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Integrative Modeling to Produce Ornamental Products

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ABSTRACT

This work is geared towards the proposition of methodologies to automate the arena of design and manufacturing for producing ornamental products (jewelry, screens, wooden carvings and stamp blocks, bottles) of different forms (stretch-formed, carved, fret-worked, lattice-worked, engraved and blow-formed). The various classes of ornamental patterns are designed and generated to make actual artifacts using different fabrication processes. The work adopts integrated product development approach which brings two different worlds of creativity, innovative designing and agile manufacturing, into contact with each other by transferring the product information from virtual representation tools to Computer Aided Manufacturing (CAM) technologies. The presented Computer Aided Design (CAD) paradigms provide the capability of designing custom-engineered ornamental products in an easy-to-use and efficient manner, using parametric design concept.

Keywords: ornamental patterns, CAD, CAM, layered manufacturing.
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1. INTRODUCTION

The creation of ornamental products is an ancient human endeavor. Ornamental products are adorned with different classes of patterns reflecting various styles of art and culture. Ornamental patterns are symmetric arrangement of shapes and can be implemented on products like jewelry, wooden carvings, furniture, stamp blocks, floor tiles, wallpapers, bricks, doors, windows, screens, textiles, utensils, beverage bottles etc. Ornamental patterns do not contribute to product's function and are mainly used only as decoration.

This work develops various forms of ornamental products with a significant amount of variations using parametric Computer-Aided Geometric Modeling methods. The design methodologies are integrated with various fabrication techniques to produce actual ornamental artifacts. The forms of ornamental products are categorized as stretch-formed, carved, fret-worked, lattice-worked, engraved, blow-formed, etc. The form and fabrication techniques executed in this work are illustrated in Tab. 1. The objective of the work is to produce computer compatible models having a wide modeling range and supported by diverse integrated manufacturing. The work adopts integrative modeling approach which can rapidly customize users' requirements by reducing the gap that traditionally exists between design and manufacturing. Integrated product development approach brings two different worlds of creativity, innovative designing and agile manufacturing, into contact by transferring and converting the product information and data among tools like Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and Layered Manufacturing (LM).

Some of the features of ornamental products are identified and adopted as guidelines. The vast majority of ornamental products exhibit symmetry. This reason must in part be tied to the practicalities of fabricating ornaments. Every ornament reflects a particular style of art and culture.

Thus, traditional patterns are generated with some new interpretations and implemented into ornamental products. Ornaments are the aesthetics products which involve artistic imagination. So, this work is directed towards incorporating the aesthetics into ornamental products through the design tools that can be used by persons having minimal artistic skills. Ornamental product generation is a specialized and time consuming endeavor. Therefore, the design paradigms capitalize the advantages of computer over hand by freeing from repetitive tasks and considerably reducing the effort and time required for designing and creating the ornamental products.

Sr. No.	**Forms of Ornamental Products**	**Ornamental Products**	**Fabrication Techniques**		
			CNC Milling	**Laser Cutter**	**Layered Manufacturing Machine**
1.	Stretch-formed	Jewelry			✓
2.	Carved	Jewelry Pendants and Bangles			✓
3.	Fret-worked	Jewelry Bangles			✓
4.	Lattice-worked	Screens		✓	
5.	Carved	Wooden Carvings		✓	
6.	Engraved	Stamp Blocks	✓	✓	
7.	Blow-formed	Bottles	✓		✓

Tab.1: Form and fabrication techniques for ornamental products.

Section 2 and 3 presents parametric voxel based ornamental patterns. Two novel methodologies are proposed and explained to produce stretch-formed and carved jewelry items. The parametric feature based ornamental patterns are presented in section 4 to create fret-worked jewelry. Section 5 deals to create traditional Islamic geometric patterns for lattice-worked screens. Section 6 produces traditional Zillij style of ornamental patterns for execution in wooden carvings. The floral class of patterns is generated for printing blocks in section 7. In section 8, an integrative modeling approach is employed to create moulds for blow formed beverage bottles.

2. STRETCH-FORMED JEWELRY

Stretch-formed jewelry is created on thin sheet metal on top of which anornamental pattern is raised with forming tools by stretching the sheet metal beyond elastic limit. The sheet metal is put under combined bending and tension stresses at the same time by clamping at edges and stretching over forming tools. The surface of sheet metal is plastically deformed to rise permanently into the jewelry pattern on the forming tools. The process is illustrated in Fig.1.The raised surface of sheet metal is reflected by light imposed on it and helps in visualization of the pattern. This form of jewelry can be applied to various types of jewelry such as pendants, rings, earrings, bracelets, etc.

The forming tools used to produce stretch-formed jewelry are dies and punches having different styles of patterns. A 3D jewelry pattern is removed and glued from stock solids (rectangular block) to model dies and punches respectively as shown in Fig. 2. These die and punch models are converted to STL format and transferred to LM machine to produce master forming components.

Alphanumeric jewelry patterns have been generated by combining parametric voxel elements in a meaningful manner. Voxel, elements are building blocks of the pattern and defined by parameterized geometry and attributes. Universally, eight voxels are defined to model any alphanumeric jewelry pattern. These voxels are grouped in a set with same geometrical and dimensional constraints as shown in Fig. 3. The pattern is created by knowing which voxel elements are required from a set and in what order they have to be placed in a 2D matrix of size 5x4. After placing the voxels at their appropriate positions in correct order, these are concatenated together to produce a pattern [13].

