Computer-Aided Design and Applications

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Flank Millable Surface Design with Conical and Barrel Tools

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ABSTRACT

This paper presents a method for creating a flank millable surface for conical and barrel tools. The approach is a generalization of an earlier approach for cyclindrical tools, where the tool is moved along two guiding curves and a NURBS approximation is created for the swept surface. The cylindirical method is first extended to conical tools, and then to general surfaces of revolution, although it has only been implemented for barrel tools. With this method, it is possible to create surfaces that can be machined accurately. These surfaces can be used in analysis software without resorting to approximations and result in better designs.

Keywords: flank milling, cylindrical and barrel tools, NURBS surfaces.
DOI: 10.3722/cadaps.2008.461-470

1. INTRODUCTION

Flank milling is an important CNC machining technique. When designing tool paths to flank mill a surface, typically a piecewise ruled surface is used to approximate the design surface, and a tool path is created by positioning the CNC tool on the guiding rails. However, in 5-axis machining, the machined surface is not a ruled surface. Thus, two approximations are being made: first, a piecewise ruled approximation to the design surface, and second, the ruled surface itself is only an approximation to what is being machined. For analysis purposes (such as required by Computational Fluid Dynamics), the piecewise ruled surface is used instead of the original design surface, since it is a better approximation to the machined surface. However, for accurate analysis, this piecewise ruled surface must be composed of many small pieces.

In an earlier paper [6], we presented a method for designing a surface with cylindrical tools that can be flanked milled exactly (or at least up to machining tolerance). The method basically finds NURBS curve approximations to several grazing curves (curves on the swept surface) and then constructs a tensor product NURBS surface containing these curves. This flank millable surface can be used in the analysis process, reducing the number of pieces required and potentially decreasing machining time. In this paper, we extend the method to general tools of revolution, and in particular, to conical and barrel tools. While we use NURBS profile curves, note that a non-NURBS profile can be approximated to a high tolerance with a NURBS profile. Finally, although we use Bedi et al.'s tool positioning method [1] in our examples, the method should work for any tool positioning method.

2. BACKGROUND

In CNC machining, a tool path is comprised of a sequence of tool positions. Each tool position defines the machine parameters for one position of the tool. The NC machine then linearly interpolates the parameters from one tool position to the next. As a sequence of tool positions are executed, the tool cuts a surface in the stock. At any given instance, there will be one curve on the tool that will be a curve on the machined surface (unless it is cut away in a later pass). This curve is known as a *grazing curve* [9]. Our method finds a NURBS curve approximation to each grazing curve and then constructs a NURBS surface containing these curves. While the tool itself has a complex cutting shape, because it is spinning at high speed it is typically treated as a surface of revolution for many analysis purposes and we do so in this paper.

We assume the reader is familiar with NURBS and Bézier curves and surfaces, which are core to our method. Here we will only review the rational Bézier representation of a circular arc. For more details on NURBS and Bézier curves and surfaces, see any spline textbook such as [3].

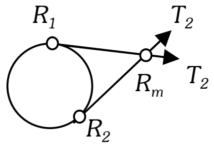


Fig. 1: Finding the rational quadratic Bézier representation a circular arc.

Referring to Fig. 1, given two points R_1 and R_2 on a circle with tangents T_1 and T_2, the three control points of a rational quadratic Bézier representation of the circular arc from R_1 to R_2 are as follows: The first and last control points are R_1 and R_2 and the middle control point R_m is the intersection of the two tangents. The weights of the first and last control points are 1, and the weight of R_m is the cosine of the half angle spanning $R_1R_mR_2$ [3].

2.1 Grazing Curves

At any tool position, the grazing curve is the set of points on the tool whose motion is perpendicular to the surface normal of the tool at those points. The grazing points on a surface of revolution can be computed using the following method, from [8]. We will assume that the tool is being swept along two guiding curves, *T(u)* and *B(u)*.

