Compound-rhythm Log-aesthetic Space Curve Segements

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ABSTRACT

This paper extends the previous work of generating monotonic-rhythm log-aesthetic space curve segments so that compound-rhythm log-aesthetic space curve segments are interactively generated. A compound-rhythm log-aesthetic space curve segment is composed of two monotonic-rhythm log-aesthetic space curve segments whose logarithmic curvature and torsion graphs are represented by two connected line segments. By presenting a method for controlling the connection point of the two curve segments and deriving the continuity condition of the logarithmic torsion graph, we show that compound-rhythm log-aesthetic space curve segments can be generated fully interactively.

1 INTRODUCTION

To design highly aesthetic surfaces, such as car bodies, the use of high quality curves is very important. In such curves, the curvature should be monotonically varying since the curvature dominates the distortion of reflected shapes on curved surfaces. Log-aesthetic curves are curves whose logarithmic curvature and torsion graphs are both linear[13]. In other words, log-aesthetic space curves are curves whose curvature and torsion are represented by simple monotonous functions of the arc length. See Fig. 1 of [14].

In this paper, we extend the previous works of generating compound-rhythm log-aesthetic planar curve segments[12] and monotonic-rhythm log-aesthetic space curve segments[13] so that compound-rhythm log-aesthetic space curves are interactively generated. Log-aesthetic space curves are curves whose logarithmic curvature and torsion graphs are both represented by straight line segments. Harada et al. originally proposed log-aesthetic planar curves[6,7] whose logarithmic curvature graphs(LCGs) are represented by straight lines. Miura derived the general formula of log-aesthetic planar curves[10]. Yoshida et al. clarified the overall shapes and the characteristics of log-aesthetic planar curves and presented a method for interactively drawing a log-aesthetic planar curve segment like a quadratic Bézier curve segment[11]. By appending logarithmic torsion graphs(LTGs) onto LCGs, Yoshida et al. proposed (monotonic-rhythm) log-aesthetic space curves whose LCGs and LTGs are both represented by straight line segments, clarified the overall shapes and the characteristics of the curves

and presented a method for interactively generating a segment[13]. The linearity of the LCG and LTG constrains that the curvature and torsion of the curve are monotonically varying.

As far as the authors know, there are not many papers for directly designing space curves, especially when the monotonicity of the curvature (and torsion) is considered. Adams proposed a method for drawing a space curve with linearly increasing and then linearly decreasing curvature and torsion with specified endpoint conditions[1]. Higashi proposed to use the evolute, which is the locus of the curvature center, to generate high quality curves[8]. The generated space curve is the intersection of two surfaces that are swept by two curves with monotonically varying curvature generated from the evolutes. There is no guarantee that the generated space curves have monotonically varying curvature and the curve shape cannot be interactively controlled. Miura proposed unit quaternion integral curves[9], which are space curves. However, we cannot interactively control the curve shape as well as curvature variation. Based on class A Bézier curves[2] proposed by Farin, Fukada et al. have proposed a method for drawing 3D class A Bézier curve segments[4] by specifying two endpoints and their tangents. They showed that typical 3D class A Bézier curves approaches the 3D extension of the logarithmic spiral, which is a special case of log-aesthetic space curves. Although the curve is not related to the monotonicity of the curvature (and torsion), Farouki et al. have recently proposed a method for generating quintic space curves[3] with rational rotation-minimizing frames with G^1 Hermite interpolation.