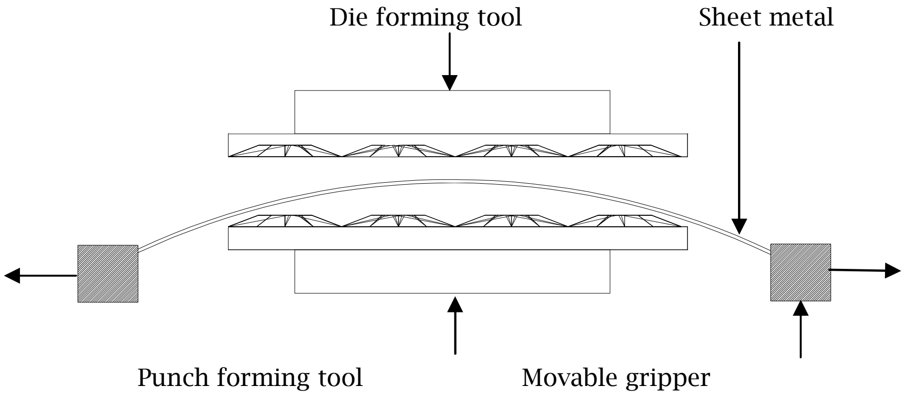


Fig.1: Stretch-forming process [13].

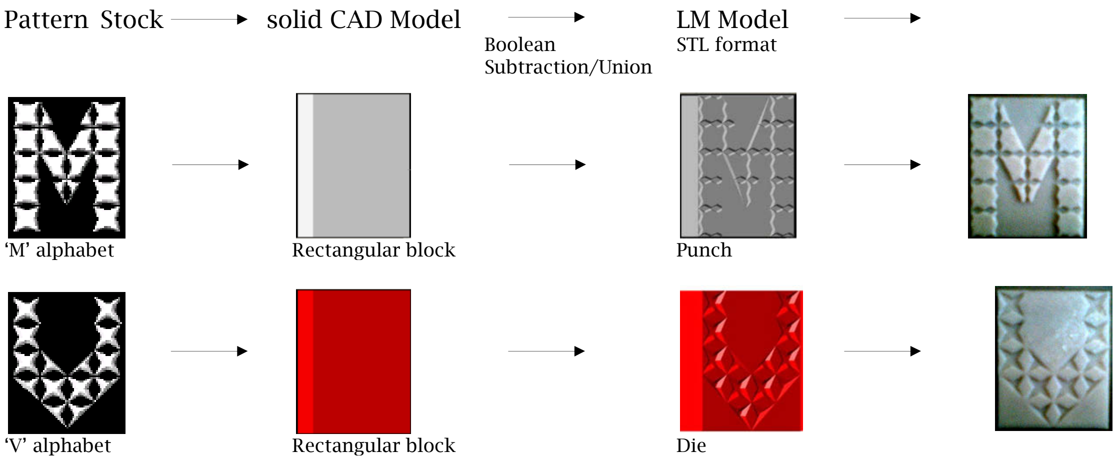


Fig. 2: CAD and LM forming components.

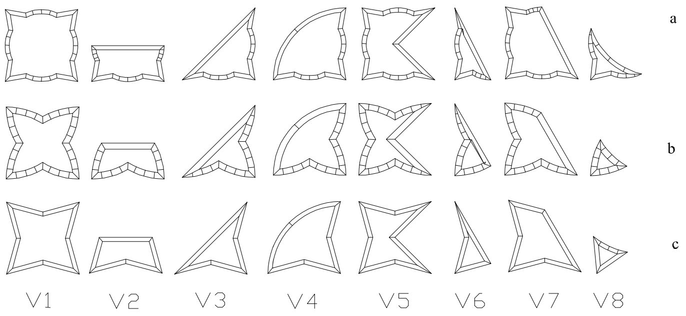


Fig. 3: Set of voxel elements for alphanumeric patterns [13].

3. CARVED JEWELRY

Carved jewelry is a form of jewelry having small internal cavities as generated with a thin sharp tool on the surface of sheet metal. This form of jewelry patterns contains repeated shapes of cavities and is

designed using voxel based technique. Parametric voxel elements are combined in a wide variety of possibilities to deliver more number of jewelry patterns with higher number of variations. Some of the voxel elements are shown in Fig. 4. By changing the modeling parameters, a large number of jewelry patterns can be produced. Some of jewelry patterns of pendants and bangles are shown in Fig. 5.
For creating the carved jewelry, a voxel signature is selected for modeling the voxel element. Then, its multiple copies are partially overlapped in a plane about an axis of rotation and are concatenated with one another. The direction and location of the axis of rotation decides the type (Pendant or Bangle) of jewelry and size of jewelry respectively. Appearance of jewelry patterns depends on the number of voxels to be overlapped about the axis of rotation. . The two examples of rendered (CAD) and fabricated (LM) models of carved jewelry are shown in Fig. 6 [14].

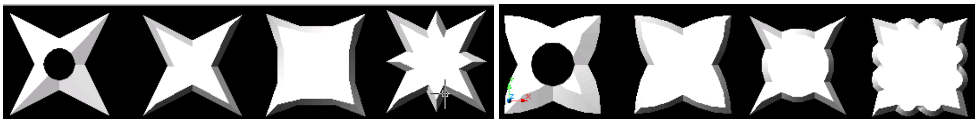


Fig. 4: Voxels elements.

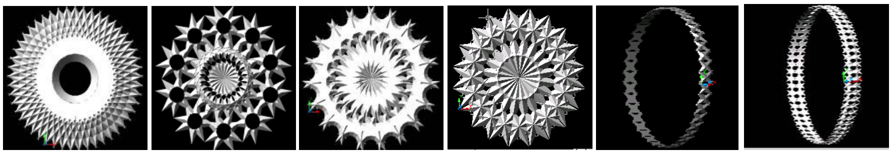
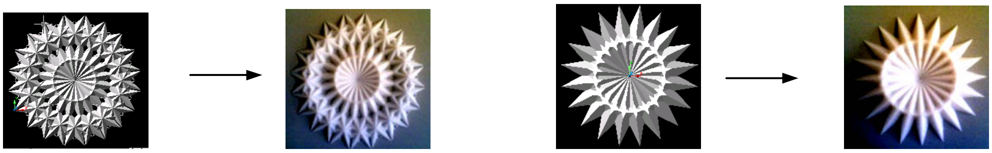


Fig. 5: Carved pendants and bangles [14].



(a) (b)

Fig.6: CAD and LM models of carved jewelry.

