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Discrete Log-Aesthetic Filter

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ABSTRACT

The aesthetic curves include the logarithmic (equiangular) spiral, clothoid, and involute curves. Although most of them are expressed only by an integral form of the tangent vector, it is possible to interactively generate and deform them and they are expected to be utilized for practical use of industrial and graphical design. However, the curvature of a log-aesthetic curve segment must be monotonically increasing or decreasing and a strong constraint as a function of arc length is imposed on its curvature. For geometrical design, it is desirable not to impose strong constraints on the designer's activity, to let him/her design freely and to embed the properties of the log-aesthetic curves for complicated curves with both increasing and decreasing curvature. Hence we develop discrete filters named discrete log-aesthetic filters to fair a sequence of points with noises and to fit it locally to log-aesthetic curves. Furthermore we extend them for surfaces.

Keywords: log-aesthetic curve and filter, digital filter, discrete curve and surface.
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1. INTRODUCTION

Thanks to the developments of the measurement and information technology, the reverse engineering is utilized which generate digital models on the computers from 3 dimensional physical models in clay or wood. Because free-form surfaces are frequently used for aesthetic design and their measurement data are usually huge and they have errors, it is very difficult to generate a high-quality digital model with smooth changes of curvature from such data.

The log-aesthetic curves include the logarithmic (equiangular) curve (the slope of the LCG: logarithmic curvature graph $\alpha = 1$), the clothoid curve ($\alpha = -1$), the circle involute ($\alpha = 2$) and Nielsen's spiral ($\alpha = 0$).Recently the generalized Cornu spiral [11] has been reported to be one of the log-aesthetic curves (when $\alpha = 1$ or $\alpha = -1$) since its curvature profile is given by a rational linear function and so its LCG gradient is given by a straight line function [10]. It is possible to generate and deform the log-aesthetic curve in real time even if they are expressed by integral forms using their unit tangent vectors as integrands ($\alpha \neq 1, 2$) and they are expected to be used in practical applications [17, 26]. However, along a segment of the log-aesthetic curve the curvature must be monotonically increased or decreased and a strong constraint depending on its arc length is imposed on its curvature. For geometric modeling, it is desirable for the designers not to give strong constraints and let them design shapes freely and methods with less constrained form are preferred to embed the properties of the log-aesthetic curve into curves whose curvature profile might both increase and decrease.

Therefore, in this paper we propose a filter named log-aesthetic filter that removes noises from a sequence of points obtained by a curve measurement, smooth them out and make the curve log-aesthetic as well. Furthermore we extend it for surfaces.

The rest of the paper is organized as follows. Section 2 describes related work and sections 3, 4, and 5 explain our filters for the plane, space curves and surface, respectively. Finally, we conclude the paper in section 6 with a discussion of future work.

2. RELATED WORK

2.1 Log-aesthetic Curve

"Aesthetic curves" were proposed by Harada et al. [7] as such curves whose logarithmic distribution diagram of curvature (LDDC) is approximated by a straight line. Miura [17] derived analytical solutions of the curves whose logarithmic curvature graph (LCG): an analytical version of the LDDC [7] are strictly given by a straight line and proposed these lines as general equations of aesthetic curves. Furthermore, Yoshida and Saito [26] analyzed the properties of the curves expressed by the general equations and developed a new method to interactively generate a curve by specifying two end points and the tangent vectors there with three control points as well as α: the slope of the straight line of the LCG. In this research, we call the curves expressed by the general equations of aesthetic curves the log-aesthetic curves.

The problems of the connection of plural log-aesthetic segments was dealt by Miura [19] and an input method of the compound-rhythm log-aesthetic curve which consist of two log-aesthetic curve segments connected with C^3 continuity was proposed by Agari [1]. Furthermore an extension of the planar log-aesthetic curve into space: the log-aesthetic curve was proposed by Miura et al. [18] and it was classified by Yoshida and Saito [27].

2.2 Discrete Filter

For the generation of high-quality surfaces used for car styling design, Farin et al. [6] proposed a surface smoothing method which for each character line of the surface, sequentially selects a point on the curve where the curvature variation criterion introduced by them is the highest in the curve and locally smooth the curve around the point. In their method, first a B-spline curve is fitted to an input sequence of points. Then they extract such a point where the difference of the third derivative, or the derivative of the curvature is the largest and remove a knot corresponding to the point. By repeating this process, they modify the shape of the curve to have a smooth curvature plot whose horizontal axis is the arc length and vertical axis is the curvature. Eck and Jaspert proposed a method to use the difference of the curvature calculated discretely as a local criterion as a fairing method of a sequence of points without B-spline curves [5]. Wagner proposed a method to smooth trajectories of robot manipulation using fourth differences of the sequence of points instead of curvature [23]. Based on the method proposed by Wagner, Higashi and Yamada made it applicable to a curve with a non-uniform knot vector by replacing fourth difference with fourth divided difference and extended it to discrete surfaces which may have defect points [8,9].