A monotonic-rhythm log-aesthetic space curve is a curve whose LCG and LTG are represented by single segments. See Fig. 1(a),(b) for the LCG and LTG of a monotonic-rhythm curve. In the figure, s, ρ, μ, Λ and Ω are the arc length, the radius of curvature, the radius of torsion, the value related to the y-intercept of LCG, and the value related to the y-intercept of LTG, respectively. We call Ω the torsion parameter. A compound-rhythm log-aesthetic space curve is a curve whose LCG and LTG are represented by two connected segments. See Fig. 1(c),(d). In other words, a compound rhythm log-aesthetic space curve is composed of two log-aesthetic space curves with G^4 continuity. It is known that the curve must be at least G^4 for the two segments in LTG to be continuous[14].
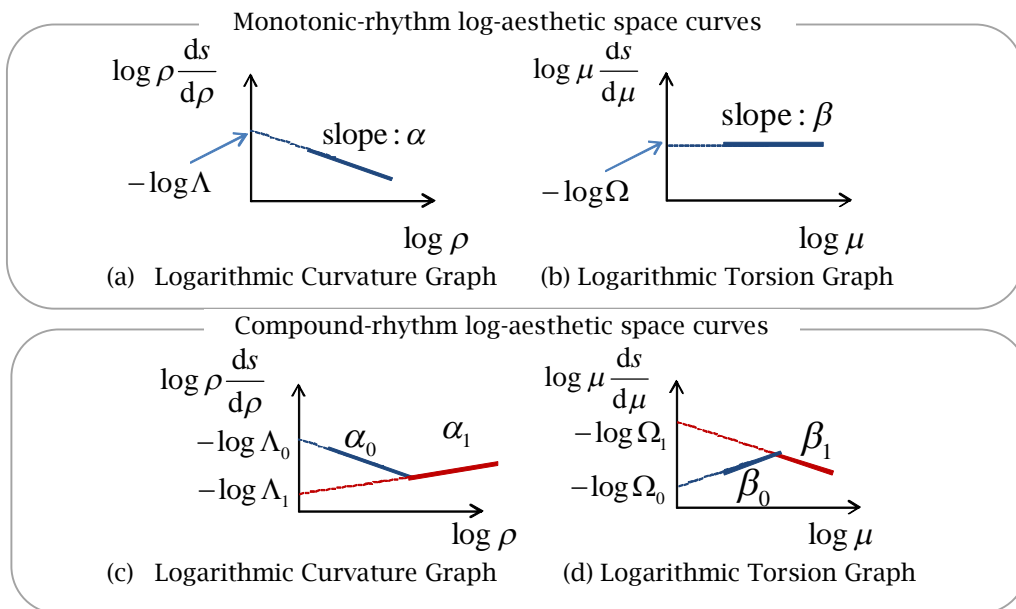

Fig. 1: Logarithmic curvature and torsion graphs.

This paper presents a method for interactively generating compound-rhythm log-aesthetic space curves [5]. In compound-rhythm log-aesthetic planar curves, the connection point of two monotonic-

rhythm log-aesthetic curves is determined by specifying the ratio of the change of the tangential angle of the first curve against the change of the tangential angle of the whole curve[12]. However, in 3D, there is no concept of tangential angle. Thus we need a different way to specify the connection point of the two curve segments. For the efficient generation of compound-rhythm log-aesthetic space curves, we show that all the parameters for the second curve segment can be derived from the continuity condition of the LCG and LTG. We have implemented our algorithm and confirmed that the curve can be generated fully interactively.

2   MONOTONIC-RHYTHM LOG-AESTHETIC SPACE CURVES

2.1   Log-aesthetic Space Curves

Monotonic-rhythm log-aesthetic space curves are curves whose LCGs and LTGs are both linear[13]. See Fig.1(a),(b). The linearity of the LCG and LTG is represented by

$$\log\left(\rho\frac{ds}{d\rho}\right) = \alpha\log\rho + c \tag{1}$$

and

$$\log\left(\mu\frac{ds}{d\mu}\right) = \beta\log\mu + d \tag{2}$$

where $c(=-\log\Lambda)$ and $d(=-\log\Omega)$ are constants. α and β are the slopes of the segments in the LCG and LTG, respectively. Monotonic-rhythm log-aesthetic space curves have two kinds of curves, which are *type* 1 and *type* 2. *Type* 1 curves are curves whose radius of curvature and radius of torsion are both monotonically increasing with respect to the arc length. In *type* 2 curves, the radius of curvature and the radius of torsion are oppositely varying.

