2(v)$

Note that no initial re-parameterization is specified as a constraint because the curves are not open.

The input curves $C_1(u)$ and $C_2(v)$ are re-parameterized into approximating polygons (or piecewise linear curves). For each polygon, the number and relative locations of the vertices are governed by the desired accuracy of approximation. In general, the higher the number of vertices, the better is the approximation. Matching is established between the vertices of the polygons.

$C_1(u)$ is re-parameterized into a set of (a + 1) points **P** as follows:

$$P = \{p_i\}_{i=0,\dots,a}\text{ ; where, } p_i = C_1(u_i)\text{ ; } u_i \in [u_{p_1}, u_{h_1-p_1}]\text{ ; } u_i < u_{i+1}\text{ ; } u_0 = u_{p_1}\text{ ; } u_a = u_{h_1-p_1}$$
$$p_0 = C_1(u_0) = C_1(u_{p_1}) \text{ and } p_n = C_1(u_a) = C_1(u_{h_1-p_1}) \tag{7}$$

Similarly, $C_2(v)$ is re-parameterized into a set of (b + 1) points Q as follows:

$$Q = \{q_j\}_{j=0,\dots,b}\text{ ; where, } q_i = C_2(v_j)\text{ ; } v_j \in [v_{p_2}, v_{h_2-p_2}]\text{ ; } v_j < v_{j+1}\text{ ; } v_0 = v_{p_2}\text{ ;}$$
$$v_b = v_{h_2-p_2} \tag{8}$$
$$q_0 = C_2(v_0) = C_2(v_{p_2}) \text{ and } q_b = C_2(v_b) = C_2(v_{h_2-p_2})$$

Note that the parameters u_i 's and v_j 's are not necessarily evenly distributed in their respective domains. Moreover, a and b are not assumed to be equal. While re- parameterizing, at every point p_i and q_j , the unit normals $N(p_i)$ and $N(q_j)$ are also calculated. Now the re-parameterized version of the function f in Eqn. (4) becomes:

$$f(p_i, q_{j(i)}) = \frac{\langle N(p_i), N(q_{j(i)}) \rangle}{\left|\overrightarrow{p_i - q_{j(i)}}\right|^2} \tag{9}$$

Therefore, the re-parameterized approximation of the original continuous optimization problem in Eqn. (6) can be expressed as:

$$Maximize \sum_{i=0}^{a}\sum_{j=0}^{b} \frac{\langle N(p_i), N(q_j) \rangle}{\left|\overrightarrow{p_i - q_j}\right|^2} \tag{10}$$

Subject to the following constraints.

1. So that $j(i) < j(i+1)$ is a valid discrete re-parameterization, two consecutive ruling lines $\overline{p_i q_{j(i)}}$ and $\overline{p_{i+1} q_{j(i+1)}}$ should not intersect each other, i.e. $j(i) < j(i+1)$.
2. No ruling line $\overline{p_i q_{j(i)}}$ should intersect the polygons, **P** and **Q**.

A good re-parameterization is where the total number of vertices and their positions relative to each other are judicially chosen. The purpose is to approximate the curve within the tolerance with a minimum number of vertices. Therefore an adaptive subdivision scheme is employed.

3. OPTIMIZATION FOR MATCHING REPARAMETERIZED GEOMETRIC FEATURES

This section describes the solution methodology to the discrete optimization problem for ruled line construction in Eqn. (10). An approach named *Greedy Ruled Line Construction* (*GRLC*) is proposed to find a set of ruling lines, *RL* that maximizes the objective function in Eqn. (10). The underlying principle of the *GRLC* approach is to construct the set

RL by adding at a time one ruling line which increases the objective function value the most. At every stage of *RL* construction, the ruling line added to *RL* is chosen from a set of candidates named *RL_candidate_list*. Each candidate in *RL_candidate_list* is called a *maximum valued ruling line* $(MVRL_i)$ which satisfies both constraints 1 and 2 in Eqn. (10). Construction of *RL* continues until all the vertices on both polygons **P** and **Q** are connected by at least one ruling line. It can be proved that this greedy approach guarantees the global optimal solution.

