Computer-Aided Design and Applications

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Input of Log-aesthetic Curve Segments with Inflection End Points and Generation of Log-aesthetic Curves with G^2 continuity

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ABSTRACT

The log-aesthetic curves include the logarithmic (equiangular) spiral, clothoid, and involute curves. Although most of them are expressed only by an integral form of the tangent vector, it is possible to interactively generate and deform them and they are expected to be utilized for practical use of industrial and graphical design. However, their input method proposed so far by use of three so-called control points can generate only an aesthetic curve segment with monotonic curvature variation and can not create a curve with the curvature-extremal point or the inflection point. Hence at first we propose a technique to input a log-aesthetic curve segment with an inflection point at its end. Then we propose a method to generate an log-aesthetic curve with G^2 continuity from a sequence of 2D points input with, for example, a liquid crystal pen tablet.

Keywords: aesthetic curve segment, G^2 continuity, inflection point, curvature-extremal point.
DOI: 10.3722/cadaps.2008.77-85

1. INTRODUCTION

"Aestehtic curves" were proposed by Harada et al. [1,2] as such curves whose logarithmic distribution diagram of curvature(LDDC) is approximated by a straight line. Miura [3,4] derived analytical solutions of the curves whose logarithmic curvature graph(LDG):an analytical version of the LDDC[1,2] are strictly given by a straight line and proposed these lines as general equations of aesthetic curves. Furthermore, Yoshida and Saito [5] analyzed the properties of the curves expressed by the general equations and developed a new method to interactively generate a curve by specifying two end points and the tangent vectors there with three control points as well as α : the slope of the straight line of the LCG. In this research, we call the curves expressed by the general equations of aesthetic curves the log-aesthetic curves. We use log-aesthetic instead of aesthetic because we would like to make clear that we are dealing with the specific curve type.

The log-aesthetic curves include the logarithmic (equiangular) curve (α =1), the clothoid curve (α =-1) and the circle involute (α =2). It is possible to generate and deform the log-aesthetic curve even if they are expressed by integral forms using their unit tangent vectors as integrands ($\alpha \neq 1$, 2) and they are expected to be used in practical applications. However, their input method proposed so far by use of three so-called control points can generate only an aesthetic curve segment with monotonic curvature variation and can not create a curve with the curvature-extremal point or the inflection point.

Hence in this paper from a sequence of the 2D points input with, for example, a liquid crystal pen tablet, we generate a cubic B-spline curve to approximate it and subdivide it at inflection and curvature-extremal points into several segments. For each segment, we determine α by approximating the slope of the LCG with a straight line and we use its slope as the initial value of α and optimize its value by some numerical method like Brent's method. We generate a log-aesthetic curve segment by use of the optimized α. Furthermore, we limit the number of the curvature-extremal points and simplify the log-aesthetic curve.

The formula in standard form II used by Yoshida and Saito [5] for the input of log-aesthetic curve segments cannot deal with the inflection point where the radius of curvature is equal to 0 because it is so normalized that the radius of curvature is equal to 1 when the arc length s is equal to 0. Hence in this paper we propose an input method of an aesthetic curve segment whose one end is an inflection point by use of the formula suitable for it. Since the curvature between two aesthetic curve segments is generally discontinuous, we propose a method to deform the curve to make the curvature continuous.

2. LOG-AESTHETIC CURVE SEGMENT WITH INFLECTION POINTS

At the inflection point of the curve, the curvature is equal to 0, the signed curvature of the curve changes its sign and the curve bends right to left or vice versa. It plays an important role for the curve design. There the radius of curvature ρ becomes infinitely large and we can not handle the inflection point adequately if we adopt formulas using ρ. In this section we review the method proposed by Yoshida and Saito using three points to input an aesthetic curve segment [5] and we propose a method to input it whose start point is an inflection point.