Modifying Eqn.(1) and (2) under the condition of the standard form[13], we can derive the following equations.

$$\rho = \begin{cases} e^{\Lambda s} & \text{if } \alpha = 0 \\ (\Lambda\alpha s + 1)^{\frac{1}{\alpha}} & \text{otherwise} \end{cases} \tag{3}$$

$$\mu = \begin{cases} e^{(\Omega s + \log v)} & \text{if } \beta = 0 \\ (\Omega\beta s + v^{\beta})^{\frac{1}{\beta}} & \text{otherwise} \end{cases} \tag{4}$$

$$\mu = \begin{cases} e^{(-\Omega s + \log v)} & \text{if } \beta = 0 \\ (-\Omega\beta s + v^{\beta})^{\frac{1}{\beta}} & \text{otherwise} \end{cases} \tag{5}$$

In case of *type* 1 curves, we use Eqn. (3) and Eqn. (4). For *type* 2 curves, we use Eqn. (3) and (5). In Eqn. (4) and (5), v is the radius of torsion at the origin. The radius of curvature ρ is always assumed to be 1 at the origin in the standard form. We can draw log-aesthetic space curves by integrating the Frenet-Serret equations using Eqn.(3) and (4) for *type* 1 curves or Eqn.(3) and (5) for *type* 2 curves.

Similarly as in log-aesthetic planar curves, we have to be careful about the range of the arc length s. See Fig. 2 in [13] for the range of s.
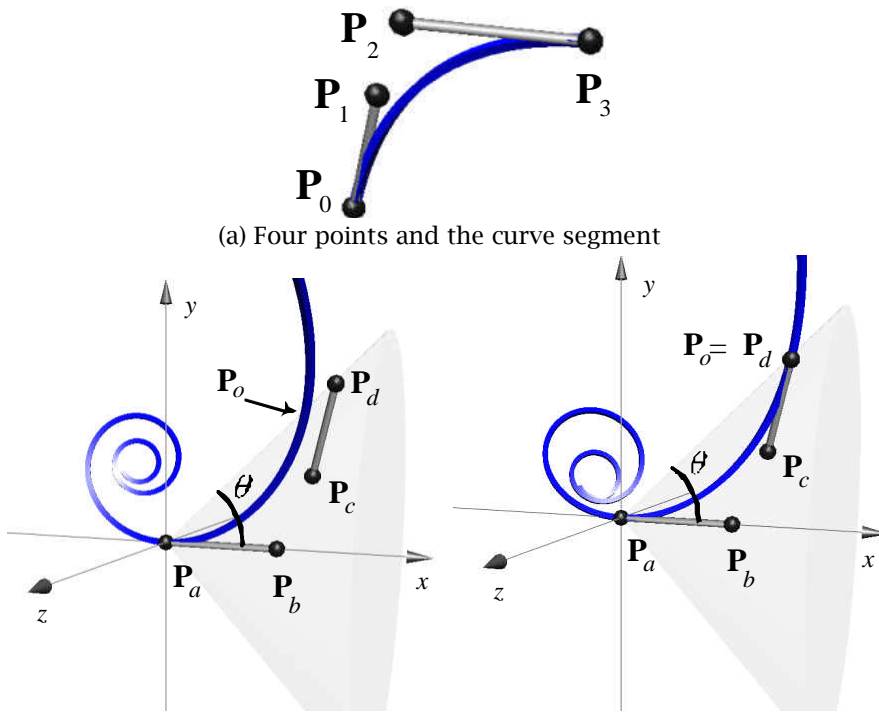
2.2   Generating a Curve Segment with Endpoint Constraints

We briefly review the method for interactively generating a monotonic-rhythm log-aesthetic space curve segment[13]. We are given two endpoints $\mathbf{P}_0$, $\mathbf{P}_3$, their unit tangent vectors $\mathbf{v}_0$, $\mathbf{v}_1$, the slopes of the LCG and LTG, α, β and the torsion parameter Ω. We specify $\mathbf{v}_0$ and $\mathbf{v}_1$ by points $\mathbf{P}_1$, $\mathbf{P}_2$. Thus $\mathbf{v}_0 = (\mathbf{P}_1 - \mathbf{P}_0)/|\mathbf{P}_1 - \mathbf{P}_0|$ **and** $\mathbf{v}_1 = (\mathbf{P}_3 - \mathbf{P}_2)/|\mathbf{P}_3 - \mathbf{P}_2|$. See Fig. 2(a). Note that in our system, the curve segment is drawn using a prism with a quadratic cross-section, so that the user can sense the torsion

of the curve. To draw a log-aesthetic space curve segment, we also need to know Λ, v and the arc length of the curve segment $s_{seg}(>0)$. Note that the curve is always drawn from $s=0$ since we use the standard form. We would like to find a monotonic-rhythm log-aesthetic space curve segment (thus we need to find Λ, v and s_{seg}) that satisfies given endpoint and tangent constraints.

