Computer-Aided Design and Applications

http://www.cadanda.com

Compound-Rhythm Log-Aesthetic Curves

Norimasa Yoshida[1] and Takafumi Saito[2]

[1]Nihon University, norimasa@acm.org
[2]Tokyo University of Agriculture and Technology, txsaito@cc.tuat.ac.jp

ABSTRACT

This paper presents an efficient and stable method for drawing compound-rhythm log-aesthetic curves. Compound-rhythm curves are curves whose logarithmic curvature graphs are represented by V-type or upside down V-type segments. We show that, once the continuity condition is derived, compound rhythm curves can be efficiently generated in a similar manner to generating monotonic rhythm curves. We also present a method for drawing compound-rhythm curves by specifying two endpoints, their tangential directions, α_0 and α_1 (which are the slopes of logarithmic curvature graphs) and the ratio r_θ of the change of the tangential angle of the curve α_0 to the change of the tangential angle of the whole curve.

Keywords: compound-rhythm log-aesthetic curves, monotonically varying curvature.
DOI: 10.3722/cadaps.2009.243-252

1. INTRODUCTION

For the generation of aesthetic surfaces, the generation of aesthetic curves themselves is very important. Log-aesthetic curves [5,8,10,11,13] are curves based on Harada's quantitative analysis of many aesthetic curves in artificial and the natural objects. Harada found that many aesthetic curves are curves whose logarithmic curvature graphs (LCGs) are almost linear. When Harada proposed the curves with linear LCGs, the curves include monotonic-rhythm curves and compound-rhythm curves [5,13]. A monotonic-rhythm curve is a curve whose LCG is represented by a single line segment. A compound-rhythm curve is a curve whose LCG is represented by two connected line segments, which are V-type or upside down V-type. Harada analyzed the characteristic lines of automobiles and he found that many of such curves are compound-rhythm curves [5]. In the previous work [10], only the generation method for monotonic-rhythm curves is clarified.

In this paper, we clarify an efficient and stable method for drawing compound-rhythm log-aesthetic curves. The contributions of this paper are the following:

(1) We show that a compound-rhythm curve can be generated by one dimensional search. We show that, once the continuity condition of a compound-rhythm curve is derived, we can efficiently generate compound-rhythm curves in a similar manner to monotonic-rhythm curves.
(2) We present a method for drawing a compound-rhythm curve by specifying two endpoints, their tangents, the slopes of the LCG α_0 and α_1 of each curve segment in the compound-rhythm curve, and the ratio r_θ of the change of the tangential angle of the curve α_0 to the change of the tangential angle of the whole curve.

The rest of the paper is organized as follows. Section 2 reviews relevant literature and log-aesthetic curves. In Section 3, we derive the continuity condition of the LCG for generating compound-rhythm curves. Section 4 presents a method for drawing compound-rhythm curves. Section 5 shows examples of compound rhythm curves. Finally, in Section 6, conclusions and future work are presented.

2. RELATED WORK AND LOG-AESTHETIC CURVES

This section reviews related work and briefly introduces log-aesthetic curves for the explanation of generating compound-rhythm curves in Section 4. More details of log-aesthetic curves, which are originally called aesthetic curves, can be found in [5,8,10,11,13].

In previous work, there is a lot of work for generating curves with monotonically varying curvature. Some of them are [1-14]. Most of the work, however, is related to generating curves with monotonically varying curvature and does not care much about the curvature variation. Log-aesthetic curves are curves in which curvature variation can be controlled by one parameter α .

A log-aesthetic curve is a curve whose LCG [5,8,12,13] is represented by a straight line whose slope is α. Let s and ρ be the arc length and the radius of curvature, respectively. We assume that the radius of curvature is monotonically increasing with respect to the arc length. Then the horizontal axis of the LCG is $\log\rho$ and the vertical axis is $\log\left(\rho\frac{\mathrm{d}s}{\mathrm{d}\rho}\right)$. The linearity of the LCG constrains that the curvature is monotonically varying. α can also be considered as the type of the curves. Depending on α, log-aesthetic curves include the Clothoid ($\alpha=-1$), Nielsen's spiral ($\alpha=0$), the logarithmic spiral ($\alpha=1$), the circle involute ($\alpha=2$) and the circle ($\alpha=\pm\infty$). In other words, log-aesthetic curves can be considered as the generalization of these curves. Log-aesthetic curves are originally proposed by Harada et.al. [5,13]. Miura derived the general formula[8] and Yoshida and Saito clarified the overall shapes and presented a method for drawing a curve segment like a quadratic Bézier curve when α is specified [10]. Yoshida and Saito have also proposed quasi-log-aesthetic curves that approximate log-aesthetic curves by rational cubic Bézier curves [11].

