

# Feature Extraction from High-density Point Clouds: Toward Automation of an Intelligent 3D Contactless Digitizing Strategy

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# ABSTRACT

This paper deals with a global intelligent 3D digitizing algorithm, which allows increasing the quality of the resulting cloud of points. Built from quality analysis and characteristic line extraction, the algorithm computes a new path belonging to an admissible space with the objective of increasing the quality of the new resulting points cloud. The extraction work is performed thanks to a scale space algorithm, based on an iterative projection algorithm and the concept of mean curvature motion (MCM). The scale space framework allows us to perform the detection at a coarse scale without any noise or digitizing error interference and to project the result back onto the original point cloud. Thus all the details of the real object's shape can be identified. Several applications illustrate geometrical feature extraction and global intelligent 3D digitizing within the context of RE. An application is also proposed to compute the distance between the real object shape and an existent CAD-model for conformity checking.

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### 1 INTRODUCTION

In many fields, most issues have as a starting point an existing physical objet. The new possibilities offered by software make essential the use of data processing. In this way, the growing numbers of applications such as reverse engineering or art involve the definition of a numerical model of the existing object. From now it is possible to obtain a digital model thanks to 3D digitizing sensors. 3D digitizing using non-contact measuring systems leads to a representation of the object's surfaces as large clouds of points. Even though the resulting clouds of points constitute a representation of the surfaces, it would be premature to regard the result as a model. Indeed, due to the system complexity, scanning without path planning may affect completeness and accuracy of the digitized data [1-3].

In the context of mechanical engineering, Reverse Engineering (RE) is the process that constructs a CAD surface model from clouds of points collected during the digitizing process. As new functional requirements of products lead to the definition of more complicated object shapes, reverse engineering plays a major role in the fields of moulds and dies for various industries such as

automotive and aeronautics [4]. RE of unknown objects consists of three main steps: part digitizing, feature extraction, surface reconstruction or CAD modelling [5,6].

As the object is unknown, the digitizing process is generally not automatic, and requires numerous scan paths according to different scan orientations [3]. As a result, the collected data consists of a large cloud of 3D independent, unorganized and non-oriented points given by their *x*,*y*,*z* coordinates. The completeness of the data as regards the object surface is not necessarily ensured. Therefore, data are pre-processed before exploitation in order to clean, filter, and reorganize the inhomogeneous and dense cloud of points [6]. However, this stage may involve the loss or the modification of some characteristic details.

The quality and automation of RE closely depends on the following stage of geometrical characteristic identification and feature extraction from the digitized data points. Most of the methods of surface reconstruction rely on finding surface contours such edges, styling lines or hole boundaries [7,8]. Some automatic methods have been developed to extract characteristic lines from discrete data. Due to the inaccuracy of 3D data and the lack of information at the neighbourhood of sharp edges, such methods generally fail when applied to points issued from a real digitizing.

Once surface boundaries or characteristic lines are identified, the cloud of points can be segmented into regions, each one corresponding to a specific surface. To each region, a CAD model is fitted to the points constituting the given region. The stage of data segmentation is seldom automatic, and more generally relies on the user's expertise.

The present paper deals with, applications of *the scale space algorithm*, a robust feature extraction method that can be used for point cloud segmentation as well as intelligent digitizing process. Basically, the feature is a Characteristic Line (CL) representative of the styling line or the 3D contour of a surface. The intelligent digitizing strategy takes advantages of the Characteristic line to support an optimal digitizing trajectory. The latter is built taking into account the capacity of the digitizing system and object's shape in order to increase the quality of the resulting cloud of points.

The paper is organized as follows. The intelligent process-planning algorithm for 3D digitizing is introduced in the next section. Section 3 is devoted to a presentation of the *space scale algorithm*. Several applications are proposed in Section 4 based on the digitizing of mechanical parts.