We transform the four points $\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2$ and $\mathbf{P}_3$ in the following manner and rename them as $\mathbf{P}_a, \mathbf{P}_b, \mathbf{P}_c$ and $\mathbf{P}_d$, respectively. We first translate the four points $\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2$ and $\mathbf{P}_3$ such that $\mathbf{P}_0$ goes to the origin, rotate them so that $\mathbf{P}_1$ is on the positive side of x-axis and $\mathbf{P}_3$ exists on the first quadrant of xy-plane. See Fig. 2(b). If we draw a log-aesthetic curve in the standard form, the positional and tangential conditions at $\mathbf{P}_a$ is always satisfied without depending on Λ, v and s_{seg}.

As shown in Fig.2(b), by rotating, through 2π about the x-axis, a half line that starts from the origin($\mathbf{P}_a$) and goes through $\mathbf{P}_d$, we can construct a cone. The angle θ in Fig. 2(b) is the angle formed by $\mathbf{P}_d - \mathbf{P}_a$ and $\mathbf{P}_b - \mathbf{P}_a$. Suppose that a log-aesthetic space curve in the standard form intersects the cone at the point $\mathbf{P}_o$. Note that there may be cases where the curve does not intersect the cone. In such a case, the curve segment cannot be drawn with the specified endpoint constraints. Let the unit tangent vector of the curve segment at $\mathbf{P}_o$ be $\mathbf{t}_e$, which we will use later. By rotating the curve segment and $\mathbf{t}_e$ about the x-axis such that $\mathbf{P}_o$ exists in the first quadrant of xy plane and then scaling the curve segment by $|\mathbf{P}_d|/|\mathbf{P}_o|$, the end point of the curve coincides with $\mathbf{P}_d$. Thus the arc length s_{seg} of the curve segment is determined by increasing s from 0 until the curve segment intersects the cone.

(a) Four points and the curve segment

(b) The curve intersects the cone at $\mathbf{P}_o$ (c) The curve of (b) is rotated and scaled

Fig. 2: Generating a monotonic-rhythm log-aesthetic space curve segments.

Let $\mathbf{t}_f = (\mathbf{P}_d - \mathbf{P}_c)/|\mathbf{P}_d - \mathbf{P}_c|$. To satisfy the tangential constraint at the end point, we need to find Λ and v such that $\mathbf{t}_e = \mathbf{t}_f$. By performing a minimization such that

$$f(\Lambda, v) = |\mathbf{t_e} \cdot \mathbf{t}_f - 1| \tag{6}$$

becomes 0, the tangential condition is satisfied at $\mathbf{P}_d$. See Fig.2(c). Thus we can generate a monotonic-rhythm log-aesthetic space curve with the specified positional and tangential conditions. Note that there may be cases where $f(\Lambda, v)$ is not defined because the curve segment does not intersect the cone or the arc length reaches its upper bound. To cope with such situation, we use the modified downhill simplex methoh[13] for the minimization.

3   THE CONTINUITY CONDITIONS OF THE LCG AND LTG

In this section, we derive the continuity condition of the line segments in the LCG and LTG.

3.1   The Continuity Condition of the LCG

A compound-rhythm log-aesthetic planar curve segment is a curve whose LCG is represented by two connected line segments[12]. See Fig.1(c). The slopes of the first and the second segments in the LCG are α_0 and α_1, respectively. For the LCG to be continuous, both $\log \rho$ and $\log(\rho\, ds/d\rho)$ of the two segments at the connection point must be equal. For $\log \rho$ to be continuous, the curvatures need to be the same at the connection point. The linearity of the two segments in the LCG is represented by

$$\log\left(\rho \frac{ds}{d\rho} \right) = \alpha_0 \log \rho + c_0 \tag{7}$$

$$\log\left(\rho \frac{ds}{d\rho} \right) = \alpha_1 \log \rho + c_1 \tag{8}$$

where c_0 and c_1 are constants. For $\log(\rho\, ds/d\rho)$ to be the same at the connection point,