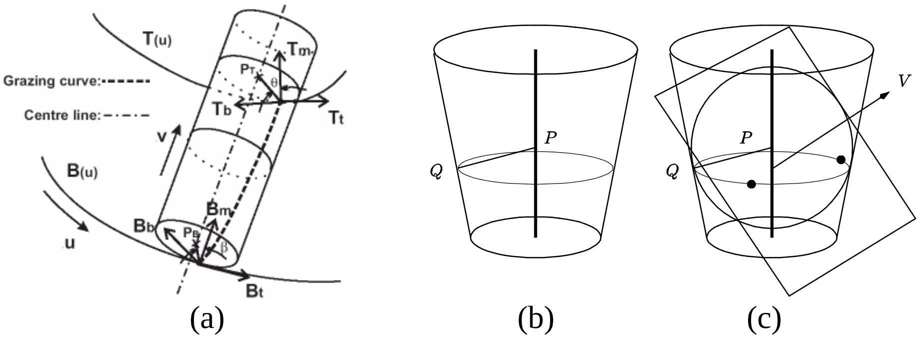


Fig. 2: (a) A tool sweeping along two guiding curves; (b,c) computing grazing points.

As shown in Fig. 2(a), if the velocity at point P_T on the tool axis is V_T and at point P_B (also on the tool axis) is V_B, then the velocity between P_B and P_T along tool axis direction can be linearly interpolated and is given by

$$V=(1-\upsilon)V_B + \upsilon\, V_T, \qquad 0 \le v \le 1.$$

Here V_T is the first derivative of the guiding curve *T(u)* at the contact point on this curve. For the solid body of the cutting tool, the velocity of this point should be the same as P_T. Similarly, V_B is the first derivative of the guiding curve *B(u)* at the bottom contact point. The coordinate between P_B and P_T along the tool axis can also be linearly interpolated and is given by

$$P=(1-\upsilon)P_B + \upsilon\, P_T. \qquad 0 \le v \le 1.$$

Given the velocity of any point on the tool axis, we can compute the grazing points on any cross section of a general surface of revolution as follows. For any point *Q* on the generating profile curve for the surface of revolution, find the point *P* on the axis of revolution such that *P–Q* is perpendicular to the profile curve at *Q* (Fig. 2(b)). Take the plane through *P* perpendicular to direction of motion *V* of *P*, and intersect this plane with the circle of revolution through *Q* (Fig. 2(c)). As proven in [8], these intersection points will be grazing points on the tool. By using a set of points on the profile curve, one obtains a set of points and piecewise linear approximation to the grazing curve.

2.2 Flank Millable Surface Design Using Cylindrical Tools

In an earlier paper [6] we designed a flank millable surface for a conical tool by constructing a sequence of Bézier curves that approximated the grazing curves at several tool positions. That method proceeds as follows. At any tool position, the tool touches each guiding curve at one location. Call these contact points T_1 and B_2 (we give these points different subscripts since the contact points typically occur at different locations on the circular cross section of the cylinder). The grazing curve will extend from T_1 to B_2. For a cylindrical tool, if we project the grazing curve into a plane perpendicular to the tool axis, then we get a circular arc. Consider the projection of the grazing curve into the planes containing T_1 and B_2 and at a third plane lying between these two planes. This gives three circular arcs. Each arc can be represented by a rational quadratic Bézier curve (Fig. 3). Note that the first control points of all three arcs (T_1, T, and T_2) lie on a line on the cylinder. Likewise the last control points of each curve lie on a second line on the cylinder. The middle control points also lie on a line, and although this line is not on the cylinder, it is parallel to the tool axis and the other two lines.

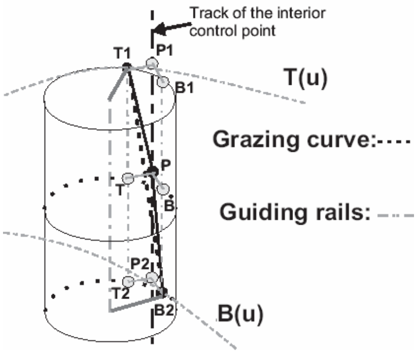


Fig. 3: A grazing curve and its control points on a cylindrical tool.