4. FRET-WORKED JEWELRY

Fret-worked is a form of jewelry, in which different ornamental patterns are either carved in low relief on a solid background, or cut out. This class of jewelry is adorned with patterns which are recurrently and symmetrically cut or carved into constructive solids in the shapes of jewelry types like pendants, bangles, rings, earrings, etc.
This part of the paper presents modeling of fret-worked jewelry bangles using the parametric feature based approach [8-9, 15, 22-24].The stock solids and fretwork patterns are two types of modeling components produced for fret-worked jewelry bangles. An instance of a stock solid is created and after that, instances of fretwork patterns are positioned on the stock. The fretwork patterns are viewed as volumes to be recurrently removed from the stock solid by Boolean operation [12].The jewelry items are parameterized to support customized design process. The variations in the jewelry models can be obtained either by changing the modeling parameters or by changing the modeling components. Some of designs of fret-worked jewelry bangles are shown in the Fig.7.
The stock solids of bangles are undecorated blanks and created by rotational sweep. The shape of stock solids depends on the shape of sweep bases which are to be revolved about an axis by 360^0. The interior surface of stock solids is kept plane to provide comfort to the user as it touches the wrist of arm. Only the exterior surface is shaped with different contours. The size of stock solid depends on the wrist of arm. Distance between the interior profile of sweep bases and the revolved axis gives the radius or size of the stock solids. The basic dimensions length and breadth of sweep bases gives the thickness and width to stock solids respectively. The exterior profile of sweep bases is defined with

variants which are constrained with in the values of the thickness and width of stock solids. Some of the stock solids are shown below in Fig. 8.

Fig. 7: Fret-worked bangles.

Fig. 8: Stock solids.

The motifs of fretwork pattern correspond to features in the feature based modeling approach. These motifs are created by translational sweep which is defined with two explicit sweeping parameters specifying the depth and direction of sweep. The depth of sweep is determined from an explicit dimension specifying the thickness of stock solid. The sweep operation begins at the exterior surface of stock solid and ends at the interior surface perpendicularly. Motifs are categorized mainly into geometric and floral and some of these are shown in Fig. 9. There exist different dimensional definitions in association with the different kinds of motifs whose shapes depend on the profile of sweep bases. The motifs are assumed to be inscribed in a circle or a rectangle. The breadth of circumscribed rectangle or diameter of circumscribed circle should be taken less than the width of stock solid. The arbitrary center point of circumscribed geometries is used to position the motif on the exterior surface of stock solid.

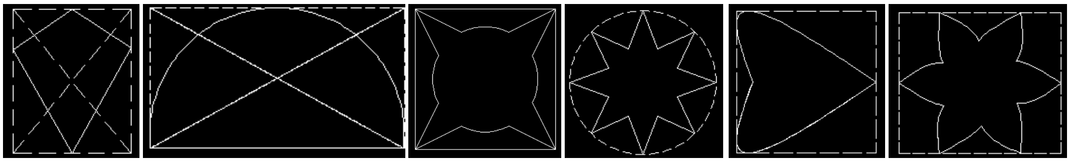


Fig 9: Fretwork motifs.

The geometric modeling techniques used in this part of work are simple, yet their results are very beautiful and inspiring. CAD in conjunction with LM technique is used to produce this form of jewelry items directly from 3D solid design models. CAD and LM model of a fretwork bangle is shown in the Fig. 10.

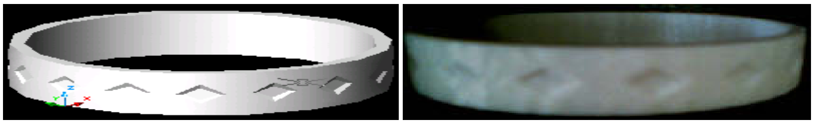


Fig. 10: CAD and LM models of Fret-worked jewelry.

5. LATTICE-WORKED SCREENS

Lattice-worked screen is flat pierced panel adorned with regular and symmetric network of pattern on wood or stone. It was extensively used in Indo-Mugal architecture as windows, room dividers, and railings around thrones, platforms, terraces, and balconies for the circulation of air and provides shelter from sunlight [16].

A parametric hierarchic based CAD paradigm is presented to generate traditional Islamic geometric ornamental patterns and to implement these patterns upon lattice-worked screens. The generated patterns are 2D in nature and executed on the planar surface of a material to produce 2½D screens. These ornamental patterns are ideally suited to computer-controlled manufacturing that can build actual world artifacts from synthetic computer descriptions.

Islamic geometric ornamental patterns are made up of repeated star shaped motifs [1-4]. These motifs are regarded as a planar map consisting of a set of vertices, having a position in the plane, and a set of edges [18-19]. The positions of vertices are formulated in terms of design parameters and pairs of vertices are connected to generate set of edges. A star motif is collection of edges that do not intersect each other except possibly at their end points and created with four design parameters. Two of these are number of rays (n) and size of star (r) corresponding to number of sides and size of the boundary polygon in which the star motif is generated respectively. Thirdly, shape of star θ corresponding to the angle formed by pair of edges generated at each midpoints of sides of the boundary polygon with vertical. Lattice-worked screens are fabricated by removing material out of one piece of 2½D planar surface, leavingthe panels with a network of line thickness of the star motifs and boundary polygons. The thickness of the star motif t_1 and boundary polygon t_2 is expressed as fourth design parameter.

Islamic star patterns are created using the description of three hierarchical level shapes which are labeled as: (1) Motifs (Lowest level shapes) (2) Compound Motifs (Intermediate level shapes) (3) Patterns (Highest level shapes). Motifs make up the compound motifs which may be the shapes in relation to patterns.

For lattice-worked screen, basic motif geometry is created from a set of line primitives by joining the defined vertices whose positions are expressed in Fig. 11 and Tab. 2. This geometry is mirrored about an arbitrary line O_1M_1. An array of geometries is created about the point O_1 for angle 2π,which results into a star shaped motif. Some of rendered star shaped motifs are shown in Fig. 12.