$$\alpha_0 \log \rho + c_0 = \alpha_1 \log \rho + c_1 \tag{9}$$

must be satisfied. Modifying Eqn.(9), we get

$$e^{-c_1} = \frac{e^{c_0}}{\rho^{\alpha_0 - \alpha_1}}. \tag{10}$$

Let $\Lambda_0 = e^{-c_0}$ and $\Lambda_1 = e^{-c_1}$. Then Eqn.(10) becomes

$$\Lambda_1 = \Lambda_0 \rho^{(\alpha_1 - \alpha_0)} \tag{11}$$

Thus, for the LCGs of two curve segments (either planar or space) to be continuous, the radius of curvature ρ at the connect must be the same and Eqn. (11) must be satisfied.

3.2   The continuity Condition of the LTG

Let the slopes of the two segments in the LTG be β_0 and β_1, respectively. See Fig.1 (d). For the LTG to be continuous, both $\log \mu$ and $\log(\mu\, ds/d\mu)$ of the two segments must be the same at the connection point of the two curve segments. The linearity of two segments in the LTG is represented by

$$\log\left(\mu \frac{ds}{d\mu} \right) = \beta_0 \log \mu + d_0 \tag{12}$$

$$\log\left(\mu\frac{\mathrm{d}s}{\mathrm{d}\mu}\right) = \beta_1\log\mu + d_1 \qquad (13)$$

where d_0 and d_1 are constants. For $\log(\mu\,\mathrm{d}s/\mathrm{d}\mu)$ to be the same at the connection point,

$$\beta_0\log\mu + d_0 = \beta_1\log\mu + d_1 \qquad (14)$$

must be satisfied. Modifying Eqn. (14), we get

$$e^{-d_1} = \frac{e^{d_0}}{\mu^{\beta_0-\beta_1}} \qquad (15)$$

Let $\Omega_0 = e^{-d_0}$ and $\Omega_1 = e^{-d_1}$. Then Eqn.(15) becomes

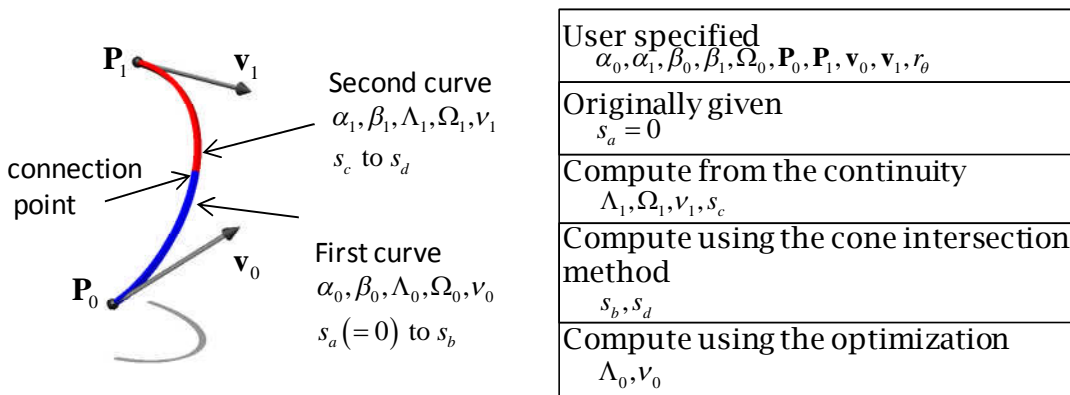$$\Omega_1 = \Omega_0\mu^{(\beta_1-\beta_0)} \qquad (16)$$

For the LTGs of the two segments to be continuous, the radius of torsion μ of the two curve segments at the connection point must be the same and Eqn. (16) must hold.

4 INTERACTIVE GENERATION OF A COMPOUND-RHYTHM LOG-AESTHETIC SPACE CURVE SEGMENT

4.1 The Parameters for Drawing a Segment

We are given two points $\mathbf{P}_0$, $\mathbf{P}_1$ that specify the two endpoints, $\mathbf{v}_0, \mathbf{v}_1$ that specify the tangential direction at $\mathbf{P}_0, \mathbf{P}_1$, respectively, $\alpha_0, \alpha_1, \beta_0$ and β_1 (See Fig. 1(c) and (d)), and the torsion parameter Ω_0. We are going to find a compound-rhythm log-aesthetic space curve segment that satisfies the endpoint constraints. In other words, we are going to find all the parameters necessary for generating the two curve segments of which the desired compound-rhythm log-aesthetic space curve segments is composed. We also provide a way to control the connection point by a user-specified parameter r_θ $(0 < r_\theta < 1)$, which we will introduce shortly.