The purpose of the methods mentioned above is for aesthetic design and they can yield curves and surfaces of a certain quality from the view point of the monotonicity of the curvature variation, or smoothness, but they do not remove the curvature instability that exists in polynomial curves like B-spline pointed out by Miura [15]. We cannot control the curvature as we do for the log-aesthetic curve.

On the other hand, in the CG field the accuracy which is required for aesthetic design is not necessary and the purpose of smoothing geometrical objects is for rendering polygonal meshes in many cases. The Researches on the processing triangular meshes has been mainly carried out. For example, applying the low-pass filter for image processing, Taubin developed a method to smooth triangular meshes [21].

The formulation of the discrete log-aesthetic filter proposed in this paper, especially for the curve looks similar to those of the techniques of curve flow that perturb points in their normal direction based on the difference approximation of derivatives of the curve and functions of the curvature. As a research on curve flow, Bruckstein et al. proposed a method to generate discrete elastic curves including the discrete clothoid curve obtained by discretizing the clothoid curve [3]. Belyaev et al. developed a method to generate nonlinear spline curves by use of difference approximation and curve flow [2]. These methods are based on the variation principle and perturb the position of the point in the normal direction to minimize an integral quantity of some function of the curvature for the whole curve. However the minimization of the integral quantity strongly depends on the boundary conditions and the final shape of the curve is nearly independent of its initial shape. It is almost impossible to reflect designer's intentions when s/he inputs a sequence of points. The discrete log-aesthetic filter does not minimize any integral quantities or perturb the positions of the points to minimize any objective functions. It finds locally the most approximate log-aesthetic curve for a given set of points and fits the points to the selected log-aesthetic curve. This is the difference of the methods which have and do not have desired shape targets as the log-aesthetic curve. Schneider and Kobbelt proposed methods to generate curves with piecewise linear curvature distributions and to generate surfaces with bilinear mean curvature distributions based on the similar idea to the curve case [22]. However the curves with piecewise linear curvature are limited to be a straight line, an circular arc, a clothoid curve and their combinations, so the generated curves by their methods are only combinations of these types of curves. The mean curvature is closely related to the bending energy of the surface and it can be related to solve the problem of minimization of bending energy (for example [24]).

There were several researches on the clothoid curve generation performed by Meek et al. [13,14], but they discussed only about the clothoid curve, which is just one type of the log-aesthetic curves and their method cannot be applied to the whole types of the log-aesthetic curves as dealt in this paper. At the current state, for surface fairing it is not possible to obtain both of capability of representation flexibility and high quality required for aesthetic design by minimizing objective functions based on the variation principle [20,25].

3. PLANE CURVE FILTER

In this section we propose a log-aesthetic filter for the planar curve given by a sequence of two dimensional points.

3.1 Formulation

We assume that the parameter of the curve to be generated is t and the curve is $C(t)$. Its curvature is calculated by

$$\kappa = \frac{\| \frac{dC(t)}{dt} \times \frac{d^2C(t)}{dt^2} \|}{\| \frac{dC(t)}{dt} \|^3} \tag{1}$$

A sequence of points of number n is assumed to be given by P_i, $i = 0,\cdots,n-1$ and the parameter interval of two parameters t_i and t_{i+1} corresponding to two consecutive points P_i and P_{i+1}, respectively is Δt_i. The curvature at a point P_i is usually supposed to be given by a reciprocal of the radius of the circle determined by three points P_{i-1}, P_i and P_{i+1} used, for example, by Schneider and Kobbelt [11] by

$$\kappa = 2\frac{\det(P_i - P_{i-1}, P_{i+1} - P_i)}{\| P_i - P_{i-1} \| \| P_{i+1} - P_i \| \| P_{i+1} - P_{i-1} \|} \tag{2}$$