2.1 Input of Aesthetic Curve Segment

As an input method of the aesthetic curve, for a given α Yoshida and Saito proposed a technique to generate an aesthetic curve segment by inputting three points to specify two end points and the tangent vectors there [5]. The formulas used for the input of the aesthetic curve segment describe the relationships among the arc length s, the radius of curvature ρ, and the direction angle of the tangent θ and the integral forms to obtain points on the curve are derived for the standard curve segment located at the standard position for the curve to go through the origin. The curve segment in the standard form is transformed similarly to have its two end points at the same positions of the two input control points and its tangents there in the same directions specified by the three control points.

2.2 Formulation of the Log-aesthetic Curve with the Inflection Start Point

As Yoshida and Saito [5] have pointed out, if we assume the length of a given curve is infinite, the necessary and sufficient condition for the aesthetic curve has an inflection point is that the slope of the LCG $\alpha < 0$. Hereafter we assume that $\alpha < 0$ for the formulation.

In the complex plane, assume that s (>0) is the arc length of a given curve, then the general equation of the aesthetic curves is given by

$$C(s) = P_0 + \int_0^s e^{ib} e^{ias^{\frac{\alpha-1}{\alpha}}} ds \tag{1}$$

where P_0 is the start point of the curve, a and b are constants, and e^{ib} is the tangent direction at the start point. As a standard form, we assume that $a > 0$, the start point is located at the origin, and the tangent at the start point is oriented in the positive direction of the real axis. Since the shape of the curve becomes similar even if we change the value of a, b can be assumed to be 1 without loss of generality and the standard form is given by

$$C(s) = \int_0^s e^{ias^{\frac{\alpha-1}{\alpha}}} ds. \tag{2}$$

The directional angle of the curve θ is equal to $s^{\frac{\alpha-1}{\alpha}}$ and the curvature of the curve κ is given by

$$\kappa = \left| \frac{d^2 C(s)}{ds^2} \right| = \frac{\alpha - 1}{\alpha} s^{-\frac{1}{\alpha}}. \tag{3}$$

Therefore for arbitrary negative value α, when $s = 0$, $\kappa = 0$. It means that the start point is an inflection point. s and θ are expressed in terms of κ by

$$s = \frac{(\alpha-1)^{\alpha}}{\alpha^{\alpha}}\kappa^{-\alpha}, \quad \theta = \frac{(\alpha-1)^{\alpha-1}}{\alpha^{\alpha-1}}\kappa^{\alpha-1}. \tag{4}$$

With θ, κ is given by

$$\kappa = \frac{(\alpha-1)}{\alpha}\theta^{\frac{1}{1-\alpha}}. \tag{5}$$

2.3 Input by Three Points

The previous subsection has described the relationships among the arc length s, the directional angle θ, and the curvature κ of the standard form of the aesthetic curve segment whose start point is an inflection point. We can specify the positions of the end points and the tangent directions there if we adopt the input method by three points to generate an aesthetic curve explained in subsection 2.1. However in case where we need further one of the end points to be an inflection point, we cannot use an arbitrary value for α because the number of the constraints is increased by one. Hence we regard α as a variable and we generate a log-aesthetic curve satisfying these conditions.

As mentioned in the previous subsection $\theta = s^{\frac{\alpha-1}{\alpha}}$ and if s is constant, θ becomes larger with the increase of α. The curvature of the other end point that is not an inflection point becomes larger and the arc length s also becomes larger (See Fig.1-3. Note that these figures shows the relationships among the directional angle θ, the curvature κ, and α of the log-aesthetic curves with the fixed end points and the different α values. The details of the figures will be explained in Section 3). Therefore the increase and decrease of α coincides with those of θ and we can determine α by the bisection search in the way similar to the calculation of Λ of the second standard form for the input of the log-aesthetic curve segment proposed by Yoshida and Saito.

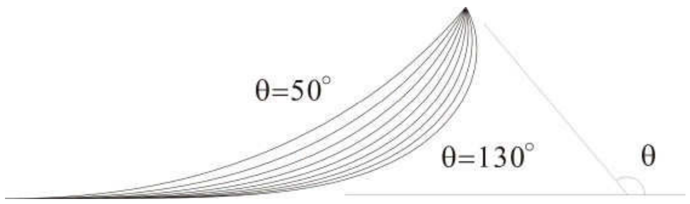


Fig. 1: A set of log-esthetic curve segments with an inflection point.