Fig. 3(a) shows a compound-rhythm log-aesthetic space curve segment as well as all the parameters necessary for generating the segment. For the first curve segment, we draw the segment using the parameters $\alpha_0, \beta_0, \Lambda_0, \Omega_0, v_0$ from the arc length s_a to s_b. For the second curve segment, we draw the segment using the parameters $\alpha_1, \beta_1, \Lambda_1, \Omega_1, v_1$ from the arc length s_c to s_d Fig. 3(b) shows how these parameters are specified or computed.

User specified
$\alpha_0, \alpha_1, \beta_0, \beta_1, \Omega_0, \mathbf{P}_0, \mathbf{P}_1, \mathbf{v}_0, \mathbf{v}_1, r_\theta$
Originally given
$s_a = 0$
Compute from the continuity
$\Lambda_1, \Omega_1, v_1, s_c$
Compute using the cone intersection method
s_b, s_d
Compute using the optimization
Λ_0, v_0

(a) Parameters for generating the first and the second curves (b) How the parameters are specified or computed

Fig. 3: Parameters for generating the curve segment.

Similarly as in monotonic-rhythm curves, we first transform the two points $\mathbf{P}_0, \mathbf{P}_1$ and $\mathbf{v}_0, \mathbf{v}_1$ as described in Sec. 2.2. After the transformation, $\mathbf{P}_0$ is the origin, $\mathbf{v}_0$ is directed toward the positive side of x-axis, and $\mathbf{P}_1$ is on the first quadrant of the xy plane. We rename the transformed points $\mathbf{P}_0, \mathbf{P}_1$ and the transformed vectors $\mathbf{v}_0, \mathbf{v}_1$ as $\mathbf{P}_a, \mathbf{P}_b$ and $\mathbf{v}_a, \mathbf{v}_b$, respectively. Let $\hat{\mathbf{v}}_a = \mathbf{v}_a / |\mathbf{v}_a|$ and $\hat{\mathbf{v}}_b = \mathbf{v}_b / |\mathbf{v}_b|$. At the start point $\mathbf{P}_a$, the positional and tangential constraints are always satisfied since the points and the vectors are transformed so that the position and the tangent vector at the start point always agree with those of the log-aesthetic space curve in the standard form.

To specify the connection point of the two segments in a compound-rhythm log-aesthetic space curves, we propose to use the ratio of the cone angle r_θ $(0 < r_\theta < 1)$. We construct a cone c_0 with its angle θ, similarly as in Sec. 2.2. We construct another cone c_1 with the same vertex and axis but the angle $\theta_0 = r_\theta \theta$. We generate the first curve segment until it intersect the cone c_1 and the second curve segment until it intersect the cone c_0. When $r_\theta = 0$, the compound-rhythm curve degenerate to the monotonic-rhythm curve with the parameters of the second curve. When $r_\theta = 1$, the compound-rhythm curve degenerate to the monotonic-rhythm curve with the parameters of the first curve. See Fig. 7, how the connection point is changed depending on r_θ. Although r_θ is not proportional to the arc length, using r_θ is an intuitive way of controlling the connection point.

4.2 Generating a Segment using Λ_0 and v_0

We show that we can generate a compound-rhythm log-aesthetic space curve segment if Λ_0 and v_0 are given. As will be described in the next section, Λ_0 and v_0 are computed using the optimization so that the tangential condition at the endpoint is satisfied.