Equation (2) is a non-linear function of the point P_i, which should be moved for fairing and in this paper we introduce the concept of central difference and calculate a discrete curvature as follows. The first derivative U_i at the point P_i by central difference is given by $(P_{i+1} - P_{i-1})/(\Delta t_{i-1} + \Delta t_i)$. If the parameter interval Δt_i are always the same, or the parameter interval is given by a constant value Δt, $U_i = (P_{i+1} - P_{i-1})/2\Delta t$. If the parameters of the point sequence are equally distributed, the discretely calculated second derivative V_i is given by $(P_{i+1} - 2P_i + P_{i-1})/(\Delta t)^2$. Hence the discrete curvature κ_i is given by

$$\kappa_i = \frac{\| U_i \times V_i \|}{\| U_i \|^3} \tag{3}$$

3.2 Curvature of Log-aesthetic Curve

We assume that ρ is the radius of curvature of a log-aesthetic curve and α the slope of its LCG (a reciprocal of the self affinity rate) , the arc length s and c, d are constants. In case of $\alpha \neq 0$,

$$\rho^{\alpha} = cs + d \tag{4}$$

and in case of $\alpha = 0$,

$$\rho = ce^{ds} \tag{5}$$

By taking the logarithm of the both sides of the above equation, we obtain

$$\log \rho = ds + \log c \tag{6}$$

In the discussion below, we deal with the case of $\alpha \neq 0$, by use of the logarithm of the radius of curvature we can treat the case of $\alpha = 0$ in a similar manner.

From Eq. (4) the curvature is given by $\kappa = 1/\rho = (cs+d)^{-\frac{1}{\alpha}}$. If we assume that for a given α specified by the designer, the curve locally satisfies the property of the log-aesthetic curve, some c_i and d_i exist such that $\kappa_i = (c_i s_i^l + d_i)^{-\frac{1}{\alpha}}$. Here we locally process $2m+1$ points including the point P_i and m points before P_i and other m points after P_i and the local arc length s_i^l is given by $s_i - s_{i-m}$ where s_i is the arc length to P_i. c_i and d_i can be determined locally using these $2m+1$ points by the least square method.

Ideally it is desirable to minimize the sum of the square of the errors of the curvatures, but it is not possible to apply the least square method because the curvature is not a linear combination of P_i's. Therefore we displace the point P_i to let it to satisfy the following equation.

$$(c_i s_i^l + d_i)^{-\frac{1}{\alpha}} = \frac{\| U_i \times V_i \|}{\| U_i \|^3} \tag{7}$$

The position of P_i is not determined uniquely by the above equation, we limit the range of its location after displacement. For example, we restrict its position on the straight line through the middle point of two points P_{i-1} and P_{i+1} whose direction is the normal vector at the point P_i. Hence if the normal vector at P_i calculated discretely is given by N_i, a point on the line P_i' is expressed by

$$P_i' = P_i + \phi N_i \cdot \tag{8}$$

We determine ϕ to satisfy Eq. (7).

3.3 Estimation of α

In practice there are two cases where the designer specifies α: the slope of the LCG and where s/he does not want to specify α and it is desirable to estimate it from a input sequence of points. The procedure to estimate α from a sequence of points is as follows.

(1) Approximate the sequence of the points by a B-spline curve.
(2) Divide it into several segments at its inflection points and those where the curvature has extremal values.
(3) Generate a LCG for each segment and estimate the slope of the graph as the α value.

3.3.1 Approximation by B-spline Curve

A sequence of points is approximated by a quadratic or cubic B-spline curve by the least square method. The objective function for the least square method is the sum of the square of the distances between the points in the sequence and the corresponding points on the B-spline curve.

3.3.2 Segmentation of B-spline Curve

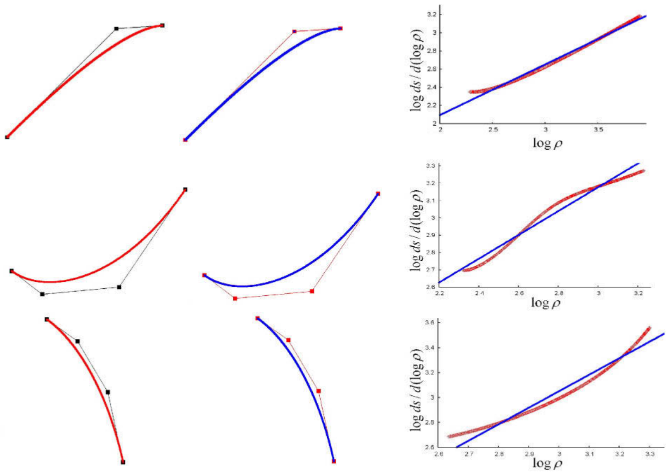
The B-spline curve generated in the previous step is subdivided into several segments at its inflection points and those where the curvature has extremal values. Each segment is equivalent to a Bezier curve and we convert each segment into it.