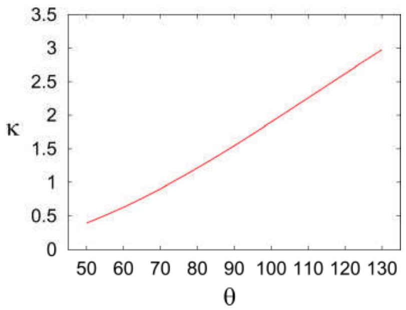


Fig. 2: Curvature plot for θ of the curves in Fig.1.

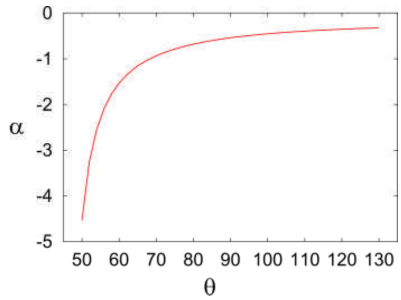


Fig. 3: Slope α plot for θ of the curves in Fig.1.

3. LOG-AESTHETIC CURVE GENERATION ALGORITHM

We generate an aesthetic curve with G^2 continuity by the following algorithm.

1) Input a sequence of points by, for example a pen tablet.
2) Generate a B-spline curve approximating the sequence of the points to smooth out to satisfy a given threshold value.
3) Subdivide the B-spline curve into several segments at the inflection points and those where its curvature has extremal values of its curvature.
4) For each segment, if none of its end points is an inflection point, determine α by approximating the LCG of the segment by a straight line to obtain the slope of the line. If one of its end points is an inflection point, regard α as a

variable and determine it for the segment to satisfy the conditions on the locations of the end points and the tangent directions there.

5) By using α determined in the previous step, generate aesthetic segments with the positional and directional conditions.

6) Since generally the curvature between two consecutive segments is discontinuous, deform the segments to make it continuous.

The log-aesthetic curves are supposed to be used for design of engineering products and it is desirable for them to be made up of a minimum number of the segments. One of the techniques of simplifying the curve is to reduce the number of the extrema to be one between two consecutive inflection points and allocate an inflection point and a point where the curvature is extremal alternately. In the formulation, we assume $\kappa \geq 0\ (\rho > 0)$, but for the input of the curve by arbitrary three points it is allowed to have a mirror reflection for the standard position and we should take care of negative curvature to distinguish between right or left turning. However, if the sign of the curvature remains the same and the amount of the change of the direction is large, we subdivide the curve not to make it exceed some constant amount (for example, 90 degrees).

3.1 Input of Points and B-spline Approximation

For the input of a sequence of points, we use, for example, a liquid-crystal pen tablet. By use of a stylus pen, we can directly input a curve as a sequence of points on the display and simultaneously the pen tablet draws the curve on it.

We approximate the sequence of the points by a B-spline curve, one of the most typical parametric curves by the least square method. We adopt cubic as the degree of the curve and the objective function of the least square method is the sum of the distances between the points and their corresponding points on the curve. If necessary, we add such constraints as the zero curvature at the end points. We start with a one-segmented curve and if the degree of approximation is not good enough and the value of the objective function is larger than the given threshold value, we increase the number of the segments one by one. If the value of the objective function becomes less than the threshold value, we stop the process. Figure 4 shows the result of the approximation. The input points are painted in black, the B-spline curve in purple and its control points are illustrated as green + signs.

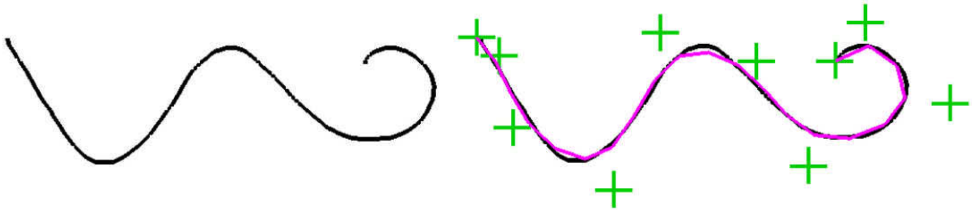


Fig. 4: B-spline approximation.