We used a quadratic rational Bézier curve (the dark dashed curve in Fig. 3) to approximate the given grazing curve with control points T_1, P, and B_2. T_1 is the top contact point between the cylindrical tool and the guiding curve $T(u)$, and B_2 is the bottom contact point between the cylindrical tool and the guiding curve $B(u)$. We need to find the interior control point P, which is done as follows.

Call the middle control point of the top arc P_1 and call the middle control point of the bottom arc P_2. We will move P along the line segment $\overline{P_1P_2}$ and find the location of P that results in the rational Bézier curve (T_1, P, B_2) with the smallest error between it and the grazing curve (where the first and last weights are 1, and the center weight is the weight of P_1 in the circular arc T_1,P_1,B_1 [3]).

Thus, for cylindrical cutting tools, the movement of the interior control point of the 3D Bézier curve is always along the locus of the interior control points of the 2D Bézier curves. The locus is a line segment, $\overline{P_1P_2}$ in Fig. 3, that is parallel to the cylindrical tool axis. For better accuracy a NURBS curve with more interior control points can be used in the circular arc representation, in which case different tracks for each interior control point can be found, each of which is a line and parallel to the tool axis. By moving each interior control point along its own track, a 3D Bézier or NURBS curve can be defined and the one with the smallest approximate error selected for use in surface design.

Our method constructed such curves at several tool positions and then constructed a tensor product NURBS surface that interpolated this sequence of curves. The tensor product surface was then shown to be flank millable to machine tolerance. See our earlier paper for details of the surface construction and for our analysis of the surface [6].

3. FLANK MILLABLE SURFACE DESIGN WITH CONICAL TOOLS

We will now generalize the method described in the previous section to conical tools. When a conical tool is used to machine a part, the grazing curve at each tool position lies on the conical tool surface (Fig. 4(a)) and its projection along the conical tool axis direction into a plane perpendicular to the tool axis is a 2D curve (Fig. 3(b)). However, this projection is not a circular arc as it was for cylindrical tools. Thus, a new technique is needed to find a NURBS representation of the 2D projection of the grazing curve and from there to find the NURBS representation of the grazing curve.

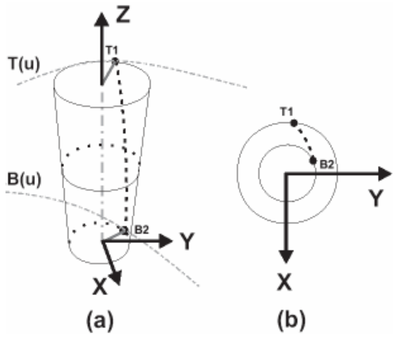


Fig. 4: (a) A conical tool with the grazing curve; (b) projection of grazing curve.

Initially a quadratic Bézier curve will be used to represent the grazing curve on the conical tool surface. More control points will be added in the subsequent surface error analysis. If a quadratic Bézier curve is used to define the grazing curve on a conical tool surface, then these top and bottom contact points, T_1 and B_2 (Fig. 4(a)), can be used as the two end control points of the quadratic Bézier curve and only its interior control point is left undecided.

To determine the interior control point, we modify the method used for cylindrical tools. The generatrix of the cone (the curve that is revolved around the cone axis to construct the surface of the cone) is not parallel to the cone axis, but we can project each point of the grazing curve on the cone surface along its generatrix direction to a plane that is perpendicular to the conical tool axis. A circular arc is obtained on this plane. Projection onto different intersection planes between the top and the bottom planes of the cone results in different circular arcs. Each 2D arc can be represented by a quadratic rational Bézier curve (Fig. 5). Our method places the control point P_x on the line $\overline{P_1P_2}$.