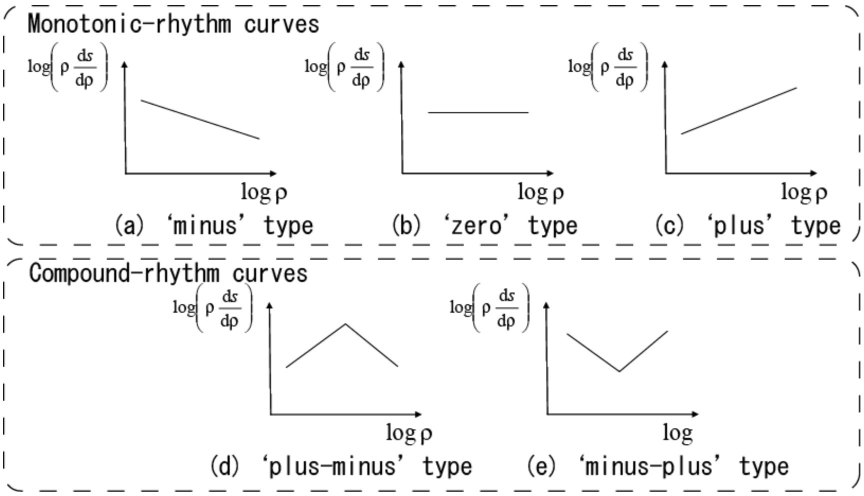


Fig. 1: Five fundamental types of log-aesthetic curves.

When Harada originally proposed log-aesthetic curves, it has 5 types of curves shown in Fig. 1. Harada called a curve whose LCG is represented by a single segment, such as Fig. 1 (a), (b) or (c), a monotonic-rhythm curve. A curve whose LCG is represented by two connected segments is called a compound-rhythm curve. In this paper, we present a method for generating compound-rhythm log-aesthetic curves.

To generate compound rhythm curves, we consider the standard form of log-aesthetic curves that are introduced in [10]. We briefly review the standard form of log-aesthetic curves. To make a log-aesthetic curve to be in the standard form, we select a reference point of the curve; translate the reference point to the origin; rotate the curve such that whose tangential direction becomes the direction of positive x -axis at the reference point; and scale the curve such that the radius of curvature at the reference point becomes 1. The reference point can be any point on the curve whose radius of curvature is not 0 nor ∞. Let the tangential angle and the slope of the straight line in the LCG be θ and α, respectively. Let i be the imaginary unit. The equation of a (monotonic-rhythm) log-aesthetic curve $\mathbf{P}_{AE}(\theta)$ [10] is given, on the complex plane, by

$$\mathbf{P}_{AE}(\theta)=\begin{cases} \int_0^\theta e^{(\Lambda+i)\varphi}d\varphi & \text{if } \alpha=1 \\ \int_0^\theta ((\alpha-1)\Lambda\varphi+1)^{\frac{1}{\alpha-1}}e^{i\varphi}d\varphi & \text{otherwise} \end{cases} \tag{2.1}$$

Here, Λ is $\frac{\mathrm{d}s}{\mathrm{d}\rho}$ at the reference point. In other words, when $\alpha \neq 1$, Λ can be considered as a parameter that chooses the reference point. When $\alpha=1$, Λ is the parameter that changes the shape of the curve. See [10] for details. Eqn. (2.1) is the equation of log-aesthetic curves formulated by the tangential angle. We also have the equation of log-aesthetic curves formulated by the arc length [10]. The relationship between the radius of curvature ρ and θ is given by

$$\rho=\begin{cases} e^{\Lambda\theta} & \text{if } \alpha=1 \\ ((\alpha-1)\Lambda\theta+1)^{\frac{1}{\alpha-1}} & \text{otherwise} \end{cases} \tag{2.2}$$

The relationship between θ and the arc length s is given by

$$\theta=\begin{cases} \frac{1-e^{\Lambda s}}{\Lambda} & \text{if } \alpha=0 \\ \frac{\log(\Lambda s+1)}{\Lambda} & \text{if } \alpha=1 \\ \frac{(\Lambda\alpha s+1)^{\left(1-\frac{1}{\alpha}\right)}-1}{\Lambda(\alpha-1)} & \text{otherwise} \end{cases} \tag{2.3}$$

In log-aesthetic curves, ρ,θ and s are related by Eqn. (2.2) and Eqn. (2.3). If one of ρ,θ and s is decided, other two parameters are automatically computed using Eqn. (2.2) and Eqn. (2.3). Again, since ρ changes from 0 to ∞, θ and s may have a upper or lower bound[10] depending on α. We show the bounds in Tab. 1.

	s	
	Lower bound	Upper bound
$\alpha<0$	-	$-\frac{1}{\Lambda\alpha}$
$\alpha=0$	-	-
$\alpha>0$	$-\frac{1}{\Lambda\alpha}$	-

	θ	
	Lower bound	Upper bound
$\alpha<1$	-	$\frac{1}{\Lambda(1-\alpha)}$
$\alpha=1$	-	-
$\alpha>1$	$\frac{1}{\Lambda(1-\alpha)}$	-

Tab. 1: The bounds of s and θ depending on α.