A maximum valued ruling line $(MVRL_i)$ represents the best match for a given vertex $\mathrm{p}_i \in \mathrm{P}$. If the ruling line $\overline{\mathrm{p}_i \mathrm{q}_j}$ satisfies both constraints in Eqn. (10) and at the same time so happens that $f(\mathrm{p}_i,\mathrm{q}_j) = \max\left\{f(\mathrm{p}_i,\mathrm{q}_j)\right\}_{j=0,\ldots,b}$ is true, then $\overline{\mathrm{p}_i \mathrm{q}_j}$ is designated as the $MVRL_i$. Since at every stage, a new ruling line is added to *RL*, the $MVRL_i$ may not always remain the same for the same p_i. This is the reason why the *RL_candidate_list* is emptied and reconstructed at every stage of *RL* construction. Below, the methods are described on how $MVRL_i$ is found while satisfying both the constraints. To describe these two methods in the general scenario, it is assumed that $MVRL_i$ is found while there are already some ruling lines in *RL*, none of which has p_i as an end point. In other words, *GRLC* method has already progressed to a stage when $MVRL_i$ will be one of the candidates in the *RL_candidate_list* and will be added to *RL set*.

The first constraint is met by a visibility checking method as defined below.

Definition: Visibility – A point $\mathrm{q}_j \in \mathrm{Q}$ is visible to p_i if the ruling line $\overline{\mathrm{p}_i \mathrm{q}_j}$ does not intersect any edges of either of the polygons P and Q.

While finding the $MVRL_i$ of the given vertex p_i, only those vertices in Q are considered which are visible to p_i. A function **IsVisible(** $\mathrm{p}_i,\mathrm{q}_j$) is defined which returns *true* only if q_j is visible to p_i. Let V_i be a subset of **Q** so that all vertices in V_i are visible to p_i.

$$\mathrm{V}_i = \left\{\mathrm{q}_j \in \mathrm{Q} \,\middle|\, \mathtt{IsVisible}(\mathrm{p}_i,\mathrm{q}_j) = true\right\}_{j=0,\ldots,b} \tag{11}$$

Fig. 2 shows how a ruling line $\overline{\mathbf{p}_i \mathbf{q}_j}$ enters RL. Fig. 2(a) explains Equation (11) where the vertex is connected by broken ruling lines to all the vertices in $\mathbf{V}_i$. The $MVRL_i$ is one among the broken lines, but not identified yet.

In order to meet the second constraint in Eqn. (10), all vertices in **P** are traversed in counterclockwise direction starting from p_{i+1} and ending at p_{i-1}. While traversing, let $\mathrm{p}_{i'}$ and $\mathrm{p}_{i''}$ be the first and last connected vertices encountered, i.e. $\overline{\mathrm{p}_{i'} \mathrm{q}_{j'}}$ and $\overline{\mathrm{p}_{i''} \mathrm{q}_{j''}}$ are two ruling lines already in *RL*. Then all the ruling lines $\overline{\mathrm{p}_i \mathrm{q}_j}; j'' \le j \le j'$ satisfy constraint 2.

A function **IsValid(** $\mathrm{p}_i,\mathrm{q}_j$) is defined which returns *true* only if $j'' \le j \le j'$. The function is so named because each $j(i), j'' \le j(i) \le j'$, qualifies to represent the discrete version of the valid re-parameterization $v_{j(i)}$. Let, R_i be a subset of **Q** containing all $\mathrm{q}_j, j'' \le j \le j'$.

$$\mathrm{R}_i = \left\{\mathrm{q}_j \in \mathrm{Q} \,\middle|\, \mathtt{IsValid}(\mathrm{p}_i,\mathrm{q}_j) = true\right\}_{j=0,\ldots,b} \tag{12}$$

Now $MVRL_i$ can be found from the set $\mathrm{V}_i \cap \mathrm{R}_i$ as shown in Fig. 2(b). If $MVRL_i = \overline{\mathrm{p}_i \mathrm{q}_j}$ then, by definition of $MVRL_i$, the condition $f(\mathrm{p}_i,\mathrm{q}_j) = \max\left(\left\{f(\mathrm{p}_i,\mathrm{q}_j)\Big|_{\mathrm{q}_j \in \mathrm{V}_i \cap \mathrm{R}_i}\right\}\right)$ holds.