# 2 INTELIGENT DIGITIZING STRATEGY FOR FREE-FORM SHAPES

The quality of the digitized data strongly depends on the strategy used. Lee and Park introduce an automatic algorithm to compute scanning parameters as scanning directions (Global Accessible Directions) or the number of scans. Their method relies on the meshing of the CAD model of the object [15]. This approach was improved by using the critical point concept in order to decrease the number of GAD [16]. Bernard computes the minimum number of part set-ups to scan the whole surface of an object, defined by a STL model, based on the visibility concepts [17]. Prieto, introduces a digitizing path planning strategy which allows to collect a cloud of points with a predefined accuracy [18]. The method consists in decreasing the digitizing noise by the identification of optimal sensor settings regarding the object shapes described by a CAD model. Zexiao *et al* [19] handle the complete issue of digitizing within the context of RE. They focus their approach to accurate edge measuring (or noisy area) for which an iterative digitizing process.

Among methods proposed in the literature, only a few ones are dedicated to unknown objects. Indeed, quite all the previous works are based on the use of an existing CAD model. In this context the proposed global intelligent digitizing strategy can be applied to an unknown objects as well as CAD models. When the CAD model is unknown, the method relies on a first digitizing of the unknown object. This first digitizing gives a cloud of points which stands for the sampled CAD model.

The experimental 3D digitizing system used to illustrate our work relies on a Coordinate Measuring Machine (CMM) equipped with a contact less laser-plane sensor mounted on a motorized indexing head as shown in Fig.1. The sensor orientations are given by two head rotations defined by the angles A and B that can only be incremented every 7.5°.



Fig. 1: Experimental digitizing system.

The sensor is moved over the surfaces according to the X,Y and Z translation axes of the CMM. The first digitizing is performed manually thanks to a joystick. Sensor orientations are chosen by the operator to ensure the surface covering. The first cloud of points is usually noisy and locally incomplete [3]. A pre-treatment is performed over this first scan to find out the quality and to extract characteristic features [3]. Characteristic lines and feature identification are performed with curvature analysis and the scale space operator called *back propagation*. This method will be detailed in the next section. At this stage, a quality analysis is performed in order to know if the expected quality of the data is reached. The data quality is figured out through two indicators: the noise and the completeness [20]. The completeness ensures the validity of a part digitizing as regards topology and accounts for the importance of the gaps existing in the point cloud. The noise is linked to data sampling errors and is generally evaluated considering the deviations between the points and a local geometrical model fitted to the points. For unsatisfactory quality zones and digitizing holes, new scan paths are automatically defined in order to increase the quality of the data. As shown in Fig. 2, the global algorithm is repeated until satisfying the quality criteria. The new scan paths, defining the intelligent digitizing trajectory, are defined by the successive orientations and positions of the sensor relative to the surface leading to the expected data quality and the data completeness [3].



Fig. 2: Global digitizing strategy.

The path planning generation can be divided in three parts: identification of the admissible digitizing space; identification of optimal sensor settings (orientation and position) for each point; optimisation of the number of sensor reorientation. Regarding the sensor technology, two positioning parameters are specially influent on the chosen quality criteria: the digitizing distance *d* and the  $\alpha$  view angle [21]. These parameters define the sensor orientation. In previous works, we have proposed a protocol to assess a digitizing system as regard noise and completeness [3], [21]. These works allows us to identify admissible intervals for *d* and  $\alpha$ , defining the admissible digitizing space. More generally, the distance d is chosen within the interval and is kept fixed for the whole new digitizing. Following, the sensor is oriented according to the local normal at the studied point so that the view angle remains within the admissible interval. This defines a visibility cone (figure 3). As the characteristics lines (CL) along which rescanning must be carried out are identified by the scale space algorithm, the local normal is easily defined for each point belonging to the CL.



Fig. 3: Optimal sensor settings.

Optimal sensor settings could lead, at the worst case, to reorientation the sensor for each point. However, reorientation is a source of error (point registration) and time loss due to the sensor technology. Thus, we optimize the number of reorientations by using the concept of visibility cones. Indeed, an admissible cone is defined for the first point according to the admissible trajectory space. As long as the next optimal sensor orientation belongs to previous cone (Fig.3), the sensor is not reoriented.

The method proposed strongly relies on the identification of the characteristic lines (CL). Hence, the digitizing trajectory follows the characteristic line. The next section will describe how these CL are identified.