In [10], Yoshida and Saito have proposed a method for drawing a monotonic-rhythm log-aesthetic curve segment by specifying two endpoints and their tangents. We are given three points $\mathbf{P}_a,\mathbf{P}_b,\mathbf{P}_c$ and α. For drawing a log-aesthetic curve segment, we search for a triangle $\mathbf{P}_0\mathbf{P}_1\mathbf{P}_2$ on the standard form that is similar to $\mathbf{P}_a\mathbf{P}_b\mathbf{P}_c$ by changing Λ in Eqn. (2. 1). If a similar triangle is found, the points on the curve in the standard form is transformed under a similarity transformation such that $\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2$ are moved to $\mathbf{P}_a, \mathbf{P}_b, \mathbf{P}_c$. To stably compute the (nearby) point at $\rho = 0$ or $\rho = \infty$, we use two configurations depending on α. See Fig. 2 for the two cases.

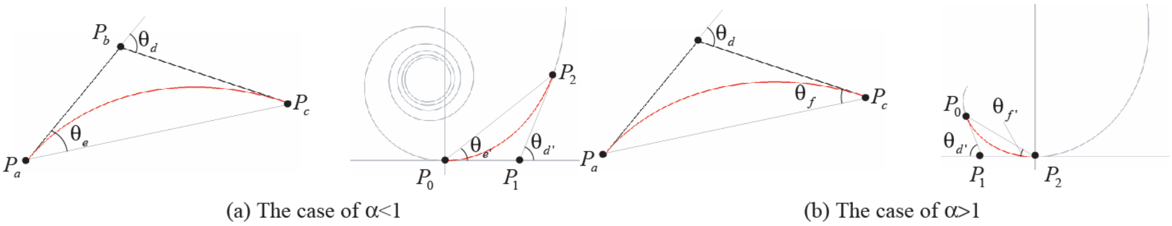


Fig. 2: Curve segments and their corresponding curve segments in the standard from.

3. The Continuity Condition of the Logarithmic Curvature Graph

This section derives the C^0 continuity condition of the two segments in the LCG. In the next section, we use this condition to generate compound rhythm curves. The continuity condition plays a key role for generating compound-rhythm curves.

We consider a compound-rhythm log-aesthetic curve segment whose LCG is represented by two segments. The slopes of the segments are α_0 and α_1, respectively. See Fig. 3. In compound-rhythm curves proposed by Harada, the signs of α_0 and α_1 are always different. See Fig. 1 (d) and (e). However, we do not assume that the signs are different. Thus α_0 and α_1 can be arbitrary.

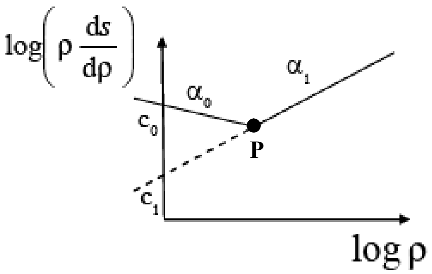


Fig. 3: Two connected segments in the LCG.

For the LCG to be continuous, both $\log \rho$ and $\log\left(\rho \frac{\mathrm{d}s}{\mathrm{d}\rho}\right)$ of the two segments must have the same value at the connection point $\mathbf{P}$. Since ρ and $\frac{\mathrm{d}\rho}{\mathrm{d}s}$ are related to the second and third derivatives of the curve respectively, the continuity of the LCG indicates that the curve should be G^3 continuous.

The linearity of two segments in the LCG is represented by

$$\log\left(\rho \frac{\mathrm{d}s}{\mathrm{d}\rho}\right) = \alpha_0 \log \rho + c_0 \tag{3.1}$$

$$\log\left(\rho \frac{\mathrm{d}s}{\mathrm{d}\rho}\right) = \alpha_1 \log \rho + c_1 \tag{3.2}$$

where c_0 and c_1 are constants. For the above two line segments to be C^0 continuous,

$$\alpha_0 \log\rho + c_0 = \alpha_1 \log\rho + c_1 \tag{3.3}$$

and ρ at the connection point must be the same. By modifying Eqn. (3.3), we get

$$e^{-c_1} = \frac{e^{c_0}}{\rho^{\alpha_0-\alpha_1}}. \tag{3.4}$$

Similar to [10], we set $\Lambda_0 = e^{-c_0}$ and $\Lambda_1 = e^{-c_1}$. Λ_0 and Λ_1 have the same meaning as Λ in Section 2. Then Eqn. (3.3) becomes