3.2 Segmentation of B-spline Curve with Curvature

In order to apply the input method of the aesthetic curve segment developed by Yoshida and Saito [5], the B-spline curve is subdivided at the inflection points and the curvature extrema. Each segment of the cubic B-spline curve is equivalent to a cubic Bézier curve and we explain how to calculate its inflection and curvature extrema.

3.2.1 Inflection points

We assume the cubic Bézier curve is given by $C(t) = (x(t), y(t))$, $0 \leq t \leq 1$ and the 1st and 2nd derivatives of $x(t)$ and $y(t)$ with respect to t is expressed by $\dot{x}(t)$, $\dot{y}(t)$, $\ddot{x}(t)$, and $\ddot{y}(t)$, respectively. We use $f(t)$ and $g(t)$ defined by $f(t) = \dot{x}(t)^2 + \dot{y}(t)^2$, $g(t) = \dot{x}(t)\ddot{y}(t) - \dot{x}(t)\dot{y}(t)$. Then the curvature with positive or negative sign κ is given by $\kappa = g(t) / f(t)^{\frac{3}{2}}$. If the curve is not degenerated, we can assume $f(t) \neq 0$ and the parameter values which give points where the curvature is equal to 0 are obtained by solving $g(t) = 0$. $g(t)$ is a cubic polynomial of the parameter t and it can be solved analytically.

3.2.2 Extremum

The extrema are obtained by solving $d\kappa(t)/dt=0$:

$$\frac{d\kappa}{dt}=\frac{2\dot{g}(t)f(t)-3g(t)\dot{f}(t)}{2f(t)^{\frac{5}{2}}}. \tag{6}$$

Hence we solve the following equation:

$$h(t)=2\dot{g}(t)f(t)-3g(t)\dot{f}(t)=0. \tag{7}$$

Because $f(t)$ is of degree 4, $h(t)$ is of degree 6. We use some numerical method to get the solutions. For example, we subdivide the interval from 0 to 1 with an equal size and for each subdivided interval, if the signs of $h(t)$ at the two ends are different, the initial value is set to be the middle point of the subdivided interval and the accuracy of the solution can be increased by Newton's method.

Figure 5 shows the control points of the cubic Bézier curve converted from the segments of the B-spline curve. In the figure, the + signs indicate the control points of the Bézier curve and the green lines are generated by connecting the control points. Figure 6 shows cubic Bézier curves subdivided at the inflections and curvature extrema.

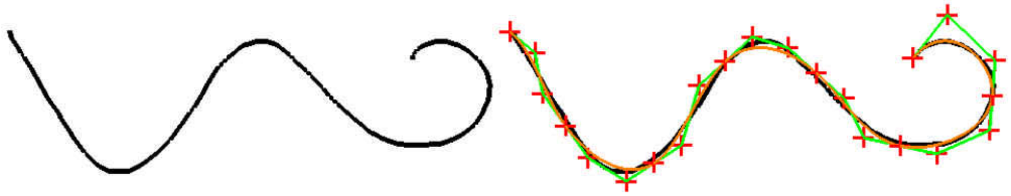


Fig. 5: Conversion to a set of cubic Bézier curves.

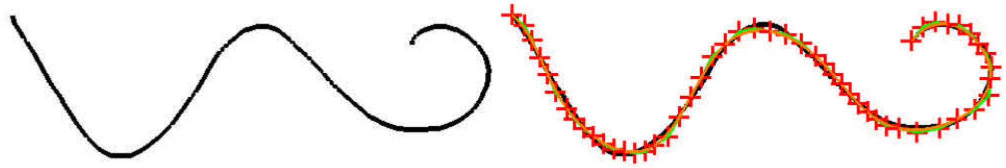


Fig. 6: Subdivision at inflection and extrema of curvature.