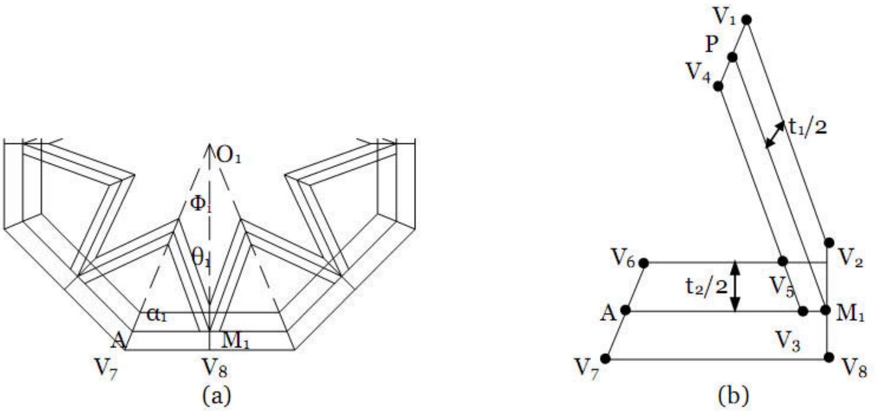


Fig. 11: Geometry of star shaped motif.

Sr. No.	Vertex	Position of the Vertex	Remarks
1.	O_1	$X_{o_1} = 0$ $Y_{o_1} = 0$	• O_1 is the center point of boundary polygon with number of sides' n_1 and size r_1.
2.	A	$X_A = -r_1 \tan(\phi_1)$ $Y_A = -r_1$	• ϕ_1 is half of angle subtended by a side of the boundary polygon with center point O_1 and equals to Π / n_1.

3.	M_1	$X_{M_1} = 0$ $Y_{M_1} = -r_1$	• M_1 is midpoint of the side AB.
4.	P	$X_P = -\sqrt{(PO_1)^2 + (Y_P)^2}$ $Y_P = \dfrac{-(PO_1)^2 + (PM_1)^2 - r_1^2}{2r_1}$	• P lies on arbitrary line AO_1 such that angle PM_1O_1 is θ_1. $PM_1 = \dfrac{r_1 \sin(\phi_1)}{\sin(\phi_1 + \theta_1)}$ and $PO_1 = \dfrac{r_1 \sin(\theta_1)}{\sin(\phi_1 + \theta_1)}$
5.	V_1	$X_{V_1} = \dfrac{-X_P + t_1 \sin(\phi_1)}{2\cos(\alpha_1 - \theta_1)}$ $Y_{V_1} = \dfrac{-Y_P + t_1 \cos(\phi_1)}{2\cos(\alpha_1 - \theta_1)}$	• α_1 is half of interior angle of the boundary polygon and equals to $\dfrac{\Pi(n_1 - 2)}{2n_1}$ • V_1 lies on arbitrary line AO_1.
6.	V_2	$X_{V_2} = 0$ $Y_{V_2} = -r_1 + \dfrac{t_1}{2\sin(\theta_1)}$	• V_2 lies on an arbitrary line M_1O_1 such that distance between lines M_1P and V_1V_2 is $t_1/2$.
7.	V_3	$X_{V_3} = -\dfrac{t_1}{2\cos(\theta_1)}$ $Y_{V_3} = -r_1$	• V_3 lies on arbitrary line AM_1.
8.	V_4	$X_{V_4} = \dfrac{-X_P - t_1 \sin(\phi_1)}{2\cos(\alpha_1 - \theta_1)}$ $Y_{V_4} = \dfrac{-Y_P - t_1 \cos(\phi_1)}{2\cos(\alpha_1 - \theta_1)}$	• V_4 lies on an arbitrary line AO_1 such that distance between lines M_1P and V_3V_4 is $t_1/2$.
9.	V_5	$X_{V_5} = 0$ $Y_{V_5} = -r_1 + \dfrac{t_2}{2}$	• V_5 lies on arbitrary line AO_1.
10.	V_6	$X_{V_6} = -r_1 \tan(\phi_1) + \dfrac{t_2}{2\tan(\alpha_1)}$ $Y_{V_6} = -r_1 + \dfrac{t_2}{2}$	• V_6 lies on an arbitrary line AO_1 such that distance between lines AM_1 and V_5V_6 is $t_2/2$.
11.	V_7	$X_{V_7} = -r_1 \tan(\phi_1) - \dfrac{t_2}{2\tan(\alpha_1)}$ $Y_{V_7} = -r_1 + \dfrac{t_2}{2}$	• V_7 lies on extending line AO_1.

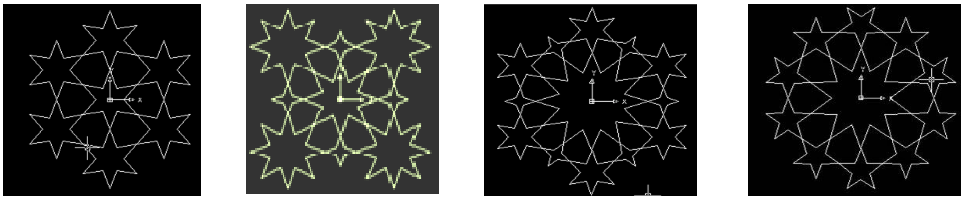
12.	V_8	$X_{V_8} = 0$ $Y_{V_8} = -r_1 - \frac{t_2}{2}$	• V_8 lies on extending line O_1M_1 such that distance between lines AM_1 and V_7V_8 is $t_2/2$.

Tab. 2: Set of vertices in the planar map of star shaped motif.

n_1=12, r_1=1, θ_1=30° t_1=0.1, t_2=0.1 | n_1=8, r_1=1, θ_1=30° $t_{1\,1}$=0.1, t_2=0.2 | n_1=6, r_1=1, θ_1=45° t_1=0.1, t_2=0.1 | n_1=10, r_1=1, θ_1=10° t_1=0.1, t_2=0.2 | n_1=8, r_1=1, θ_1=20° t_1=0.1, t_2=0.2

Fig. 12: Star shaped motifs.