$$\Lambda_1 = \Lambda_0 \rho^{(\alpha_1-\alpha_0)}. \tag{3.5}$$

Therefore, for the two segments to be C^0 continuous in the LCG, ρ at the connection point of the two curve segments must be the same and Eqn. (3.5) must be satisfied. Note that the continuity in the LCG does not necessarily mean that the curve is C^0 or G^1 continuous. Suppose that there are two curve segments whose LCG is continuous and G^3 continuous. Translating and/or rotating one of the curve segments does not change their LCG. LCG is only related to the second and the third derivatives of the curves, which are related to the curvature and the derivative of curvature with respect to the arc length, respectively.

For generating a compound-rhythm curve, the two curve segments in the compound rhythm curve should be G^2 continuous and Eqn. (3.5) must be satisfied. As will be explained in the next section, G^2 continuity can be easily achieved. Thus the continuity condition of the LCG reduces to Eqn. (3.5).

4. Generation of Compound Rhythm Curves

This section presents a method for drawing a compound-rhythm curve when the two endpoints, their tangents, the slopes of the LCGs of the two curve segments α_0 and α_1 and the ratio r_θ of the change of the tangential angle of the curve α_0 with respect to the change of the tangential angle of the curve α_1. The two endpoints and their tangents are specified by three points $\mathbf{a}_0, \mathbf{a}_1$ and $\mathbf{a}_2$ like a quadratic Bézier curve. See Fig. 4.

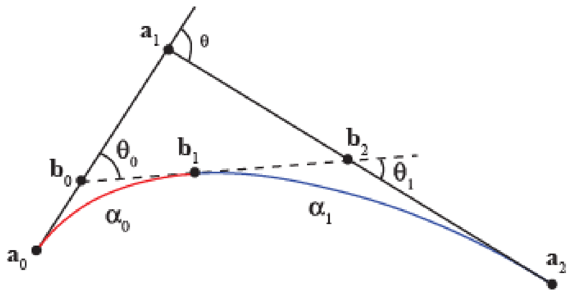


Fig. 4: Generation of a compound-rhythm curve.

Referring to Fig. 4, the change of tangential angle θ of the whole curve is computed as the angle formed by the vectors $\mathbf{a}_1 - \mathbf{a}_0$ and $\mathbf{a}_2 - \mathbf{a}_1$. θ_0 and θ_1 are computed by $\theta_0 = r_\theta \cdot \theta$ and $\theta_1 = \theta - \theta_0$, respectively.

Without loss of generality, we assume that $|\mathbf{a}_1 - \mathbf{a}_0| \le |\mathbf{a}_2 - \mathbf{a}_1|$. If this inequality does not hold, we just swap the coordinates of $\mathbf{a}_0$ and $\mathbf{a}_2$. With this assumption, the radius of curvature monotonically increases from $\mathbf{a}_0$ to $\mathbf{a}_2$. The radius of curvature is the smallest at $\mathbf{a}_0$ and the largest at $\mathbf{a}_2$.

Similar to a monotonic-rhythm curve, a compound-rhythm curve is generated by searching for the similar triangle formed by a compound-rhythm curve on the standard form to the triangle formed by

$\mathbf{a}_0, \mathbf{a}_1$ and $\mathbf{a}_2$. If the similar triangle is found, the points on the compound-rhythm curve in the standard form are transformed such that the vertices of the similar triangle are transformed to $\mathbf{a}_0, \mathbf{a}_1$ and $\mathbf{a}_2$ under a similarity transformation.

For monotonic-rhythm curves, we have used two configurations to stably compute a curve that have a nearby point at $\rho = 0$ or $\rho = \infty$. When $\alpha \le 1$, we have used the configuration shown in Fig.2 (a). When $\alpha > 1$, we have used the configuration shown in Fig. 2 (b). Due to the same reason as monotonic-rhythm curves, we also use two configurations for drawing compound-rhythm curves depending on α_0.