3.3 Generation of Log-aesthetic Curve

Each of the cubic Bézier curves obtained by subdividing at the inflections and the curvature extrema is approximated by a log-aesthetic curve. In order to apply Yoshida and Saito's method [5], we calculate the intersection point between two lines who have one of the two end points of the curve and whose directions are equal to the tangent vectors there. By this method, between two consecutive segments the tangent becomes continuous, but the curvature does generally discontinuous.

3.4 Deformation for Curvature Continuity: Between Two Segments

In this subsection, we discuss how to connect two consecutive segments with G^2 continuity and in the following subsection, we explain how to guarantee G^2 continuity for the whole curve with three or more segments.

Figure 7 shows a log-aesthetic curve with two log-aesthetic curve segments one of whose end points is an inflection. Both of the end points of the log-aesthetic curve are inflection points and the log-aesthetic curve segments are defined by the control points shown as the vertices of the polygonal line in the figure.

Since we use all the parameters including α for the generation of the log-aesthetic curve segment, for the continuity of curvature it is necessary to change the positions of the control points. Hence there are two possible deformation ways for G^2 continuity to keep the original shape of the curve as much as possible: 1) translate the common end points of the two segments, 2) incline the tangent direction there. As designers generally think that the position of the end point is more important that the tangent direction there, in this research we change the tangent direction at the common end points of the segments.

Since the log-aesthetic curve segment is generated by scaling some segment of the standard form, it is not possible to analytically calculate the curvature at the end points for given control points and we have to numerically determine the transformation matrix for the scaling to evaluate the curvature. However as shown in Fig.1-3, it is generally possible the curvature at the end points κ increases (decreases) according to the directional angle θ for the left-turning (right-turning) curve segment.

Figure 1 shows a set of log-aesthetic curve segments one of whose end points is an inflection point located at $(0,0)$ and the other is at $(\sqrt{2}+1,1)$, the directional angle of the tangent vector there is given by θ, changing from 50 degrees to 130 degrees. If the directional angle is 45 degrees, the distance between the 1st and the 2nd control points is equal to that between the 2nd and the 3rd and we should generate a circular arc. Figures 2 and 3 plot the curvature κ and α for θ, respectively. As we can see from these graphs, both of κ and α increases in accord with θ. If the relationships among three variables qualitatively remain the same, in case the difference between the curvatures at the joint is given as shown in Fig.8, as the curvature of the first segment is relatively smaller that that of the second segment, we can turn the tangent direction counterclockwise and increase the curvature of the 1st segment and decrease that of the second segment. Hence we can efficiently make them equal by bisectional search in a short time. The processing time is about 0.5 seconds on a PC with a Pentium 4, 3.2 GHz.

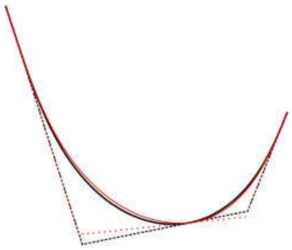


Fig. 7: Two segments with inflection end points.

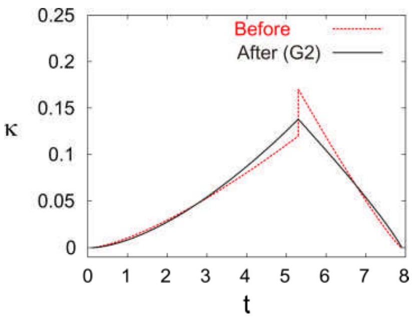


Fig. 8: Curves in Fig.7 before and after deformation.

3.5 Deformation for Curvature Continuity: Whole Curve

In the explanation of the previous subsection, we took a log-aesthetic curve segment one of whose end points is an inflection point as an example and the curvature at one of the end points is fixed to be zero. Even if another segment is connected there, the effect of the deformation for the continuity of curvature is limited inside the two segments and it does not affect the continuity of curvature to other segments outside. However for a general log-aesthetic curve segment, if we deform the segment for the continuity of curvature at one end point, that at the other end point is generally broken.