Suppose that Λ_0 and v_0 are known. We draw the first curve segment until it intersects the cone c_1. In Fig. 4(a), the first curve segment intersects the cone c_1 at $\mathbf{P}_q$. We will compute the parameters Λ_1, Ω_1, s_c and v_1 for the second curve segment from the continuity condition. Since we know the radius of the curvature ρ and the radius of torsion μ of the first curve segment at the connection point, we can compute Λ_1 and Ω_1 using Eqn.(11) and (16), respectively. s_c and v_1 are computed from the condition that ρ and μ of the first curve at the end point must be the same as those of the second curve at the start point. Solving Eqn.(3) with respect to the arc length, we get

$$s_c = \begin{cases} \dfrac{\log \rho_c}{\Lambda_1} & \text{if } \alpha_1 = 0 \\[2mm] \dfrac{\rho_c^{\alpha} - 1}{\Lambda_1 \alpha_1} & \text{otherwise} \end{cases}. \tag{17}$$

Here ρ_c is the radius of curvature at the connection point. Solving Eqn.(4) and (5) with respect to v, we get

$$v_1 = \begin{cases} e^{(\log \mu_c - \Omega_1 s_c)} & \beta_1 = 0 \\ (\mu_c^{\beta_1} - \Omega_1 \beta_1 s_c)^{1/\beta_1} & \text{otherwise} \end{cases}, \tag{17}$$

$$v_1 = \begin{cases} e^{(\log \mu_c + \Omega_1 s_c)} & \beta_1 = 0 \\ (\mu_c^{\beta_1} + \Omega_1 \beta_1 s_c)^{1/\beta_1} & \text{otherwise} \end{cases}. \tag{18}$$

Here μ_c is the radius of curvature at the connection point. For *type* 1 curves, we use Eqn.(17). For *type* 2 curves, we use Eqn.(18). Now we have computed all the parameters of the second curve, except

for s_d. s_d is determined by drawing the second curve until it intersects the cone c_0. In Fig. 4(a), the second curve segment intersects the cone at $\mathbf{P}_o$. Note that the second curve needs to be translated and rotated so that the positions and Frenet-Serret frames of the first and second curves agree with each other at the connection point. By an appropriate rotation and scaling, the generated compound-rhythm log-aesthetic space curve segment always satisfies the endpoint constraints except for the tangential constraints at the end point $\mathbf{P}_b$ without depending on the values of Λ_0 and ν_0.

4.3   Finding a Curve Segment Satisfying Endpoint Constraints

We rotate the curve segment generated in Sec. 4.2 such that $\mathbf{P}_o$ exists in the first quadrant of xy plane and scale the curve segment by $|\mathbf{P}_b|/|\mathbf{P}_o|$. Now $\mathbf{P}_o$ is in the same position as $\mathbf{P}_b$. The rotated curve segments with the overall shapes are shown in Fig. 4(b). Let the tangent vector of the second curve segment at $\mathbf{P}_o$ be $\mathbf{t}_e$. The user specified tangent at $\mathbf{P}_b$ is $\hat{\mathbf{v}}_b$. We need to find Λ_0 and ν_0 such that $\mathbf{t}_e = \hat{\mathbf{v}}_b$. By performing a minimization such that

$$f(\Lambda_0, \nu_0) = |\mathbf{t_e} \cdot \hat{\mathbf{v}}_b - 1| \tag{19}$$

becomes 0, the tangential condition is satisfied. We use the modified downhill simplex method for the minimization since $f(\Lambda_0, \nu_0)$ may not be defined similarly as in monotonic-rhythm curves.
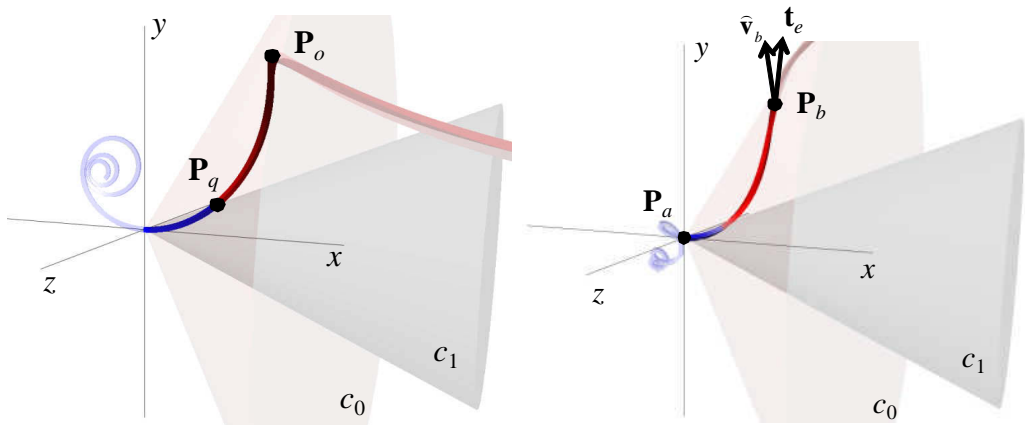

(a)   The curve segments intersect the cones   (b)   The rotated and scaled curve segment
Fig. 4: Compound-rhythm log-aesthetic space curve segments and the cones.