We first consider the case of $\alpha_0 < 1$. We do not assume anything about α_1. Thus α_1 can be an arbitrary value. Currently, we do not know the value of Λ_0. Since Λ_0 must be between 0 and $\frac{1}{\theta_0(1-\alpha_0)}$ (derived from Tab. 1), we assume, for example, $\Lambda_0 = \frac{1}{2\theta_0(1-\alpha_0)}$. As will be explained shortly, finding a compound-rhythm curve reduces finding an appropriate Λ_0. Since we know θ_0, we can draw an log-aesthetic curve of α_0 using Eqn. (2.1) from $\theta = 0$ to θ_0. Now in Fig. 5(a), the curve is drawn from $\mathbf{p}_0$ to $\mathbf{p}_2$. $\mathbf{p}_1$ is the intersection of tangent lines at $\mathbf{p}_0$ and $\mathbf{p}_2$. Using Eqn. (2.2), we compute the radius of curvature ρ at θ_0. Now we can compute Λ_1 using Eqn. (3.5). Using the computed ρ and Λ_1 guarantee that the second and third derivatives of the two curves with respect to the arc length are the same. Using Λ_1 and ρ at $\mathbf{p}_2$ ($\theta = \theta_0$), we compute the starting tangential angle θ_s of the curve α_1 using Eqn. (2.3). We compute the curve of α_1 using Eqn. (2.1) from θ_s to $\theta_s + \theta_1$ on the standard form, translate the curve such that the starting point becomes $\mathbf{p}_2$ and rotate the curve such that the tangential direction at the starting point becomes equal to the tangential direction of the curve α_0 at $\mathbf{p}_2$. The translation and rotation guarantee that the two curves will be G^2 continuous. Now, the curve α_1 from $\mathbf{p}_2$ to $\mathbf{p}_4$ is drawn as shown in Fig. 5(a). $\mathbf{p}_3$ is the intersection point of tangent lines of the curve α_1 at $\mathbf{p}_2$ and $\mathbf{p}_4$. $\mathbf{p}_5$ is the intersection of the tangent lines at $\mathbf{p}_0$ and $\mathbf{p}_4$. If the triangle formed by $\mathbf{p}_0, \mathbf{p}_5, \mathbf{p}_4$ and the triangle formed by $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ are similar, we can draw a compound-rhythm curve by transforming the curve on the standard form such that $\mathbf{p}_0, \mathbf{p}_5, \mathbf{p}_4$ are transformed to $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ under a similarity transformation. We search for a similar triangle by changing Λ_0. Since the angle formed by $\mathbf{p}_5 - \mathbf{p}_0$ and $\mathbf{p}_4 - \mathbf{p}_5$ and the angle formed by $\mathbf{a}_1 - \mathbf{a}_0$ and $\mathbf{a}_2 - \mathbf{a}_1$ are always the same, the similarity of the triangles can be checked by comparing the angle formed by $\mathbf{p}_5 - \mathbf{p}_0$ and $\mathbf{p}_4 - \mathbf{p}_0$ and the angle formed by $\mathbf{a}_1 - \mathbf{a}_0$ and $\mathbf{a}_2 - \mathbf{a}_0$. If the two angles are the same, we can say that the two triangles are similar. Similarly as monotonic rhythm curves, we use the bisection method to find Λ_0. When $\alpha_0 < 1$, the range of Λ_0 is $0 \le \Lambda_0 \le \frac{1}{\theta_0(1-\alpha_0)}$. When $\alpha_0 = 1$, the range of Λ_0 is $0 \le \Lambda_0 \le \infty$. When $\alpha_0 = 1$, the bisection method is extended so that Λ_0 can be arbitrarily large.

A compound rhythm curve with $\alpha_0 > 1$ and arbitrary α_1 can be generated in a similar manner except that the configuration shown in Fig. 5 (b) is used. We assume that Λ_0 is known and draw the curve of α_0 from $\theta = 0$ to $-\theta_0$. The curve is drawn from $\mathbf{p}_2$ to $\mathbf{p}_0$ as shown in Fig. 5 (b). $\mathbf{p}_1$ is the intersection of tangent lines of the curve α_0 at $\mathbf{p}_0$ and $\mathbf{p}_2$. Using Eqn. (3.5), we compute Λ_1. ρ at $\mathbf{p}_2$ is always 1 because of the constraint of the standard form. We compute the starting tangential angle θ_s of the curve α_1 using Eqn. (2.3). We then compute the curve of α_1 using Eqn. (2.1) from θ_s to $\theta_s + \theta_1$ on the standard form and rotate the curve such that the tangential direction at the starting point becomes equal to the tangential direction of the curve α_0 at $\mathbf{p}_2$. As shown in Fig. 5 (b), we can draw the curve α_1 from $\mathbf{p}_2$ to $\mathbf{p}_4$. $\mathbf{p}_3$ is the intersection point of tangent lines of the curve α_1 at $\mathbf{p}_2$ and $\mathbf{p}_4$. $\mathbf{p}_5$ is the intersection of the tangent lines at $\mathbf{p}_0$ and $\mathbf{p}_4$. If the triangle formed by $\mathbf{p}_0,\mathbf{p}_5,\mathbf{p}_4$ and the triangle formed by $\mathbf{a}_0,\mathbf{a}_1,\mathbf{a}_2$ are similar, we can draw a compound-rhythm curve by transforming the curve on the standard form such that $\mathbf{p}_0,\mathbf{p}_5,\mathbf{p}_4$ are transformed to $\mathbf{a}_0,\mathbf{a}_1,\mathbf{a}_2$ under a similarity transformation. The search for a similar triangle is performed by changing Λ_0 using the bisection method. We change Λ_0 within the range of $0 \le \Lambda_0 \le \dfrac{1}{\theta_0(\alpha_0 - 1)}$.