The compound motif comprises a shape in which two same or different motifs are surrounded by a central motif. One ray of each surrounded motif is in contact with a ray of the central motif. One side of the boundary polygon of each surrounded motif is also in contact with a side of the boundary polygon of central motif. Each surrounded motif is in contact with its neighboring one, only if sum of interior angles (=360°(n−2)/2n) of all the three boundary polygons (one central and two surrounded) is 360°. From aesthetic and symmetry point of view, same shapes of surrounded motifs are placed at alternate positions of the rays of central motif. With the above said considerations, some of compound motifs are shown in Fig. 13. The central and surrounded motifs may have different or same number of rays, shape and thickness parameters, but, size of the surrounded motifs depend upon the size and number of rays of the central motif.



n_1=6, n_2=6, n_3=6 | n_1=8, n_2=8, n_3=4 | n_1=12, n_2=6, n_3=4 | n_1=6, n_2=6, n_3=6

α_1=120°, α_2=120°, α_3=120° | α_1=135°, α_2=135°, α_3=90° | α_1=150°, α_2=120°, α_3=90° | α_1=150°, α_2=120°, α_3=90°

Fig. 13: Cases of compound motifs: each surrounded motif is in contact with its neighbor.

Referring to Fig. 14, geometry of surrounded motif is created from a set of edges by joining the defined vertices whose positions are formulated in Tab. 3. This geometry is mirrored about an arbitrary line O_1M_1. An array of all generated geometries is created about point O_2 for angle 2π. By following the same procedure, second surrounded motif is generated and rotated about the point O_1 by an angle $2\phi_1$ to place at consequent ray position of the central motif. Multiple copies (equal to half of number of rays of central star motif) of both the surrounded motifs are created and placed at alternate ray of the central motif. Some of rendered compound star motifs are shown in Fig. 15.
Patterns are composed of motifs which are multiplied and ordered in regular sequences. A rectangular array of motif is generated by defining the following parameters: (1) Number of rows (R) (2) Number of columns (C) (3) Angle of array (∂) (4) Row offset (RO) (5) Column offset (CO). The first two parameters

are taken according to the size of the surface of the screen to be filled. The row and column offsets parameters depend on the size of generated motif and angle of array taken. Some of rendered star shaped patterns are shown in Fig. 16.
The ornamental patterns are rendered and submitted to computer controlled laser cutting machine. Laser cutting is used as it utilizes the tremendous power of light to obtain precise cuts with minimal waste or burring and leaves a clean finish. The depth of cut produced can be varied by laser cutting as it depends on the power of the light being focused through the laser lens. Two of the rendered Islamic geometric patterns are used for cutting using computer controlled laser cutting machine, and are shown in Fig.17.

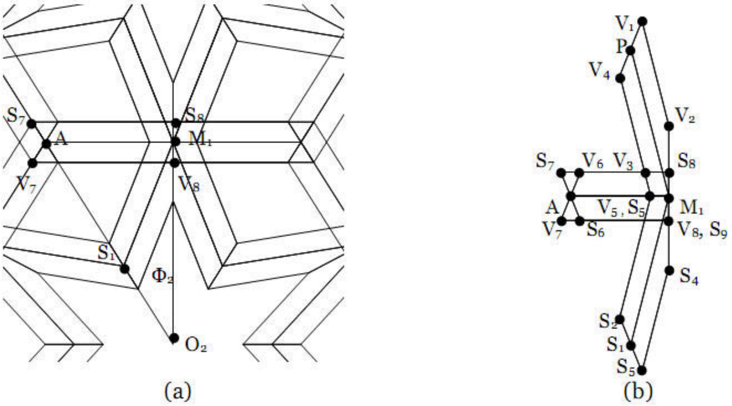


Fig.14 Geometry of star shaped compound motif.

Sr. No.	Vertex	Position of the vertex	Remarks
1.	O_2	$X_{o_2} = -r_1 - r_2$ $Y_{o_2} = 0$	• O_2 is center point of the 1st surrounded boundary polygon. • $r_2 = \frac{r_1 \tan(\phi_1)}{\tan(\phi_2)}$ and $\phi_2 = \prod / n_2$
2.	S_1	$X_{S_1} = -\sqrt{(S_1O_2)^2 + (Y_{S_1})^2}$ $Y_{S_1} = \frac{-(S_1M_1)^2 + (S_1O_2)^2 - r_2{}^2 - 2r_1r_2}{2r_2}$	• S_1 lies on arbitrary line AO_2 such that angle $S_1M_1O_2$ is the contact angle θ_2 $S_1M_1 = \frac{r_2 \sin(\phi_1)}{\sin(\phi_2 + \theta_2)}$ $PO_1 = \frac{r_2 \sin(\theta_2)}{\sin(\phi_2 + \theta_2)}$
3.	S_2	$X_{S_2} = \frac{-X_{S_1} - t_1 \sin(\phi_2)}{2\cos(\alpha_2 - \theta_2)}$ $Y_{S_2} = \frac{-Y_{S_1} + t_1 \cos(\phi_2)}{2\cos(\alpha_2 - \theta_2)}$	• α_2 is half of interior angle of the boundary polygon and equals to $\frac{\Pi(n_2 - 2)}{2n_2}$ • S_2 lies on arbitrary line AO_2.