#### **3** SCALE SPACE ALGORITHM

In accordance with the edge detection paradigm in image processing, extracting the crest lines on point clouds constitutes a 3D shape analysis [9]. Ridge Lines are defined by the loci of points where the maximal principal curvature takes a positive maximum along its curvature line. The loci of points where the minimal principal curvature reaches a negative minimum along its curvature line [10-12]. Thus, crest lines (ridge or valley lines) detection is linked to curvature computation. Most curvature estimation methods use a quadratic or polynomial regression, leading to instability in the case of noisy data. A solution to this instability would be to denoise the surface. However, the initial point positions would then be lost, and preserving the initial geometrical information is crucial in our applications (noise estimation in particular). As was shown in [27] estimating robust information while preserving the initial point positions can be done using a scale space approach.

Numerous scale space (or multiscale) approaches for processing surfaces (defined as point clouds or meshes) have been proposed. Among others, Pauly [13] proposed detecting feature points and fitting surfaces in the feature point neighbourhoods. The number of fitted surfaces yields the feature type. This classification is done at various scales (i.e. for various neighbourhood radius), which yields a measure of the feature scale. But this method does not introduce a proper scale space evolution. Thus, it would be interesting to use a method that detects features at a given scale but gives the result on the initial point set. We iterate a simple and robust smoothing operator that approximates one step of the Mean Curvature Motion (MCM). Given a surface S then for all P in S, the Mean Curvature Motion is expressed as:

$$\frac{dP}{dt} = H(P)\vec{n}(P) \tag{1}$$

where H(P) is the mean curvature of point P and n(p) is the normal to S at P.

Each MCM step corresponds to a scale going from a fine scale (the original point cloud) to a coarse scale (the smoothed point cloud). The MCM operator consists in a simple projection: projecting each point onto its local regression plane. Once all points have been projected, the operator is applied to the smooth surface yielding an iterative smoothing. This iterative process is called scale-space since each iteration corresponds to a scale. The scale space definition is very robust since it only relies on an order one approximation of the surface. It also yields a very fast implementation.

#### 3.1 Curvature Computation

Surface curvature computation has been widely investigated [13-14]. Curvature can be estimated by computing the surface tensor [24], by surface regression or by fitting curves to the cloud of points [25]. Voronoi cell covariance analysis could also be used to compute the principal curvature direction [26]. Consider a surface point P. We define the neighbourhood of P as the set of the points  $P_i$  such that  $(P_i - P) < r$ . Radius r is the only parameter of the scale space iteration. Let N be the cardinal of the neighbourhood. The barycentre of the neighbourhood is defined by:

$$m = \frac{1}{N} \sum_{i=1}^{N} p_i \tag{2}$$

and the covariance matrix

$$C = \sum_{i=1}^{N} (p_i - m)^T . (p_i - m)$$
(3)

Thus, C is a 3x3 symmetric matrix. Principal component analysis of C yields the neighbourhood regression plane: it is the plane passing through m and orthogonal to the eigenvector v corresponding to the least eigenvalue of C. One can prove [27] that projecting P onto this local regression plane induces a motion that is asymptotically equivalent to the mean curvature motion. A direct consequence of this motion is that the curvature can be found as the displacement amplitude along the normal at P. One can also prove that the eigenvector v is a good approximation to the normal direction at point P.



Fig. 4: (a) Curvatures of a 26-face polyhedron-measuring artefact, (b) curvature of a pump carter.

# 3.2 Back Propagation

At each step, we can keep track of the motion of each point. Thus the backward scale space is trivial. If we consider a point  $P_t$  at step t and its evolution  $P_{t+1}$  at step t+1, we can then define the sequence by Eq. (4):

$$d_p(t) = P_{t+1} - P_t \tag{4}$$

The reverse scale space operator (or back propagation operator) is then defined by Eq.(5):

$$P_t^{-1}(P_{t+1}) = P_{t+1} - d_p(t)$$
(5)

Using this operator, it is easy to go backward in the scale space from step t to 0 (i.e. the initial point cloud). In this case, the operator becomes even simpler:

$$P_t^{-1}(P_t) = P_0 (6)$$

As we just need to store the initial position for each point.

# 4 APPLICATIONS

# 4.1 Feature Extraction to Increase Digitized Data Quality

Let us consider the simple case of the Reverse Engineering of a pump carter (Fig.5). After a first digitizing of the object, the digitized cloud of points collected is noisy, inhomogeneous and present digitizing gaps (holes). In particular, the outward borderline presents a strong lack of points.