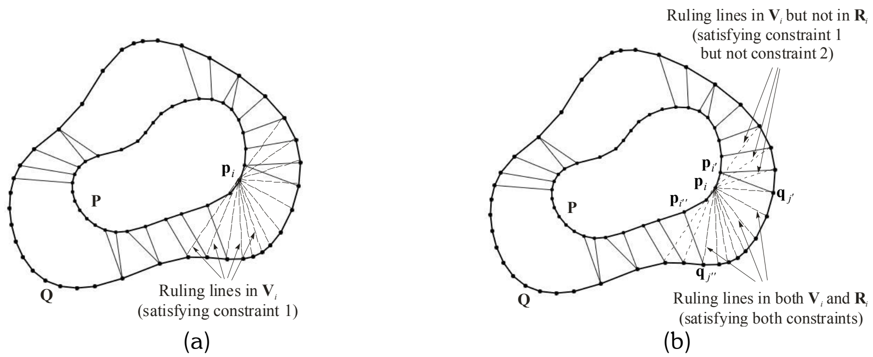

Fig. 2: (a) Visible vertices to $\mathbf{p}_i$ satisfying first constraint (b) finding ruling lines satisfying both constraints 1 and 2.

Since both polygons are closed, no initial match conditions are specified. The greedy approach constructs the *RL_candidate_list*. Among all candidates in *RL_candidate_list*, the one with the maximum value is chosen and added to *RL*. The end vertices of this ruling line are marked as connected. Then the candidate set is emptied and a fresh set is reconstructed excluding all the ruling lines that are already in *RL*. Again the "best" one is chosen from the set and stored in *RL*. This is performed repeatedly until all the vertices of both polygons are connected by at least one ruling line.

It is possible that there can exist one-to-many matching of the vertices. This happens when the curvature of the curves differ significantly at the vertices. It is neither intuitive nor visually pleasing that one vertex on one directrix matches with many points on the other directrix. This means that while material blending, one vertex of the source will metamorphose into an arc on the target and vice versa. Therefore, a vertex insertion method is developed that "spreads out" the ruling lines so that all ruling lines have one-to-one correspondence. This is achieved by inserting more vertices near the vertex with degree more than one and connecting each of them with the ruling lines as shown in Fig. 3.

If p_i and p_{i+1} are two consecutive vertices on P, both of which may have degrees more than one. Let, out of all the ruling lines connected to p_i, the line $\overline{\mathrm{p}_i\mathrm{q}_j}$ has the highest function value. Similarly, out of all the ruling lines connected to p_{i+1}, the line $\overline{\mathrm{p}_{i+1}\mathrm{q}_{j+k}}$ has the highest function value. Therefore, the $k-1$ points between q_j and q_{j+k} have to be detached from their connections on **P** and be relocated because the two ruling lines $\overline{\mathrm{p}_i\mathrm{q}_j}$ and $\overline{\mathrm{p}_{i+1}\mathrm{q}_{j+k}}$ are the locally best matches. The points $\mathrm{q}_{j+1},\ldots,\mathrm{q}_{j+k-1}$ were connected to either p_i or p_{i+1} because there were no other points available in-between.

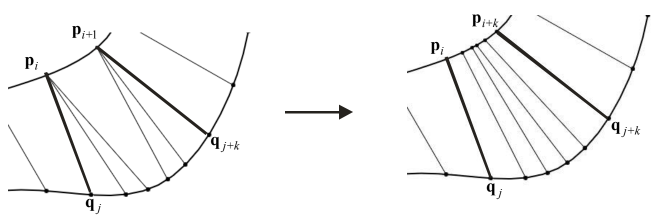

Fig. 3: The ruling lines with highest function value are identified and inserted.