3.3.3 Estimation of α from LCG

An LCG is drawn for each Bezier curve and the slope of the graph is estimated as the α value by generating an approximating line for the graph with the least square method. Figure 1(a) shows several Bezier curves of quadratic and cubic and Fig.1(c) shows their LCGs and the straight lines whose slopes are estimated α values. Figure 1(b) shows log-aesthetic curves generated with estimated α values, the constraints on the positions of the end points and the directions of the tangent vectors there.

3.3.4 Application Examples of Plane Curve Filter

Figure 2 shows an example of the application of the log-aesthetic filter for a sequence of points generated from a planar quadratic Bezier curve. Figure 2(a) shows 20 points painted in sky blue which are sampled from a quadratic Bezier curve whose control points are located at each place where the sign + is marked and relocated by adding maximum 0.5% random noises relative to the total length of the curve to their x and y coordinates.



(a) Bezier curves (b) Log-aesthetic curves (c) LCGs and their approximate lines with α's

Fig. 1: Estimation of α.

For the purpose of the comparison, we draw a log-aesthetic curve with $\alpha = 0.5$ in blue generated from the control points of the Bezier curve. Figure 2(b) shows a curve after filtering. Figure 2(c) shows a graph of ρ^{α} for arc length calculated from the sequence of the noise-added points in red and the graph is strongly vibrating because of the addition of the noises. The pink line is an approximation of the graph obtained by the least square method and the blue line is the graph of ρ^{α} for the arc length of the log-aesthetic curve. We specified $\alpha = 0.5$ for the filter and the points moved onto the log-aesthetic curve and its discrete logarithmic curvature graph also moved onto that of the log-aesthetic curve as shown in Fig. 2(d). The program ran on a PC with Pentium 4, 3.2GHz and we performed the filter about 250 times until the vibration of the LCG stopped and became almost completely stable for this example. One filtering operation can be done very fast and the total processing time is about 0.2 seconds although the number of iterations is large.

As Fig. 3(a) indicates, since the filter can be applied locally, it can be applied to not only sequences of points similar to one segment of the log-aesthetic curve, but also those whose directional angle changes more than 180 degrees. Furthermore, it can be applied to those which have inflection points and whose curvature changes its sign from positive to negative or vice versa as shown in Fig. 3(b). At the inflection point since the radius of curvature becomes infinite, the filter does not displace the points in its neighborhood.

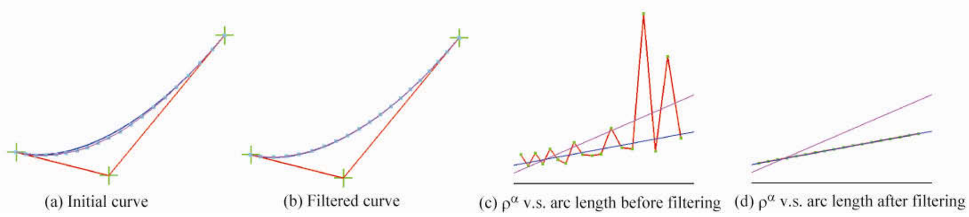


Fig. 2: An application example of the planar filter to a discrete curve generated form a quadratic Bezier curve.

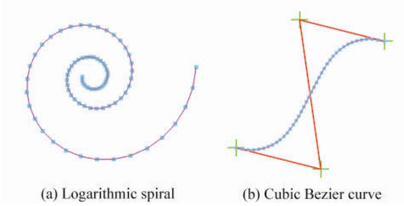


Fig. 3: Application examples to discrete curves generated form a logarithmic spiral and a cubic Bezier curve.

For the examples shown so far, we apply the log-aesthetic filter for all the points of the curve at once as usual filter operations. We calculate an approximant line and displace all the points onto the line to make them to have specified curvature values. However, if a point is far from the line in the sense of the curvature value, the locations of the points sometimes may diverge. Hence we adopt the same strategy used by Higashi and Yamada [8,9] for the following examples. We move a point which is the most far from the approximant line onto it and then calculate a new approximant line. We repeat these processes. By this strategy, the filtering operation becomes much more robust. Note that the times written for the following figures are counting the numbers of each one point process.