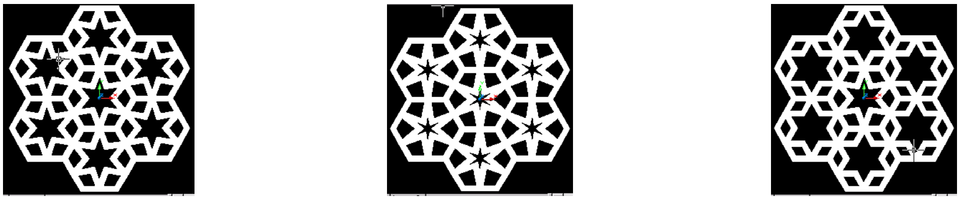
4.	S_5	$X_{S_5} = -\dfrac{t_1}{2\cos(\theta_2)}$ $Y_{S_5} = -r_1$	• S_5 lies on an arbitrary line AM_1 such that distance between lines S_1M_1 and S_2S_5 is $t_1/2$.
5.	S_3	$X_{S_3} = \dfrac{-X_{S_1} + t_1\sin(\phi_2)}{2\cos(\alpha_2 - \theta_2)}$ $Y_{S_3} = \dfrac{-Y_{S_1} - t_1\cos(\phi_2)}{2\cos(\alpha_2 - \theta_2)}$	• S_3 lies on arbitrary line AO_2.
6.	S_4	$X_{S_4} = 0$ $Y_{S_4} = -r_1 - \dfrac{t_1}{2\sin(\theta_2)}$	• S_4 lies on an arbitrary line O_2M_1 such that distance between lines S_1M_1 and S_2S_5 is $t_2/2$.
7.	S_6	$X_{S_7} = -r_1\tan(\phi_2) + \dfrac{t_2}{2\tan(\alpha_2)}$ $Y_{S_7} = -r_1 - \dfrac{t_2}{2}$	• S_6 lies on arbitrary line AO_2.
8.	S_9	$X_{S_9} = 0$ $Y_{S_9} = -r_1 - \dfrac{t_2}{2}$	• S_9 lies on an extending line O_2M_1 such that distance between lines AM_1 and S_6S_9 is $t_2/2$.
9.	S_7	$X_{S_7} = -r_1\tan(\phi_2) - \dfrac{t_2}{2\tan(\alpha_2)}$ $Y_{S_7} = -r_1 + \dfrac{t_2}{2}$	• S_7 lies on extending line AO_2.
10.	S_8	$X_{S_8} = 0$ $Y_{S_8} = -r_1 + \dfrac{t_2}{2}$	• S_8 lies on extending line O_2M_1 such that distance between lines AM_1 and S_7S_8 is $t_2/2$.

Tab. 3: Set of vertices in the planar map of star shaped compound motifs.

6. ENGRAVED STAMP-BLOCKS

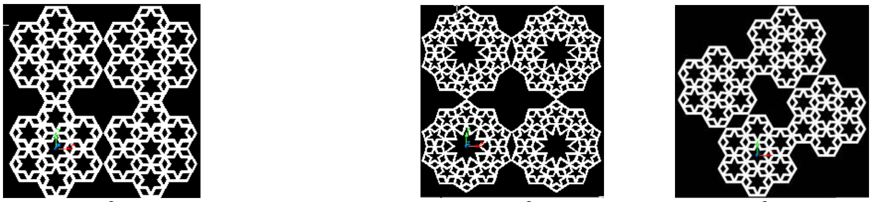
India has been renowned for its printing and embroidery on cloths using the hand engraved stamp-blocks which is shown in Fig. 18 [21, 25]. This part of the work replaces hand engraving with CNC milling/engraving machine and empowers the craftsmen to engrave patterns on a block, digitally created with the help of a CAD system.

Engraving is the practice of incising an ornamental pattern onto a hard surface, leaving a shallow groove in the geometry of pattern. The grooved surface is used to produce mould of rubber chemical compound. The rubber mould, when taken out of the surface, is replica of the engraved pattern and pasted on a wooden block to use it as a stamp. Patterns are printed on the cloth using stamp-blocks and then various stitches or embroideries can be employed to outline the pattern.



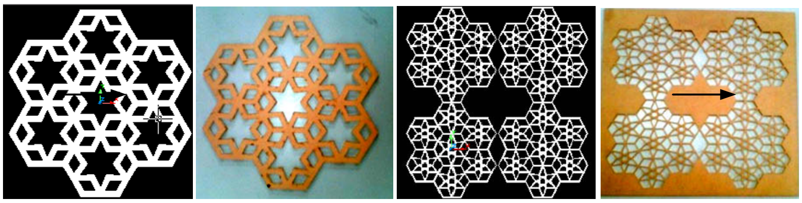
n_1=6, r_1=1, θ_1=20°, t_1=0. 2, t_2=0.2 n_1=6, r_1=1, θ_1=10°, t_1=0.2, t_2=0.2 n_1=6, r_1=1, θ_1=30°, t_1=0.2, t_2=0.2 n_2 =6, θ_2=20°, n_3=6, θ_3=20° n_2=6, θ_2=10°, n_3=6, θ_3=10° n_2=6, θ_2=30°, n_3=6, θ_3=10°

Fig. 15: Star shaped compound motifs.



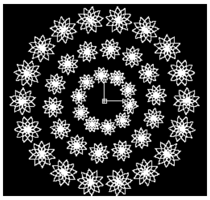
R=2, C=2, ∂=0 RO=4.82, CO=4.82 R=2, C=2, ∂=0 RO=4.12, CO=3.92 R=2, C=2, ∂=30° RO=6.0, CO=6.0

Fig. 16: Islamic geometric patterns for lattice-worked screens.



(a) (b)

Fig.17: CAD models and patterns cut by computer controlled laser cutting machine.



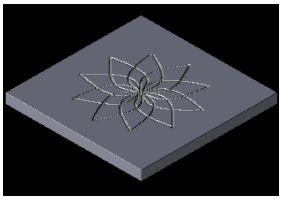


Fig. 18: A Traditional stamp-block [25]. Fig. 19: The floral pattern. Fig. 20: Engraved floral motif.