Therefore it is desirable to adopt some numerical optimization method whose variables are the direction angles between two consecutive segments and objective function is the sum of the differences between the curvatures at the end points without using the derivatives of the objective function such as Powell method [6]. However numerical optimization methods with many variables generally need a lot of processing time and they are much even worse if the derivatives of the objective function are not available. Hence we apply the method mentioned in the previous subsection to the joint where the difference of the curvature is the largest one by one and stop the process if all of the difference of the curvatures becomes less than a threshold value.

The results obtained by this technique are shown in Fig. 9. The curvature profiles before and after deformation of the curve in Fig. 9 are plotted in Fig. 10. As this figure shows, the curvature is continuous at every joint between two consecutive segments is G^2 continuity is guaranteed for the whole curve. The processing time for curves with more than ten segments including the 14-segment curve in Fig. 9 is 5 to 10 seconds on a PC whose CPU is Pentium 4, 3.2

GHz. Techniques of the adjustment of the direction angles to avoid the vibration of the directional angles and speed up the convergence are those of future work.

Figure 11 shows a log-aesthetic curve generated from the example in Fig.4. Figure 12 plots the curvature, inflection points, and curvature extrema of the B-spline curve and those of the corresponding log-aesthetic curve.

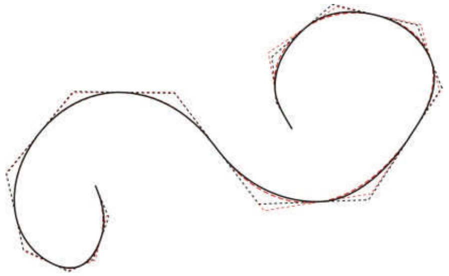


Fig. 9: Curves before (red) and after (black) deformation.

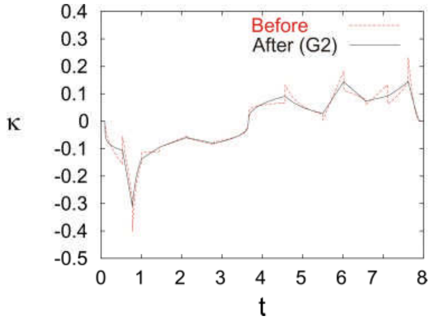


Fig. 10: Curvature profiles of the curves in Fig. 9.

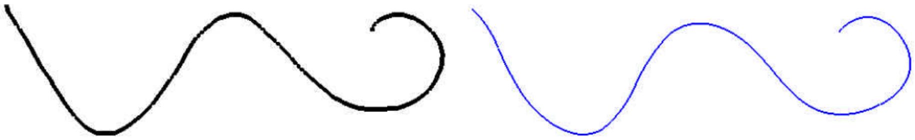


Fig. 11: Generated log-aesthetic curve.

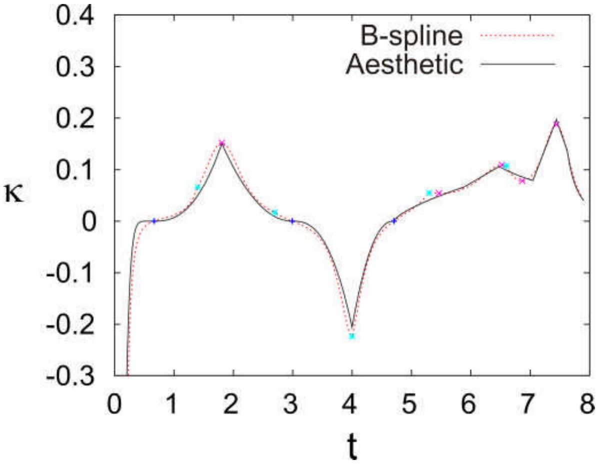
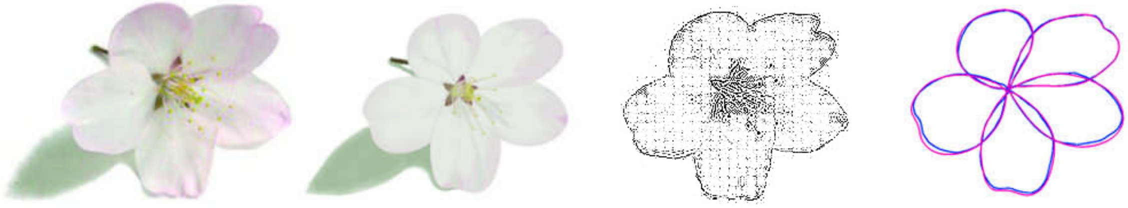


Fig. 12: Curvature, inflection points (+) and curvature extrema (×) of the B-spline and the log-aesthetic curves.