Figure 4 shows examples of the application of the log-aesthetic filter to a discrete logarithmic spiral with different application times. We added 0.5 % noises relative to the whole curve length to the x and y coordinates of each point at the maximum and the initial discrete curve is illustrated in Fig.4(a) with the radius of curvature drawn as blue lines. As this figure shows, the radius of curvature becomes smoother when we increase the application times. Figures 5-7 compare the performance of two filters, one is the four divided difference (FDD) filter proposed by Higashi and Yamada [8,9] to make the four divided difference to be equal to 0 and the other is the log-aesthetic (LA) filter. Figure 4 shows the effects of the change of the number of iterations of the log-aesthetic filter. Figure 5 shows several curvature graphs obtained by the two filters with different iteration numbers. As you can see from the figure, the FDD filter cannot remove small vibrations of the curvature even if the number of iterations is very large, but the LA filter remove them completely at the reasonable iteration number. Figure 6 compares the smoothness of the results of two types of the filtering. The blue lines indicate the radii of curvature. We can control m explained in subsection 3.2 to change the final shape of the log-aesthetic filtering. Figure 7 shows the effects of the change of m. If we use a smaller m, the deformation of the curve is also smaller. One of the problems of the FDD filter is that it is not possible to control the amount of deformation when we apply it repeatedly. On the other hand, we can control it easily by changing the value of m.

4. SPACE CURVE FILTER

The curve which satisfies the general equation of the aesthetic curve, or the log-aesthetic curve is restricted to be planar, but Miura et al. [9] extended the concept of self-affinity of the plane curve into three dimensional space and the curve which has the self-affinity was proposed as the space log-aesthetic curve. Yoshida and Saito analyzed the properties of the space log-aesthetic curve and they classified it into several types [13].

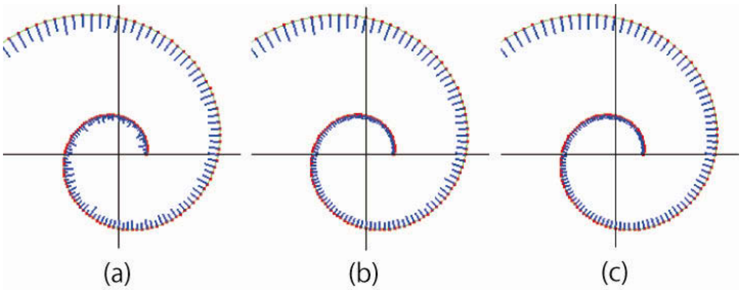


Fig. 4: Application example of a discrete logarithmic spiral. (a) Initial curve, (b) LA filter 50 times, (c) 300 times.

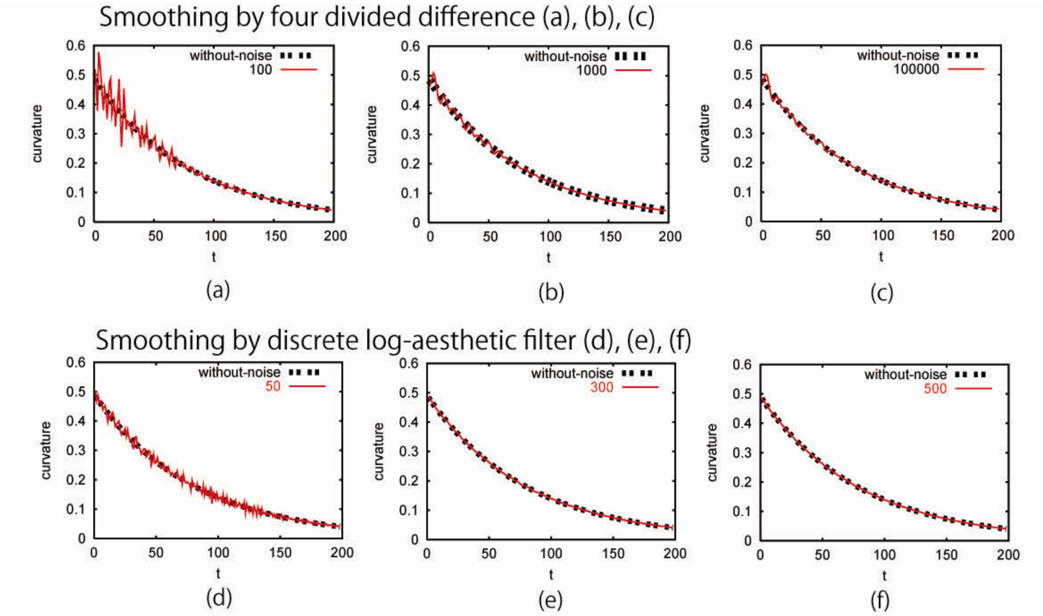


Fig. 5: Curvature graphs obtained by applying four divided difference = 0 and LA filter.