Pattern styles on the stamp-blocks vary from region to region and are based on the traditions. The Indian traditional floral patterns are generated using a hierarchic model, which makes abundant use of Primitive Instancing. Primitive Instancing reuses the same primitive (shape used to construct a pattern) at multiple locations in the pattern [6, 20]. The primitive has a fixed description but the individual instances carry other variable parameter information such as size, position and orientation. Hierarchy deals with dividing the pattern into several level shapes and can be represented as a rooted tree with nodes representing components of the pattern as shapes. Primitives used in the floral patterns are curves. Motif (leaf) is created by joining two curves at start and end. Compound motif (flower) is created using a polar array with a number of leaf motifs. The transformations define positions of motifs with respect to a pattern. The fit points of curves decide the shape of motifs. A sample floral pattern is shown in Fig. 19. These patterns are essentially 2D. The G-codes for these patterns can be generated using the CAM system. Fig. 20 shows an engraved floral motif.

7. CARVED WOODEN BLOCKS

Creating elaborate patterns in the wood with the help of carving tools is called wood carving. Traditionally, wood carving involves chiseling of wooden blocks that can be fit in the furniture, window or door to provide a finished and distinguished decoration.

This work generates traditional Zillij style of geometric ornamental patterns with potential to produce a variety using computer aided geometric modeling. These patterns are not made using random, but are drawn within the constraints of mathematical symmetry and the laws of proportion [5]. These patterns show rotational symmetry around a central point. In addition, these patterns look the same when reflected. Such types of patterns are shown in Fig. 21. The basic shapes in these patterns are stars and rosettes surrounded by a ring of regular polygons [7, 11, 27]. Such shapes with five, six, eight, ten, twelve and sixteen rays occur most frequently.

These rendered patterns are executed on flat wooden surface to produce wooden carvings in a computer controlled manufacturing environment. The modeling process for the ornamental patterns starts by devising mathematical descriptions and formulations using ubiquitous polygonal technique. CAD programming interfaces are used to turn the mathematical descriptions of the patterns into computer program which render the patterns on the computer screen. These geometrical patterns provide precise information to guide the paths of cutting tools and are in harmony with computer controlled manufacturing.

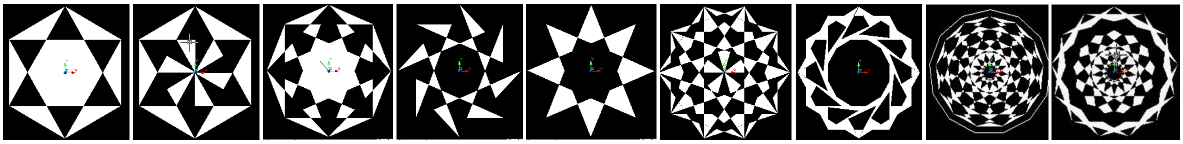


Fig. 21: The Zillij style of patterns.

The representation formalism for Zillij style patterns uses a classical geometric methodology known as the polygonal technique. This methodology makes use of diagonals grids drawn with different number of sides of regular polygons as shown in Fig 22. The patterns are created by selecting the regions of irregular polygons that have been resulted from the network of diagonals of a regular polygon. The vertices of these irregular polygons are identified on the basis of the points of intersection of diagonals of the n-sided regular polygon. The vertex numbers 'V_i' are put as parameters to find a point of intersection of two diagonals. The coordinates of an intersection point'N' obtained by joining the vertex numbers $V_1 - V_2$ and $V_3 - V_4$ of the regular polygon in the grid can be found by following equations. For example, b, c and d are the points of intersection of two diagonals 1-6 & 3-10, 1-4 & 3-10 and 1-4 & 2-9 respectively as shown in Fig. 22.

The rendered patterns are submitted to laser cutting machine which uses a beam of light as the carving tool, so no part of the machinery actually touches the material being carved. One of the patterns carved on the laser cutter is shown in Fig. 23. This pattern does not elaborate on laser cutting methodology but just presents a case study in the form of computer assisted manufacturing of a carved ornamental pattern.

8. BLOW-FORMED BOTTLES

Blow-formed bottles are constructed by placing hot, hollow and thermoplastic preform between two halves of a blow-mould, closing the mould, and then blowing the preform against the inside wall surface of the mould cavity. Blow-formed bottles are commonly made from Poly Ethylene Terephthalate (PET) and used in packaging of drinks, foodstuffs, and personal care products. Most of the PET bottles available in the market consists circumferential ribs [10]. Such types of bottles give almost similar appearances thus, decrease commercial value of the product filled in the bottle. The aesthetically appealing bottles create an effect on the purchasing choices made by consumers. So, it is desirable to adorn the PET bottles in an artistic manner to provide a level of appeal to customers.

$$X_N = \frac{r(m_{N_1}\sin(a_{v_1}) - (m_{N_2}\sin(a_{v_3}) + \cos(a_{v_3}) - \cos(a_{v_1}))}{m_{N_1} - m_{N_2}}$$

$$Y_N = m_{N_1}(X_N - r\sin(a_{v_1}) + r\cos(a_{v_1})$$

$$m_{N_1} = \frac{\cos(a_{v_2}) - \cos(a_{v_1})}{\sin(a_{v_2}) - \sin(a_{v_1})}$$

$$m_{N_2} = \frac{\cos(a_{v_4}) - \cos(a_{v_3})}{\sin(a_{v_4}) - \sin(a_{v_3})}$$

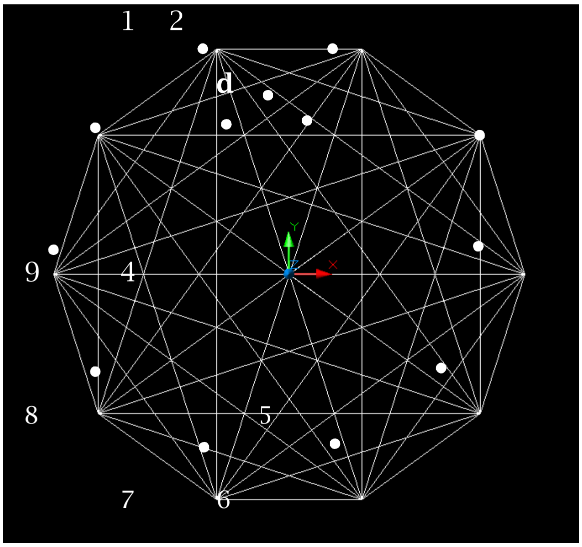


Fig. 22: The Diagonals grid representing vertex numbers and intersection points.