Fig. 5: Illustrated case: (a) pump carter, (b) first digitizing cloud of points.

After the first digitizing, the Scale Space Algorithm is applied in order to identify the Characteristic Lines and the frontiers of the digitizing gaps (Fig.6). As previously exposed, the Scale Space algorithm also allows the evaluation of the local normal at each point of the Characteristic Line.

The new path planning is computed in order to complete the digitizing gaps and to decrease the digitizing noise [3]. The illustration is proposed here for the digitizing of the outward borderline, with the objective of completeness. The new trajectory is defined according to normals defined in Fig. 6(b). For this application, d is chosen equal to 100mm and  $\alpha$  belongs to [0°,60°]. The new points cloud is obtained considering 7 sensor orientations. The Fig. 7(a) shows the new digitized point cloud for the outward borderline, enhancing the completeness of the points as regards the object surfaces.



Fig. 6: Results of the Scale Space Algorithm: (a) Characteristic line extraction, (b) Normal evaluation for the outside borderline.



Fig.7: (a) new path planning simulation, (b) the actual final points cloud generated by the application of the 3D intelligent path planning.

# 4.2 Geometrical Feature Extraction for Cloud Point Segmentation

The scale space algorithm is a robust method to compute curvature directly digitized data points and a method to track points back to their original positions. Thus, it is possible to handle two related problems: inflexion line computation and the ridge-valley classification. An immediate application is geometric features extraction by point cloud segmentation.

The curvature zero-crossings define the inflexion lines. They are therefore equivalent to the "zerocrossing of Laplacian" proposed by Marr and Hildreth [22]. These closed curves segment the cloud into zones of ridges (areas of positive mean curvature points) and valleys (areas of negative mean curvature points) zones. The scale space motion yields, at each step, a curvature which is therefore scale-dependent. The sign analysis of this multi-scale curvature leads to a binary classification in ridge and valley zones at each scale. Figure 8 illustrates the scale-dependence of the extracted zone. The classification is performed in Fig. 8(b). Several projections lead to the final coarse scale classification (Fig. 8(c-e)). Thanks to the back propagation operator, the segmentation is tracked back to the original point position, Fig.8(f). Thus the global geometric properties are identified at the desired scale.



Fig. 8: Classification of digitized points in ridge and valley zones: (a) real object, (b) initial segmentation, (c) (d) (e) first, second and third projection, (f) back projection at the original position.

At each scale, curvature level lines can be generated on the scale space mesh of the point original samples. In order to extract a  $\alpha$ -level line ( $\alpha \in IR$ ), we detect edges whose extremities *P1 P2* have curvatures *H*(*P1*) and *H*(*P2*) such that (*H*(*P1*) -  $\alpha$ ). (*H*(*P2*) -  $\alpha$ ) < 0. Line vertices are linearly interpolated along these edges and linked using mesh connectivity information. When a real object's edge is digitized, it is almost impossible to know which points of the scan, represent the real edge. Choosing a low  $\alpha$  allows for segmenting the point cloud into flat and sharp areas. This segmentation ensures that an area classified as flat corresponds to a real flat part with no edge. Thus, the segmentation becomes more reliable (Fig. 9).



Fig. 9: (a) extraction of level line 0.12 of a spherical faced measurement artefact, (b) segmentation in flat and non-flat zones.

Once, the surface is segmented, the surface reconstruction becomes easier. As Illustrated with the example of the pump carter (Fig. 10), a characteristic line corresponds to a 3D contour, a frontier of a set of points to which a model can be fit. For instance, points belonging to the blue region can be fit by a plane surface, whereas points corresponding to the green region can be fit by another plan. Therefore, the next stage would consist in finding, in automatically, the model of the feature corresponding to the segmented surface.



Fig.10: Geometrical feature segmentation of the pump carter.

#### 4.3 Error Maps

Scale space algorithm allows us to define an extraction of geometrical features by point cloud segmentation. Thus, each data point belongs to an identified geometric feature. When the CAD-model of the digitized object is known, it is possible to compare the real object to its CAD-model. By this way an error map is drawn up.