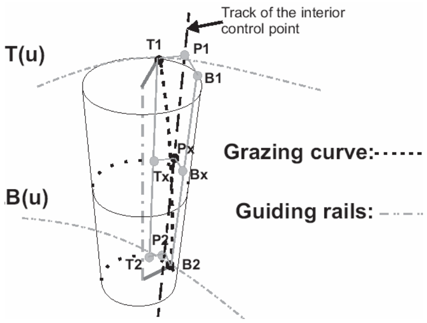


Fig. 5: The projection of the grazing curve and its control points.

This method is a generalization of the method used for cylindrical tools in Section 2.2 and the remaining details are essentially the same: the interior control point(s) position is moved up and down the segment $\overline{P_1P_2}$ to find the spline curve that best approximates the grazing curve. See [4] for additional details.

3.1 Example of Approximating a Grazing Curve on a Conicial Tool

An example is given to test the proposed method; this example is also used in the remaining tests in the paper. The control points for the top guiding rail are $P^T1=(75, 15, -5)$, $P^T2=(30, 30, -5)$, $P^T3=(0, 60, -5)$ (Fig. 6, left), for the bottom guiding rail are $P^B0=(60, 0, -45)$, $P^B1=(30, 30, -45)$, $P^B2=(15, 75, -45)$ (Fig 6, middle), with the two guiding rails shown superimposed on the right of Fig. 6.

For this example, the parameters of the conical cutter and the control points are

$$B_2 = [R_b \cos(\pi/6), R_b \sin(\pi/6), 0] \quad T_1 = [R_t \cos(\pi/3), R_t \sin(\pi/3), h]$$
$$V_b = [-R_b \sin(\pi/6), R_b \cos(\pi/6), 0] \quad V_t = [-R_t \sin(\pi/3), R_t \cos(\pi/3), 0]$$
$$w_t=1,\ w_x=\cos(\pi/12),\ w_b=1,\ R_t=6,\ R_b=4,\ h=45$$

where R_t and R_b are the top and bottom radius of the cone; w_t, w_x and w_b are the weights of the control points B_2, P_x and T_1; h is the effective contact length along the axis of the conical cutter; V_b and V_t are velocities at B_2 and T_1, where the directions are along line directions of each circle and their magnitudes are velocities.

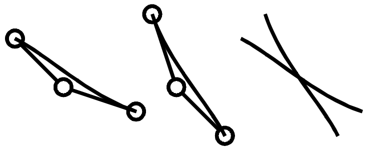


Fig. 6: Control points for top and bottom guiding curves (in mm).

Bedi et al.'s tool positioning method [1] is used to position this conical tool at a location on the grazing curves. The three control points, B_2, P_x, T_1, and their weights define a quadratic rational Bézier curve.. We calculated the deviation between the given grazing curve and the approximate Bézier curve with the interior control point P_x set to the middle of $\overline{P_1P_2}$; Fig. 7(a) is a plot of this error. The shape of the error curve is not symmetric, and the maximum error occurs near v=0.6 and is less than 0.135.

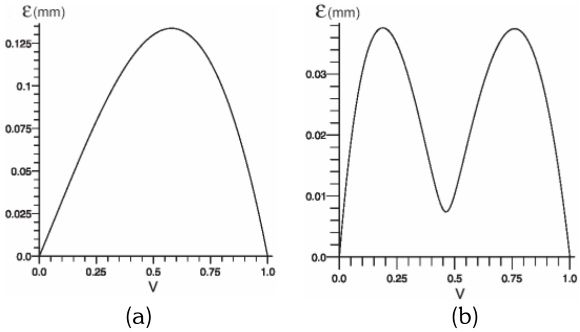


Fig. 7: Deviation along the grazing curve (a) P_x in the middle; (b) P_x not in the middle.

To reduce the maximum error, we optimized the position of P_x along $\overline{P_1P_2}$. The value of P_x that resulted in the smallest curve error was then used to define the grazing curve. The distribution of the error along the grazing curve is plotted in Fig. 7(b). This reduces the maximum curve error to around 0.038.