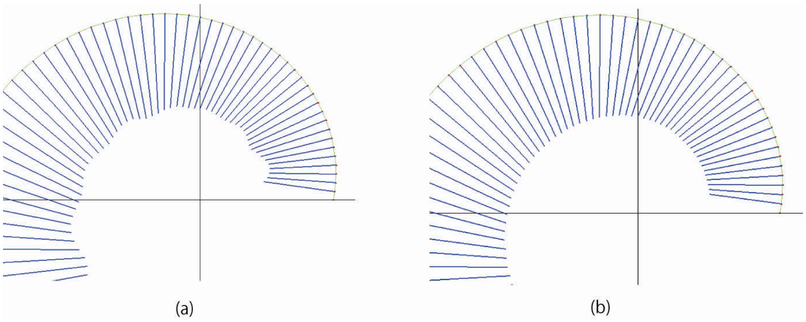


Fig. 6: Comparison of four divided difference = 0 and LA filter. (a) FDD, 100,000 times, (b) LF filter 500 times.

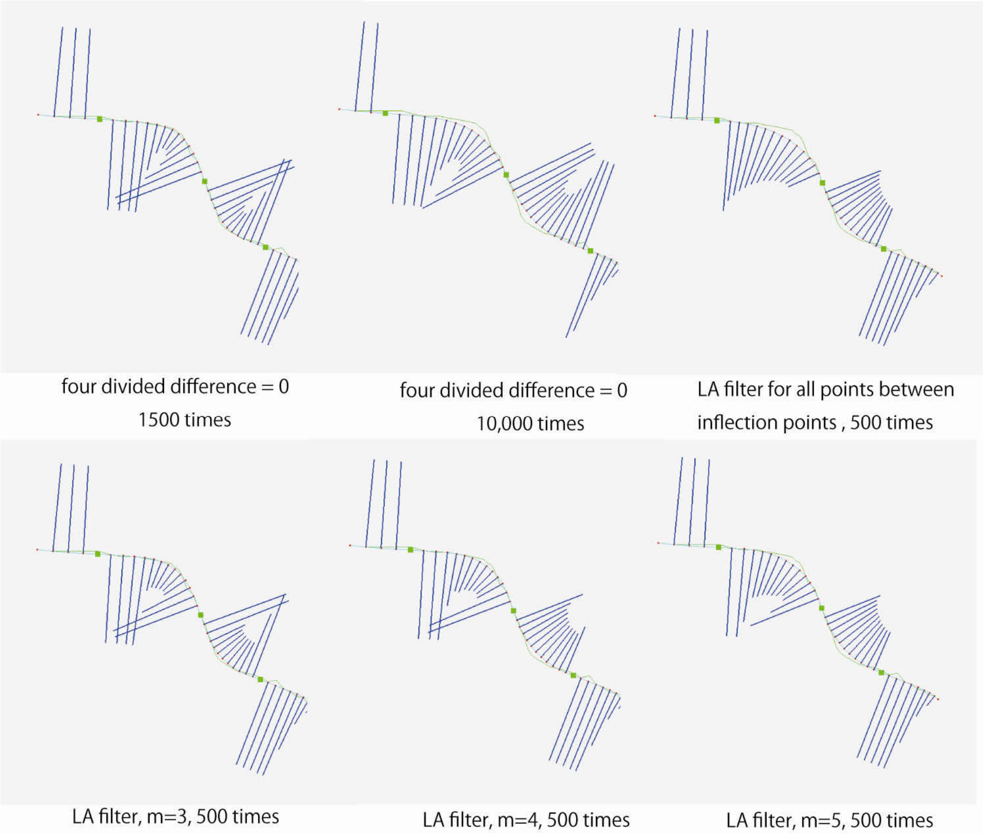


Fig. 7: Comparison of four divided difference = 0 and LA filter.