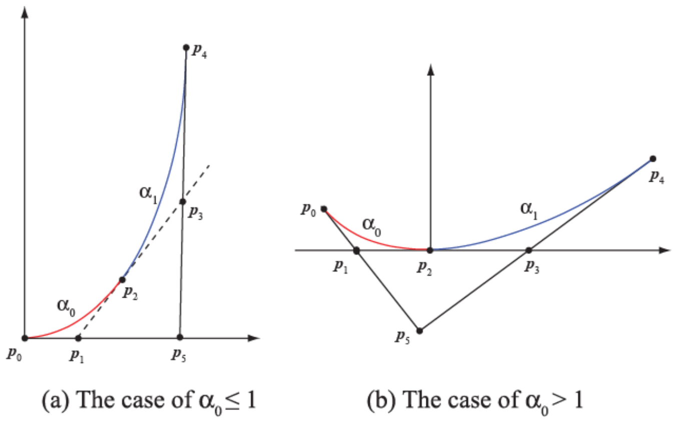


Fig. 5: Two configurations for generating compound-rhythm curves.

In the above, we do not talk about the bounds of $\theta_s + \theta_1$, which is very important for the stable computation. When $\alpha_1 < 1$, $\theta_s + \theta_1$ have an upper bound. Note that this bound does not depend on α_0. If $\alpha_1 < 1$, $\theta_s + \theta_1 < \dfrac{1}{\Lambda_1(1-\alpha_1)}$ must hold without depending on α_0. If this inequality does not hold, we search for a smaller Λ_0 in the bisection method such that Λ_1 computed by Eqn.(3.5) gets smaller.

Fig. 6 shows a process for finding a similar triangle. Fig. 6 (a) show the positions of $\mathbf{a}_0,\mathbf{a}_1,\mathbf{a}_2$. The search for a similar triangle is performed by changing Λ_0 using the bisection method. As the curves change from Fig. 6 (b) to (f), the triangles formed by $\mathbf{p}_0,\mathbf{p}_1,\mathbf{p}_2$ get similar to the triangle formed by $\mathbf{a}_0,\mathbf{a}_1,\mathbf{a}_2$.