If the maximum error shown in Fig. 7(b) is still too high, more control points can be used to approximate the grazing curve by adding more control points to the circular arcs using knot insertion [3]. Fig. 8 shows a four control point rational quadratic B-spline curve being used to approximate the grazing curve.

3.2 Approximating a Grazing Surface

After creating the grazing curves approximations, a NURBS surface can be constructed. This NURBS surface is used to approximate the given grazing surface. The details to generate the NURBS surface (the flank millable surface) from the NURBS curves are described in the references [4, 6].

3.3 Examples of Flank Millable Surfaces for Conical Tools

We give some examples in this section to demonstrate the proposed flank millable surface design method with conical tools. The design starts with the two guiding curves shown in Fig. 6, where the degrees are two and where the knot vector for both of them is [0,0,0, 1,1,1].

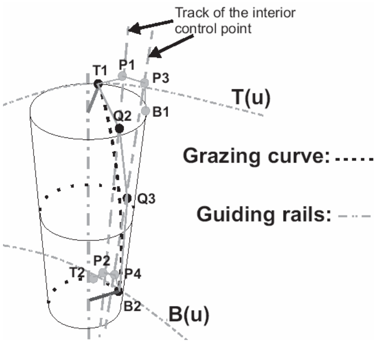


Fig. 8: A grazing curve with its four control points.

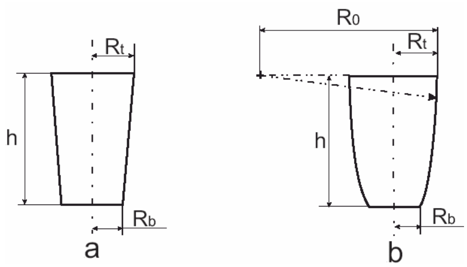


Fig. 9: (a) a conical tool. (b) a barrel tool.

The geometry of the conical tool is given in Fig. 9(a), with R_t=7, R_b=4 and h=50. Bedi et al.'s tool positioning method [1] is used to position the conical tool and generate the tool path. NURBS approximations to the grazing curves are created, and the grazing curves are used to build the flank millable surface [4]. Each grazing curve is defined with a rational Bézier curve. Three tool positions, u=0, u=0.48 and u=1, were used to find the interior control points and their weights of the flank millable surface (Fig. 10). A bi-quadratic rational Bézier surface that can be flank milled was constructed. The maximum error in our NURBS approximation to the grazing surface is around 0.048. If this error exceeds the user defined tolerance, more control points can be added in u and/or v directions to control the surface error.

To illustrate the effect of increasing the number of control points, the number of control points is increased from three by three to various values up to four by five and the corresponding NURBS surfaces are created. The maximum surface errors are tabulated in Tab. 1. From the table, it can be seen that the surface error is reduced when the number of control points are increased.

4. FLANK MILLABLE SURFACE DESIGN WITH TOOLS OF REVOLUTION

In the previous section, a method to design a flank millable surface with a conical tool was developed. We now generalize this method to tools defined by general surfaces of revolution, although we have only tested it on barrel tools. The key for the generalization is to model the grazing curve on the tool surface of revolution. If each grazing curve on the tool can be expressed as a NURBS curve, the flank millable surface can be built using the techniques developed in [4,6].

The key observation is that in the construction for cylindrical and conical tools, the six control points *T1*, *P1*, *B1*, *T2*, *P2*, *B2*, form a linear by quadratic NURBS patch *S* for the section of the tool between the two grazing points, T_1 and

B_2, where these two grazing points are also the first and last control points of our NURBS approximation to the grazing curve. The remaining control point P of the grazing curve was found by treating S's column of interior control points (P_1, P_2) as defining a linear curve and locating P along this line. This will also be our method for a general surface of revolution: we construct a NURBS patch S for the section of the tool bounded by the two points of contact (T_1, B_2), and then locate each interior control point of the grazing curve on the NURBS curve formed by an interior column of control points of S. The next section illustrates this process for a barrel tool.