The insertion of points between p_i and p_{i+1} are done according to proportional parametric increments of q_j and q_{j+k}. Let the parameter associated with q_{j+l} be v_{j+l}, $l = 0,\ldots,k$. Since $k-1$ vertices are going to be inserted between p_i and p_{i+1}, index $i+1$ will increase to $i+k$. Let the parameters associated with p_i and p_{i+k} be u_i and u_{i+k}, respectively. Then $k-1$ points are sampled from $\mathbf{C}_1$ between p_i and p_{i+k} as follows.

$$p_{i+l} = C_1(u_{i+l}) \qquad \textbf{where,} \quad u_{i+l} = u_i + (u_{i+k} - u_i)\frac{v_{j+l} - v_j}{v_{j+k} - v_j}; l = 1,\ldots,k-1 \tag{13}$$

Using the proportional parameter increment approach of inserting points is only a discrete way of re-parameterization only. The vertex insertion is done in two stages. The first stage involves marching along C_1 and inserting points using the equation 10. Later, at the second stage points are inserted on C_2 using the same procedure discussed above.

4. IMPLEMENTATION AND EXAMPLES

The above methodology is implemented in Microsoft Visual C++. OpenGL library functions are used for visualization. GLUI library functions are used for user interfacing. In Fig. 4, the *RL* set contained 44 ruling lines, and the value of the objective function was 5.00185. The redundant ruling lines are identified and removed which resulted in a reduction in the objective function value to 3.43399. After the two pass vertex insertion, the final *RL* set contains 44 ruling lines, each of which represents one-to-one matching. The objective function value increased to 3.5000.

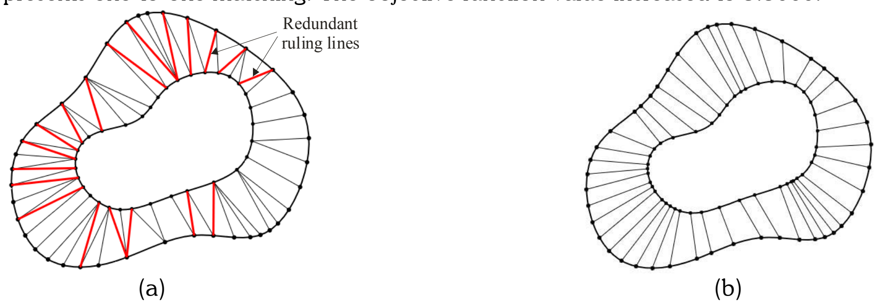

Fig. 4: (a) Ruling line set RL with redundant lines (b) final ruled line set after two passes of point insertion. Each ruling line represents one-to-one matching between the directrices.

In order to compare the results of Fig. 3, both directrices $\mathbf{C}_1$(u) and $\mathbf{C}_2$(v) are re-parameterized using the *uniform parameterization*. The vertices are connected one-to-one parametrically to generate the ruling lines, as shown in Fig. 5 (a). The result is a twisted ruled surface, and the value of the objective function is a low 1.18029. As a naïve approach to untwist the surface, the ruling lines are shifted by a constant number of vertices and the objective function values are recorded. The highest objective function value observed is 3.01518 for a 4-vertex shift, as shown in Fig. 5 (b). This is much less than the result in Fig. 4.

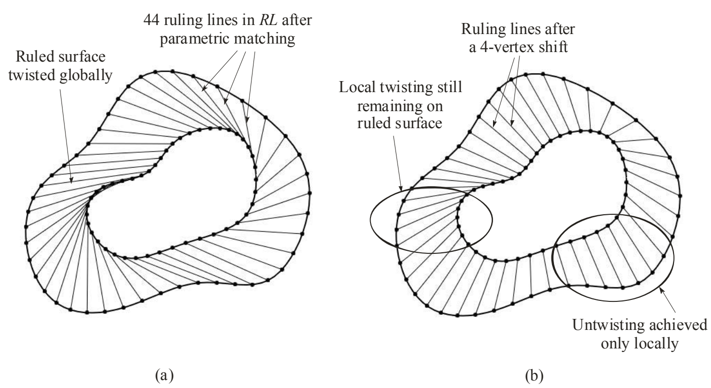

Fig. 5: Comparison of results with *GRLC* with parametric matching, (a) globally twisted ruled surface produced, (b) global untwisting is not guaranteed by *k*-shift of ruling lines.