The idea is to compute the distance between each point and its theoretical position in the CADmodel. To create the error map between the data points and the CAD-model, we first have to bring both in a same frame. That can be made in two steps: a global registration and a local one (as initial point clouds are far from each other). The first registration is using a CAD-software (CATIA V5<sup>©</sup> in our case), and consists in making in coincidence surfaces of the digitized data with corresponding surfaces of the CAD-model. The goal of this first step is to move the point cloud very close to the CAD-model. This first stage is simplified by the use of the scale space algorithm, allowing the matching of extracted surface contours to the corresponding CAD-model.

Once that step is implemented, the local registration consists in minimizing the distances between the digitized point cloud and the CAD-model. To implement that second step the CAD-model is meshed part with a high density (distance between vertices: 0.1mm).

In fact, the local registration relies on the Small Displacement Torsor method [23]. Let  $M_{i}$  be a point of the cloud of points, let  $M_{th}$  be the corresponding theoretical point of the CAD model, and let  $\xi_{i}$  be the distance both points, the objective is to find the local transformation so that local deviations are minimized (Fig. 11). The theoretical point  $M_{th}$  is the projection of the point  $M_{i}$  on the plane surface defined by its three nearest neighbors in the CAD model point cloud. We are allowed to make this approximation because we assume that the CAD model point cloud is perfect (noiseless, dense, etc.).



Fig. 11: Distances between the scanned point and the theoretical point.

The SDT method consists in applying a small displacement  $D_{MThi}$  at each point so that the point cloud approaches as best as possible the CAD model according to the least-square criterion:

$$\mathbf{e}_{i} = \xi_{i} - \vec{\mathbf{D}}_{MThi} \cdot \vec{\mathbf{N}}_{i} \tag{7}$$

The whole movement can be described using the small displacement of a particular point O,  $D_0(u,v,w)$ , and  $R(\alpha, \beta, \gamma)$  the small rotation vector of the whole surface. This yields to:

$$\mathbf{e}_{i} = \xi_{i} - (\vec{\mathbf{D}}_{O} \cdot \vec{\mathbf{N}}_{i} + \vec{\mathbf{R}} \times \overrightarrow{OM}_{Thi})$$
(8)

Finally, according to the least-square criterion, the optimization scheme leads to find (u, v, w,  $\alpha$ ,  $\beta$ ,  $\gamma$ ) so that  $W = \sum e_i^2$  is minimal. This leads to solve a linear system of 6 unknowns to 6 equations [23]. Once the six parameters are determined, we can evaluate the distance between the noisy scanned point and the theoretical one of the CAD model according to Eq. 8), giving the error map.



Figure 12, shows an application of error map computation for a mechanical complex part after the global registration. The point cloud contains 4000000 points for which all the distances have been computed to generate the error map. Points for which the calculated errors are greater than 0.2mm are not significant and probably result from a digitizing error due to accessibility problems (depth of hole greater than the sensor capacities).

The interest of this SDT method is that it is very easy to compute, it is very fast and we know exactly what is done. This algorithm is sufficient to implement the registration of a point cloud with a CAD model. In fact, the registration error is maybe ten times under the digitalization error.

# 5 CONCLUSION

In this paper, we introduce a global intelligent strategy which brings an answer to problems raised by digitizing of a free form objects. The final aim is to generate a point cloud whose quality is in agreement with the given application. In This way, after a first digitalizing (without any path-process consideration), a pre-processing step is applied. This step consists in a quality assessment and in the characteristic line extraction. If the quality is not reached, a new intelligent path planning is computed in order to increase the quality of points cloud. The new digitizing is carried out along the identified characteristics lines. Those characteristic lines are identified from a smoothed points cloud at the desired scale thanks to the robust scale space algorithm based on the property of the mean curvature motion (MCM). Smoothing the point cloud allows for the extraction of high quality features. Thanks to the back propagation operator introduced, indentified lines and features are tracked back to the original data position. This way, the geometrical details present in the original scanned data are not lost. The application of this method to data segmentation within the context of Reverse engineering highlights all the utility of the scale space algorithm. Thanks to the scaling, surface boundaries are extracted with good precision allowing further surface modelling.

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