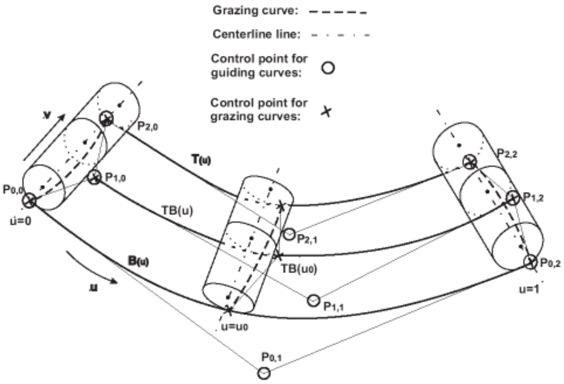


Fig. 10: Tool positioned at three locations on guiding curves.

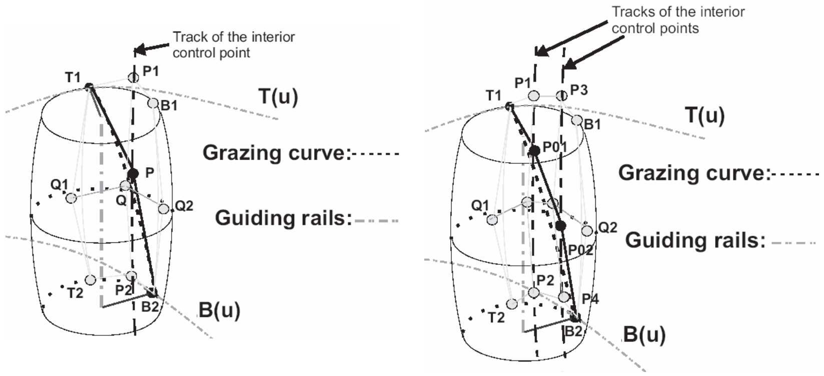
CPs	3×3	3×4	3×5	4×3	4×4	4×5
ε_{max}	0.048	0.046	0.044	0.0185	0.0155	0.011

Tab. 1: Errors for different NURBS surface. C.Ps: Control Points.

4.1 Modeling of the Grazing Curve on a Barrel Tool

For a tool with a generalized NURBS profile of revolution, the technique for conical tools is generalized and is illustrated in Fig. 11(a) where for simplicity we assume that the profile curve is the arc of a circle in NURBS form. Our method begins by first constructing a NURBS representation for the section of the tool spanned by the two grazing points, T_1 and B_2. For cylindrical and conical tools, only two control points are needed to represent the profile for the surface of revolution (a line). For a more general tool, more control points are needed for the profile curve. This complicates our construction somewhat, since in the NURBS patch for the portion of the tool of interest, we now need to construct control points that do not lie on the patch boundary, as well as find weights for these control points.

The control points of the patch are decided by the control points of the profiles, and the control points of the top and the bottom circular arcs. The two boundary columns of control points for this patch, T_1, Q_1, T_2, B_1, Q_2, B_2, are known. The three interior control points, P_1, Q and P_2 (whose position is determined so that each row of control points forms an arc) are used to define a NURBS curve approximation of the grazing curve from T_1 to B_2 (see Fig. 11(a)). The weights of control points P_1 and P_2 are set to the cosine of the half angle $\angle T_1P_1B_1$, and the weight of the control point Q equals to the product of cosine of the half angle $\angle P_1QP_2$ and the cosine of the half angle $\angle T_1P_1B_1$. (The weight of Q comes from multiplying the weight of the control point in the same row of the profile arc with the weight of the control point in the same column of the cross section arc). This NURBS curve (with control points P_1, Q, P_2) is the track $\overset{\frown}{P_1P_2}$ of the interior control point P of the NURBS curve used to approximate the grazing curve.



(a) (b)

Fig. 11: Grazing curve and its control points on a tool of revolution (a) one track (b) two tracks.

To build the NURBS representation of the grazing curve, the contact points T_1 and B_2 are used directly. The interior control point P is moved along the track $\overset{\frown}{P_1P_2}$ to find the position such that the maximum error between the grazing curve and the approximate NURBS curve is minimized. The weight of the interior point P is the cosine of the half angle $\angle T_1P_1B_1$, the weight of the profile arc.