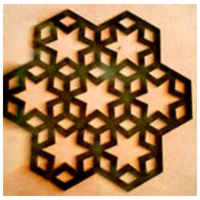
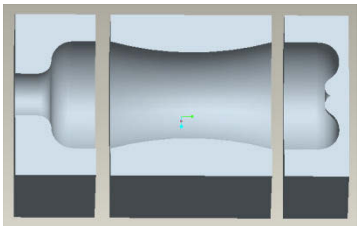


Fig. 23: Carved pattern on the laser cutter.

The blow mould is production tool for PET bottles and made in two same halves. A parametric modeling approach is employed within an integrated environment to create and customize blow moulds having bottle shaped cavities decorated with different styles of ornamental patterns. A library of 3D planar ornamental patterns is equipped, from which a pattern is selected, bended and then glued to mould cavity having different contours. Rendered CAD models of decorated moulds can be used to

produce prototypes using LM technology for showing to customers [26]. Moreover, NC programs can be automatically generated for the machining of decorated blow mould cavities for production purposes.
Two types of modeling components are created. Firstly, undecorated three parts of the mould having cavities corresponding to the three portions of the bottle (top, middle and bottom), as shown in Fig. 24.The two parts of the mould cavities take the shape of top portion of the bottle dome and base of bottle. Another part includes the middle portion of bottle which is an adjacent circumferential portion of top bottle dome. These undecorated parts of the mould are created by rotational sweep and their cavity shapes depend upon the profiles of sweep bases. Secondly, 3D ornamental patterns corresponding to decoration for the middle part of mould cavity as shown in Fig. 25.
The Toroidal Bend feature of a CAD modeling software is used to bend the 3D solid form of ornamental pattern into toroidal (revolved) shape. This bending feature rotates each of the parallel planes around the intersection of the neutral plane and the end surface of the pattern by the angle specified. The neutral plane defines the theoretical plane of zero deformation (elongation or compression) and is considered along the sectional thickness of the pattern.
The planar pattern is bent toroidally into the cavity shape of middle part of bottle mould by defining bend profile and angle. The shape of the cross section of the toroid or bend profile is created from a chain of entities like spline, arc, line, etc. A 3-point arc is selected as bending profile shown in Fig. 26. Two end points of the arc are fixed on extreme surfaces of the pattern and third point defines radius and centre of bending curvature. The two parallel planes at end surfaces of pattern are selected to pull towards each other at an angle of bend equal to 180 degrees about an axis passing through the center of curvature to generate the cavity shape of bottle mould half.
Referring to Fig. 27, the modeling process starts with selecting a pattern, then bending toroidally into the cavity shape. The process is followed by creating the stock solid of the middle section of the mould by sweeping a cross section along the trajectories of bent pattern using Variable Section Sweep feature of the system to control the section's orientation, rotation, and geometry.



Top Middle Bottom

Fig. 24: The three undecorated parts of a blow mould.

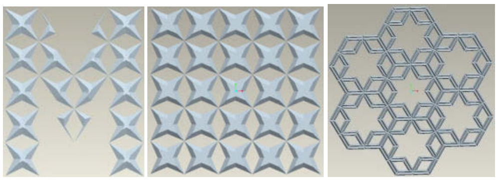
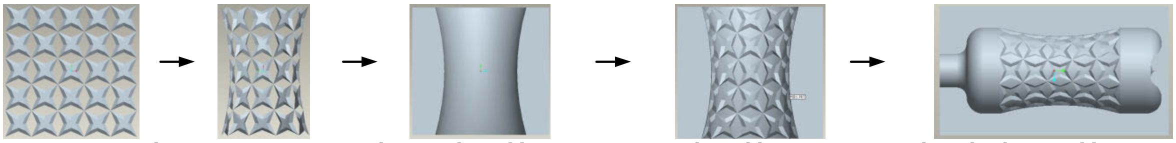


Fig. 25: The ornamental patterns.

Fig. 26: The bending profile corresponding to the cavity shape of middle part of mould.



Ornamental Pattern Bent PatternUndecorated Mould Section Decorated Mould SectionDecorated Bottle Blow Mould

Fig. 27: Modeling to produce decorated mould for blow-formed bottles.

9. CONCLUDING REMARKS

This work presents new CAD paradigms to develop products in the domain of personal embellishment, architectural ornaments as well as interior decoration. The idea is to use the advances in the field of Computer Aided Geometric Design and Computer Aided Manufacturing towards product ideation. The work is in the direction of integrated product development, as it proposes methodologies and tools to build up products and integrates CAD-CAM technologies. In this work, the designs of various ornamental products belonging to different eras are generated automatically with the help of algorithms developed and their fabrication methodologies are proposed so as to obtain the designers' objects.

This work explores different forms of ornamental patterns. A few novel approaches are proposed to produce stretch formed, carved and fret-worked jewelry. An algorithm is presented to generate Islamic geometric patterns using the parameterized motifs. This algorithm produces traditional style of lattice-worked screens by piercing a surface of wood or stone. The work also generates traditional style of Zillij patterns for wooden carvings. The Indian traditional floral patterns are also created to embed on printing stamp blocks for embroidery work on fabrics.

The geometric modeling techniques used in this work are simple, yet their results are very beautiful and inspiring. Most of the ornamental products are non functional, but decorative and important, because they strongly influence human beings' emotions and product's attractiveness. Such attributes are expressed via different forms of ornamental patterns. The goal of producing different forms of ornamental products has been achieved concisely and unambiguously in this work through CAD paradigms in unified manner.

The proposed CAD paradigms produce computer compatible models that support diverse integrated manufacturing. They possess a wide modeling range and capable to represent complete ornamental products and not just shapes which can be easily interfaced with computer controlled fabrication methods such as CNC milling, laser cutting and engraving, water-jet cutting and Layered Manufacturing Technologies.

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