For a space curve $C(t)$ parameterized by s, let its unit tangent vector be T, unit principal normal vector N, and unit binormal vector B. These vectors are related by the Frenet-Serret formula as follows:

$$\begin{pmatrix} T'(s) \\ N'(s) \\ B'(s) \end{pmatrix} = \begin{bmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{bmatrix} \begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix} \tag{9}$$

where κ and τ are the curvature and torsion, respectively.

Since the curvature and torsion, or their reciprocals: the radius of curvature and radius of torsion can be independently specified, with respect to the radius of torsion $\mu = 1/\tau$, we assume that the following equation is satisfied:

$$\log(\mu \frac{ds}{d\mu}) = \beta \log \mu + C' \tag{10}$$

where β and C' are constants. Hence $\mu^{\beta-1} ds / d\mu = c_1$ assumed to be satisfied where c^1 is a constant.

4.1 General Equation of Log-aesthetic Space Curve

If we assume both of the slope of the logarithmic curvature graph α and β introduced above are not equal to 0, the radius of curvature ρ and the radius of torsion μ of the log-aesthetic space curve are given by

$$\rho^{\alpha} = cs + d, \tag{11}$$

$$\mu^{\beta} = gs + h. \tag{12}$$

where s is the arc length and Eq.(11) is the same as Eq.(4). α, β, c, d, g and h are constants and the shape of the curve depends on these constants.

4.2 Implementation of Space Curve Filter

For a space curve, since there are two equations (11) and (12) to be satisfied, we need two parameters. Hence similar to the plane curve case, we restrict the position of the currently processing point P_i on the plane through the middle point of two points P_{i-1} and P_{i+1}, and which includes both of the normal and binormal vectors at the point P_i. A new position P_i' of the point P_i after filtering is expressed by

$$P_i' = P_i + \phi N_i + \varphi B_i. \tag{13}$$

First, ϕ is determined by Eq.(11). Then φ is determined by Eq.(12). The third derivative can be calculated by 4 points: two points before and two points after the point P_i without P_i itself. Hence Eq.(13) becomes a quadratic function of φ and two φ's generally satisfy the equation. We adopt such a φ that its absolute value is smaller, or the amount of its displacement is shorter.

5. SURFACE FILTER

In this section we discuss about log-aesthetic type filters for the discrete surface. Hence we think about a filter corresponding to the circular arc, which can be regarded as the simplest log-aesthetic curve as its radius of curvature is constant. As the log-aesthetic curve specifies the relationship between the radius of curvature and the arc length, it is necessary to decide some quantity in differential geometry corresponding to them. For a curve among the arc length s, the curvature κ and the arc length of its indicatrix of tangents σ there is such a relationship that $\kappa = \lim_{s\to 0} \sigma / s$. This is quite similar to a relationship among the area S, the Gaussian curvature K and the area of the Gauss map S' of a surface $S(u,v)$ such that $K = \lim_{S\to 0} S'/S$ [2]. Therefore we correspond the surface with a constant Gaussian curvature to the circular arc with a constant curvature. If we use this correspondence, the fundamental equation of aesthetic curves $\rho^{\alpha-1} d\rho / ds = C$ [9] corresponds to $(1/K)^{\alpha-1} d(1/K)/dS = C$. Here we remark that

$$\lim_{S\to 0} \frac{S'}{S} = \frac{\iint |N_u \times N_v| du dv}{\iint |S_u \times S_v| du dv} = K. \tag{14}$$

In the above equation, N_u and N_v are the derivatives of the unit normal of the surface with respect to u and v, respectively.

Another type curvature for the surface is the mean curvature which is as important as the Gaussian curvature. For example, if the mean curvature H is always equal to 0 on the surface, it is called the minimal surface and a very important surface especially in calculus of variations [2]. A thin membrane generated with an arbitrarily shaped wireframe is always a minimal surface. However its Gaussian curvature K is always equal to 0 and it is not possible to generate raised surfaces frequently used in aesthetic design. The minimal surface is not suitable for many design practices. In the field of geometric modeling, for example Schneider and Kobbelt proposed a method for faring surfaces in which they made the mean curvature distribution to be bilinear as an extension of the discrete Clothoid curve whose second derivative of the curvature is equal to 0 [13]. Research efforts should be made on filters using the mean curvature in the future.