4.2 Example of Approximating a Grazing Curve on a Barrel Tool

A simple example is given to exhibit the generalized curve design method. A barrel tool is used to test the proposed method. The geometry of the barrel tool is shown in Fig. 9(b) and the cutting tool parameters for this example are (see Fig. 11(a))

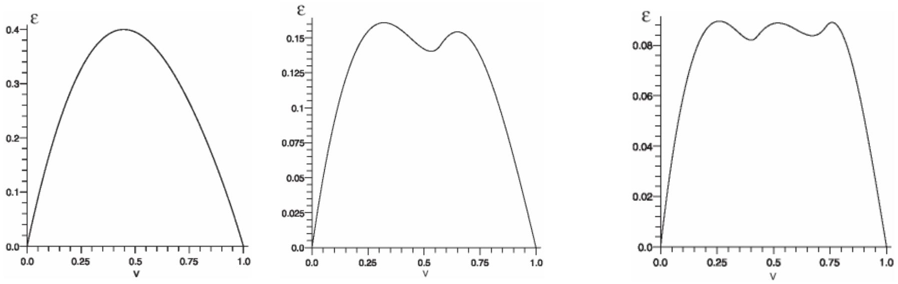
$$B_2 = [R_b\cos(\pi/6), R_b\sin(\pi/6), 0] \quad T_1 = [R_t\cos(\pi/3), R_t\sin(\pi/3), h]$$
$$V_b = [-R_b\sin(\pi/6), R_b\cos(\pi/6), 0] \quad V_t = [-R_t\sin(\pi/3), R_t\cos(\pi/3), 0]$$
$$w_t=1,\ w_x=\cos(\pi/12),\ w_b=1,\ R_t=5,\ R_b=8,\ R_0=339,\ h=45,$$

where R_t and R_b are radius of the top and the bottom of the tool; R_0 is the radius of the generatrix; w_t, w_x and w_b are the weights of the control points T_1, P and B_2; h is the effective contact length along the axis of the barrel cutter; V_t and V_b are velocities at points T_1 and B_2, where the directions are along line directions of each circle and their magnitudes are velocities.

Bedi et al.'s tool positioning method [1] was used to position this tool at a single location on the grazing curves. The ``projection'' of this grazing curve along the profile of the tool onto the top and the bottom planes of the barrel are 2D arcs. These arcs and the profile curves between them compose a patch on the surface of the barrel. The control net of this patch with weights can be calculated as described in Section 4.1 and in particular, the interior control points P_1, Q and P_2 are defined. These interior control points are used to construct a B-spline curve that is the track of the middle control point of the approximate grazing curve. In this example, the middle control point is P.

Using T_1, P and B_2, a NURBS curve is built to approximate the grazing curve. The error between the grazing curve and its approximate NURBS curve is calculated and is minimized by moving the control point P along the curve P_1, Q, P_2; we have plotted the error of this approximation to the grazing curve in Fig. 12(a).

From Fig. 12(a), it can be seen that the maximum curve error is around 0.4, which is too large for general engineering applications. To reduce the maximum curve error, more control points are needed. In this study, we increased the control points to four (Fig. 11(b)), and then to five (Fig. 11(c)). We used degree two NURBS curves to approximate the grazing curves for all three examples. The knot vectors were [0,0,0, 0.5, 1,1,1] for the four control point curve and [0,0,0, 0.33, 0.67, 1,1,1] for the five control point curve. We see from the plots that the maximum error decreases from 0.4 to 0.17 to 0.09 as the number of control points are increased from three to four to five.



(a) (b) (c)

Fig. 12: Deviation along the grazing curve (a) three control points; (b) four control points; (c) five control points.

4.3 Modeling of the Grazing Surface

After the grazing curve is defined, it can be used to generate a NURBS surface to approximate the grazing surface. The approximate grazing curves at a few tool positions are used to skin a NURBS surface.