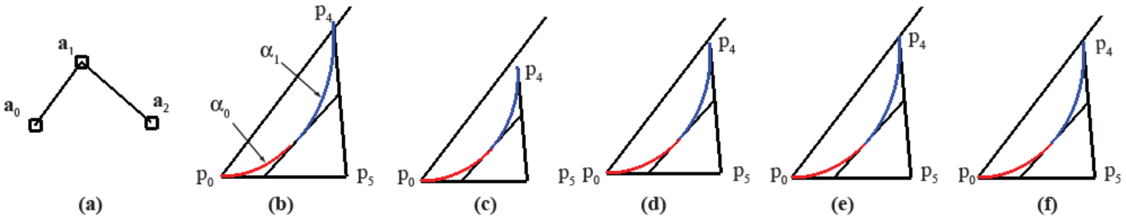
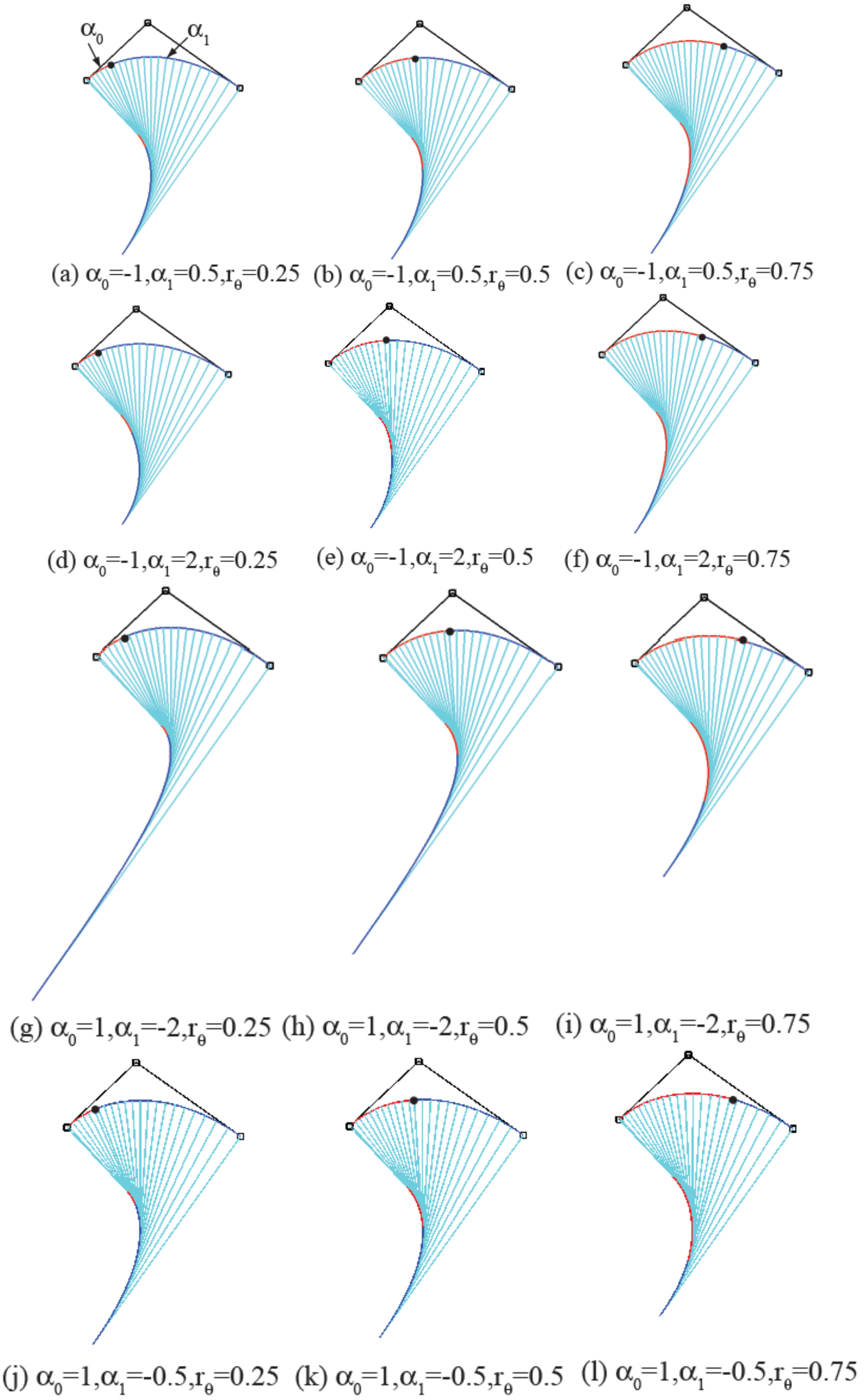


Fig. 6: A search process for generating a compound-rhythm curve ($\alpha_0 = -1$, $\alpha_1 = 1.5$).

5. RESULTS

We show examples of compound-rhythm curves generated by the proposed method. Fig. 7 shows various compound-rhythm curves and their evolutes with the same endpoints and the tangents.

Evolutes are curves that are the loci of the center of the curvature. A black dot on a curve indicates the connection point of the curve α_0 and the curve α_1. As was explained earlier, compound-rhythm



(a) $\alpha_0=-1, \alpha_1=0.5, r_\theta=0.25$ (b) $\alpha_0=-1, \alpha_1=0.5, r_\theta=0.5$ (c) $\alpha_0=-1, \alpha_1=0.5, r_\theta=0.75$

(d) $\alpha_0=-1, \alpha_1=2, r_\theta=0.25$ (e) $\alpha_0=-1, \alpha_1=2, r_\theta=0.5$ (f) $\alpha_0=-1, \alpha_1=2, r_\theta=0.75$

(g) $\alpha_0=1, \alpha_1=-2, r_\theta=0.25$ (h) $\alpha_0=1, \alpha_1=-2, r_\theta=0.5$ (i) $\alpha_0=1, \alpha_1=-2, r_\theta=0.75$

(j) $\alpha_0=1, \alpha_1=-0.5, r_\theta=0.25$ (k) $\alpha_0=1, \alpha_1=-0.5, r_\theta=0.5$ (l) $\alpha_0=1, \alpha_1=-0.5, r_\theta=0.75$

Fig. 7: Various compound-rhythm curves with the same end points and tangents.

curves are G^3 continuous. From Fig. 7, we can see that the curves are G^3 continuous since their evolutes are G^1 continuous.

Fig. 8 shows another example of compound-rhythm curves. Fig.8 (a),(b),(e) and (f) show curves in which both α_0 and α_1 being positive or negative.