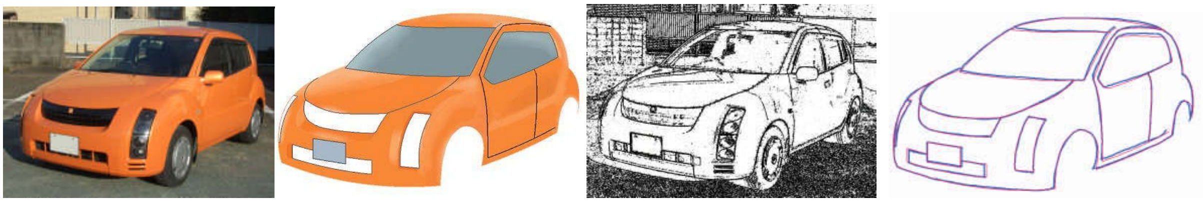
4. IMPLEMENTATION AND DISCUSSIONS

Using log-aesthetic curves generated by our system, we created Fig.13(b) based on a photo of the flower petals shown in Fig.13(a). The curves are log-aesthetic and the painting was directly done with a liquid-crystal pen tablet. Figure 13(c) shows the outline of the flower petals extracted by image processing. The outlines of the flower petals drawn with the B-spline and the log-aesthetic curves are compared in Fig.13(d). The B-spline curve is generated by tracing the outline shown in Fig.13(c). For a natural object, we can deform the outline of the flower petals with creases to be fair by use of the log-aesthetic curve.



(a) Cherry blossom (photo) (b) With log-aesthetic curve (c) Outline (d) B-spline (blue) and log-A curve (red)
Fig. 13: Drawing with log-aesthetic curves (natural object).

Figures 14 and 15 show several examples of the usage of the log-aesthetic curves to express the outlines of a car body and a musical instrument. For the industrial products, we can reconstruct their outlines by use of the log-aesthetic curves.



(a) Car (photo) (b) With log-aesthetic curve (c) Outline (d) B-spline (blue) and log-A curve (red)
Fig. 14: Drawing with log-aesthetic curves (industrial design: car body).

(a) Violin (photo) (b) With log-aesthetic curve (c) Outline (d) B-spline (blue) and log-A curve (red)
Fig. 15: Drawing with log-aesthetic curve (industrial design: musical instrument).

5. CONCLUSIONS

In this paper, we have proposed a method to generate log-aesthetic curves which have inflection points and curvature extrema. We input a sequence of points as a curve by use of a liquid-crystal, approximate it by a B-spline curve, subdivide the B-spline curve at curvature-extremal points, then determine α of each segment from its logarithmic curvature graph and again approximate each segment of the subdivided B-spline curve by a log-aesthetic curve to generate a curve. If one of the end points of the segment is an inflection point, we regard α as one of the variables and propose a method to generate a log-aesthetic curve keeping the tangent conditions at the end points. We propose another technique to limit the number of the curvature extrema and generate a log-aesthetic curve with less number of the segments. Furthermore, we devise a technique to deform the curve to make the curvature between two consecutive segments continuous and another to guarantee G^2 continuity for the whole curve. The generated log-aesthetic curve

has the continuity of curvature and each segment of the curve can have a different α value. If the curvature monotonically increases or decreases for two consecutive segments and the signs of their α are different, we can generate a compounded-rhythm curve.
For future work, we will devise a generation method of the log-aesthetic curve in consideration for the pressure and the inclination, and the speed of the input pen and develop a styling CAD system using the log-aesthetic curve.

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