The directrices can be considered as key curves and a series of metamorphosis curves (iso-property curves) for heterogeneous object modeling can be generated between them by using the ruled lines. Fig. 6 shows the example where the ruling line set is divided into 5 uniform segments (material blending).

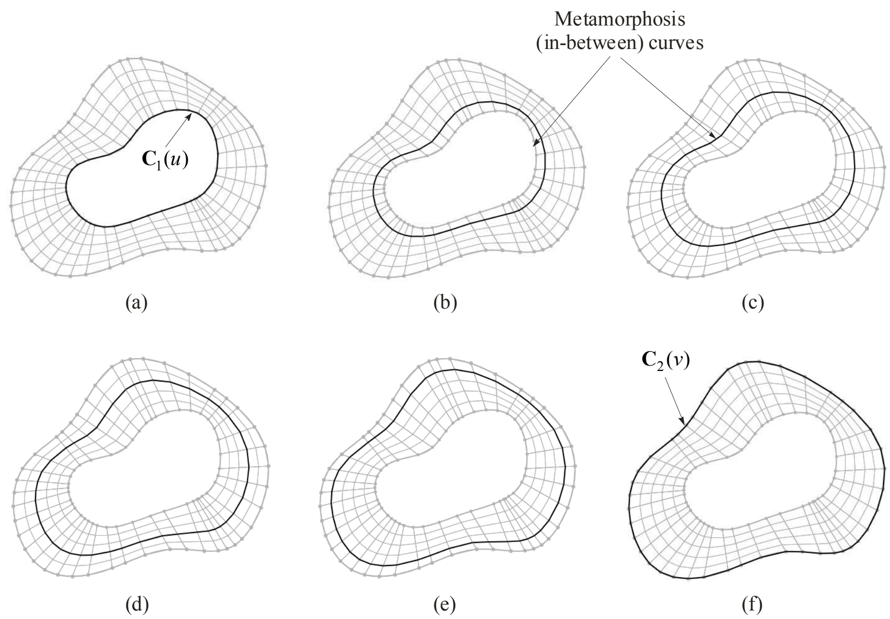

Fig. 6: Metamorphosis between the curves $\mathbf{C}_1$ and $\mathbf{C}_2$ using ruled line set with *five intermediate curves, i.e.* $j = 0\ldots 5$, (a) $j = 0$ is same as the first generator curve, (b) $j = 1$, (c) $j = 2$, (d) $j = 3$, (e) $j = 4$, (f) $j = 5$, is same as the second generator curve.

The matching method described in this paper can be directly used to match the material governing features for designing a heterogeneous object [13]. In order to perform material blending, a lofting is done between the material governing features, where it was assumed that the freeform curves representing the features are already re-parameterized and therefore already matched. Therefore, no twisting or stretching is observed in the "elements." The discrete curve matching approach can be considered a generalization of the lofting process presented in [13].

5. CONCLUSIONS

In this paper, a new method is proposed to generate matching lines between two material governing features (directrices) for heterogeneous object modeling. Each connecting line represents a match between two points on the directrices. A new method named Greedy Ruling Line Construction (GRLC) is developed to match the directices such that their material features (properties) are blended each other along their matched normal direction. The developed mGRLC method generates non-self-intersecting and non-twisted connecting material governing features. By subdividing the connecting lines into equal number of segments, a series of intermediate iso-property curves are obtained to represent the material metamorphosis between the governing material features. The developed methods can also be used in several other applications such as getting smooth transition between two given 2-D curves for animation, shape blending, morphing, approximation of 3-D solid object models from planar curves, multi-axis NC machining of ruled surfaces.

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