5.1 Surface with Constant Gaussian Curvature

If the maximum and minimum curvatures are constant, both of the Gaussian and mean curvatures are constant since they are their multiplication and a half of their summation, respectively. Hence the plane, spherical and cylindrical surfaces are classified into the surface with a constant Gaussian curvature because the maximum and minimum curvature are constant. Furthermore if one of them is equal to 0, the Gaussian curvature does not depend on the other curvature and the developable surfaces including the conical and tangent surfaces are those with a constant Gaussian curvature, in these case with especially $K = 0$. The surfaces of revolution with a constant Gaussian curvature include a surface like rugby ball and barrel shape surfaces in case of $K > 0$ and the pseudosphere in case of $K < 0$ [2].

5.2 Bilinear Coons Patch Bounded by Log-aesthetic Curves

We show several examples of the bilinear Coons patch bounded by four log-aesthetic curves with a constant Gaussian curvature. Figure 4 shows a bilinear Coons patch and its reflection lines (zebra rendering) from three different views. Two opposite sides of the patches are the same log aesthetic curve and the other two sides are straight lines. This patch is a ruled surface and its Gaussian curvature is always equal to zero. The zebra patterns in this figure are smoothly changing and we observe that the surface is of high quality.

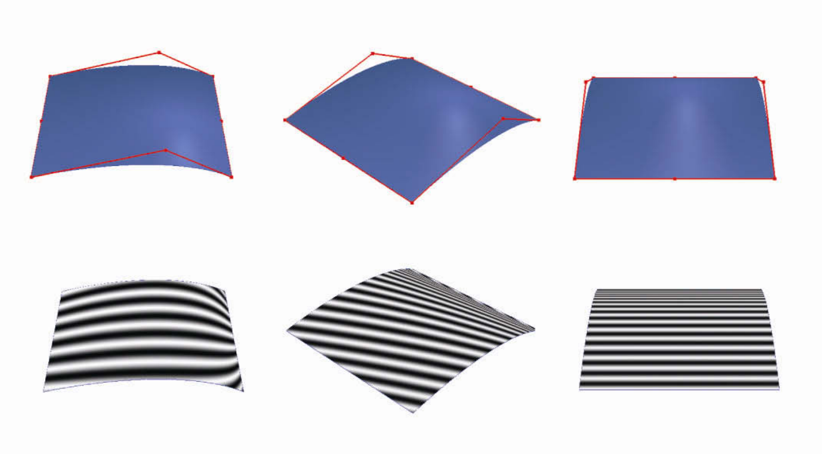


Fig. 8: A bilinear Coons patch bounded by four log-aesthetic curves.

5.3 Implementation of Surface Filter

We assume that a given discrete surface consists of regularly connected points P_{ij}, $0 \le i \le m$, $0 \le j \le n$. The Gaussian curvature of the surface is given by $eg - f^2/(EG - F^2)$ where E, F and G are the coefficients of the first fundamental form and e, f and g are those of the second fundamental form. In order to implement a surface filter, it is necessary to determine first and second derivatives and the unit normal vectors discretely. Similar to the curve cases, we adopt the concept of central difference for the first derivatives and use consecutive three points to calculate the second derivative in each parameter direction. The other second derivative S_{uv} can be calculated by four points around P_{ij} by central difference. The normal is calculated by a weighted sum of the normal vectors of the triangular polygons around P_{ij} taking their areas as weights and we assume its direction does not change by a filtering operation. We let P_{ijc} be a point obtained as an average of 8 points sharing four sided polygons with the current point P_{ij} and the point after filtering P_{ij}'. We restrict its displacement only in the normal direction N_{ij}. Then

$$P_{ij}' = P_{ijc} + \phi N_{ij}. \tag{15}$$

We calculate an average C of the Gaussian curvatures around the point P_{ij} and determine the value of ϕ to make the Gaussian curvature of the point P_{ij} to be identical to C. The equation $K = C$ is a quadratic function of ϕ. We adopt such a ϕ that its absolute value is smaller, or the amount of its displacement is shorter.

6. CONCLUSIONS

In this paper, we have proposed the log-aesthetic filters for discretely defined plane and space curves and surfaces to remove noises, fair them and make them to have the properties of the log-aesthetic curve. We have implemented the discrete log-aesthetic filter for plane curves and confirmed its validity by applying it to several discrete curves. The discrete log-aesthetic filer for plane curves is not a filter based on the curve flow to optimize some objective function by calculus of variations, but it fits a discrete curve to some targeted log-aesthetic curve and the filtering operations can be performed very rapidly.

As future work, we will implement the log-aesthetic filters for space curves and surfaces and apply them to various examples and improve their performance.

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