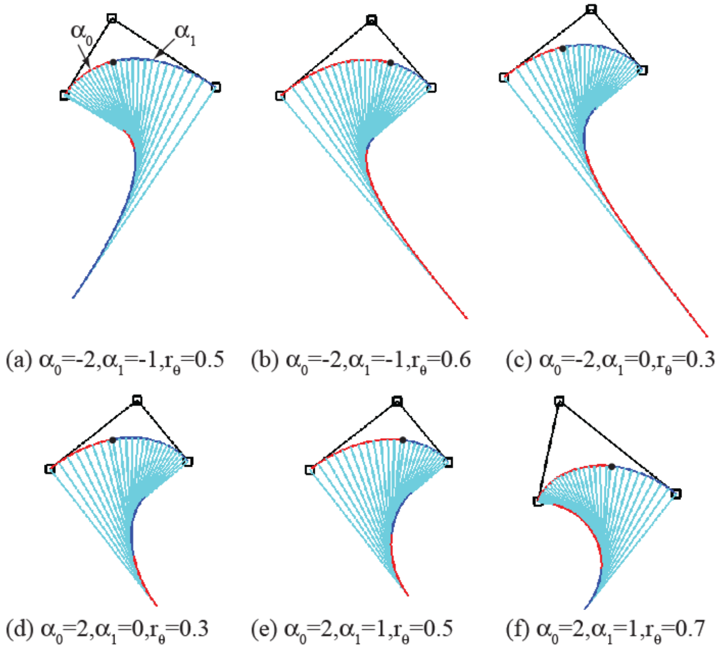


Fig. 8: Another example of compound-rhythm curves.

6. CONCLUSIONS

We have presented a method for drawing compound-rhythm log-aesthetic curves by specifying two endpoints, their tangents, the slopes (α_0 and α_1) of the straight lines in the LCG, the ratio r_θ of the change of the tangential angle of the curve α_0 with respect to the change of the tangential angle of the curve α_1. We have derived the continuity condition of the LCG and using the continuity condition, we showed that the generation of compound-rhythm curves reduces to one-dimensional search.

Due to the constraint of monotonically varying curvature, the position of three points, α_0, α_1 and r_θ dictate whether a compound-rhythm curve is drawn or not. One direction of future research is clarifying the drawable regions depending on these parameters. Another area of future research includes the approximation by free-form curves, such as NURBS or Bézier curves, and the extension to 3D log-aesthetic curves.

7. REFERENCES

[1] Dietz, A. D.; Piper B.: Interpolation with cubic spirals, Computer Aided Geometric Design, 21(2), 2004, 165-180.
[2] Dietz, A. D.; Piper, B.; Sebe, E.; Rational cubic spirals, Computer-Aided Design, 40(1), 2008, 3-12.
[3] Farin, G: Class A Bézier Curves, Computer-Aided Geometric Design, 23(7), 2006, 573-581.
[4] Frey, W. H.; Field, D. A.: Designing Bézier conic segments with monotone curvature, Computer Aided Geometric Design, 17(6), 2000, 457-483.
[5] Harada, T.; Yoshimoto, F.; Moriyama, M.: An aesthetic curve in the field of industrial design. In: Proceedings of IEEE Symposium on Visual Languages, IEEE Computer Society Press, New York, 1999, 38–47.

[6] Higashi, M., Kaneko, K., Hosaka, M.: Generation of high quality curve and surface with smoothly varying curvature, Eurographics, 79-92, 1998.
[7] Mineur, Y.: A shape controlled fitting method for Bézier curves, Computer Aided Geometric Design, 15(9), 879-891, 1998.
[8] Miura, K. T.: A general equation of aesthetic curves and its self-affinity, Computer-Aided Design and Applications, 3(1-4), 2006, 457–464.
[9] Sapidis, N. S.; Frey, W. H.: Controlling the curvature of quadratic Bézier curve, Computer Aided Geometric Design, 9 (2), 85-91, 1992.
[10] Yoshida, N.; Saito, T.: Interactive Aesthetic Curve Segments, The Visual Computer (Proc. of Pacific Graphics), 22(9-11), 2006, 896-905.
[11] Yoshida, N.; Saito, T.: Quasi-Aesthetic Curves in Rational Cubic Bézier Forms, Computer Aided Design and Applications, 4(1-4), 2007, 477-486.
[12] Yoshida, N.; Hiraiwa, T.; Saito, T.: Interactive Control of Planer Class A Bézier Curves using Logarithmic Curvature Graphs, Computer Aided Design and Applications, 5(1-4), 2008, 121-130.
[13] Yoshimoto, F.; Harada, T.: Analysis of the characteristics of curves in natural and factory products. In: Proc. of the 2nd IASTED International Conference on Visualization, Imaging and Image Processing, 2002, 276–281.
[14] Wang, Y.; Zhao, B.; Zhang, L.; Xu, J.; Wang, K.; Wang, S.: Designing fair curves using monotone curvature pieces, Computer Aided Geometric Design, 21 (5), 2004, 515-527.