4.4 Examples of Flank Millable Surfaces for the Barrel Tool

Examples are given in this section to demonstrate the proposed flank millable surface design method. The surface design starts with the two guiding curves shown in Fig. 6. The degrees of the guiding curves are two and the knot vectors of both are [0,0,0, 1,1,1].

A barrel tool is used to machine the designed surface. The geometry of the tool is shown in Fig. 9(b) and has R_t=7, R_b=4, R_0=418.7 and h=50. Bedi et al.'s tool positioning method is used to position the tool and generate the tool path. Grazing curves at three tool positions, u=0, u=0.48 and u=1, are used to generate each four control point quadratic approximate NURBS curve (with knot vector [0,0,0, 0.5, 1,1,1]), and consequently to design a four by three bi-quadratic NURBS surface using the method given in [4].

The maximum surface error is around 0.145, which is too large for general engineering applications. To reduce the maximum error, we used more control points in the control net of the surface by using knot insertion on the rows of the NURBS surface. In particular, two NURBS surfaces, a four by four and a four by five, were developed to check the maximum surface error. The control points in the guiding curve direction (the u direction) are kept the same, i.e., three, but the control points in the v direction are increased from four to five and then to six using knot insertion. The degree for both of surfaces is not changed. The knot vector in the v direction is [0,0,0, 0.33, 0.67, 1,1,1] for the five by three surface and [0,0,0, 0.25, 0.5, 0.75, 1,1,1] for the six by three surface.

Using the proposed surface design method, two more NURBS surfaces were designed. The deviation between the grazing surface and the flank millable surface are computed and are listed in Tab. 2.

C.P.s(uxv)	4×3	4×4	4×5
ε_{max}	0.145	0.079	0.048

Tab. 2: Errors for different NURBS surface. C.Ps: Control Points.

5. DISCUSSION

In this paper, a method to design flank millable surfaces was developed. The proposed method can effectively control the designed surface error and create a NURBS surface that can be flank milled in a single tool pass to machine tolerance.

Compared to the polynomial composition method presented in an earlier paper [7], the proposed method has a slightly different process to approximate the grazing surface. A high degree flank millable surface is needed in [7], and the approximate polynomial curves exactly lie on the tools of revolution. For the proposed method, the approximate curve is not on the tool of revolution, but is of lower degree and fewer control points are needed to define each grazing curve and the final surface. For example, for the barrel tool example, the surface constructed in this paper is degree 2×2 and has 20 control points, while the polynomial composition method was degree 8×2 and required 27 control points. While the new method has higher error on the barrel tool example (0.048 vs. 0.024 for the polynomial composition method), with additional control points, the error can be decreased to the desired level [4].

We can also compare the proposed general flank millable surface design method with the least squares method [5]. Both of the proposed methods and the least squares method can achieve a quality flank millable surface. The proposed approach has some runtime advantages, but it is somewhat less accurate than the least squares approach [4]. The least squares method is sensitive to a number of parameters such as location of points, knots and the number of control points. A successful use of the method requires knowledge of least squares and surface design. However, the users of the flank millable surfaces are designers and typically do not have a strong background in surface mathematics, which can cause problems in the use of the least squares method. In comparison the proposed method is more robust in use and does not require a deep knowledge of the mathematics of surfaces.

Finally, instead of using rational curves, polynomial curves can also be used to approximate the grazing curves. In particular, a four control point cubic Bézier curve can be used to approximate a 2D arc [2]. Thus, instead of the rational B-spline curves in Fig. 8, for example, a cubic polynomial curve can be used to define the projection of the grazing curve on the top and the bottom planes of the cone. As a result, the generated approximating curve is a polynomial curve or a NUBS curve and the resulting surface is polynomial instead of rational. As before, more control points can be added (using the knot insertion) to increase the accuracy of the NUBS approximating curve. For a discussion of these polynomial methods to construct a flank millable surface, see [4].

6. REFERENCES

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