5   RESULTS

Fig. 5 and 6 show various compound-rhythm log-aesthetic space curve segments with their curvature and torsion plots. To help understand the shape of space curves in the 2D figures 5 and 6, the shadows of the curves are also shown. Fig. 5 shows *type* 1 curves where the radius of curvature and the radius of torsion are both monotonically increasing. Fig. 6 show *type* 2 curves where the radius of curvature and the radius of torsion are oppositely varying.   Fig. 7 shows the effect of changing r_θ.

Similarly as in monotonic-rhythm curves, the curve segment may not be constructible depending on the use-specified parameters. From our experience of drawing the curve segment, we found that the space of user-specified parameters in which a curve segment can be drawn is larger for compound-rhythm curves than monotonic-rhythm curves.
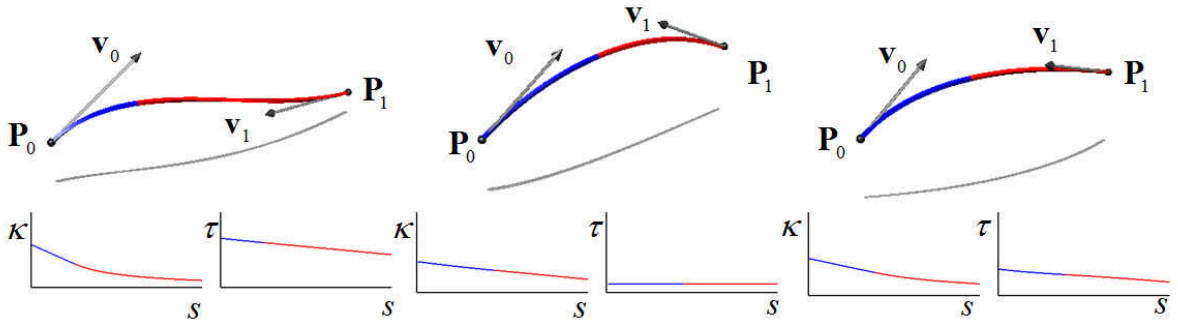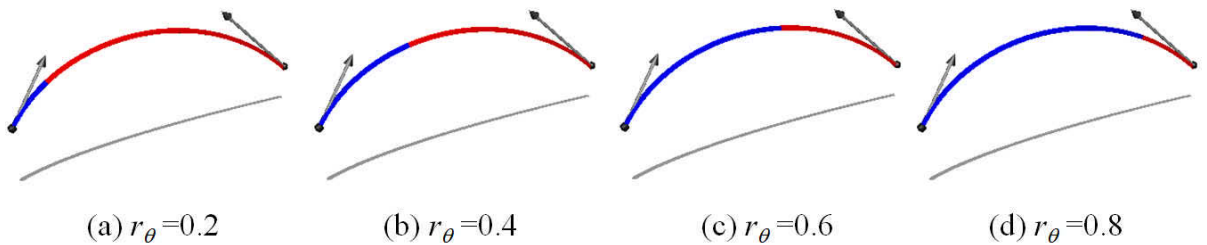
(a) $\alpha_0 = -1, \alpha_1 = 1, \beta_0 = -1, \beta_1 = 1, \Omega_0 = 0.1$ (b) $\alpha_0 = 0, \alpha_1 = -1, \beta_0 = 2, \beta_1 = 1, \Omega_0 = 0.1$ (c) $\alpha_0 = -1, \alpha_1 = 1, \beta_0 = 2, \beta_1 = -1, \Omega_0 = 0.6$
Fig. 5: Compound-rhythm log-aesthetic space curves (*type* 1).
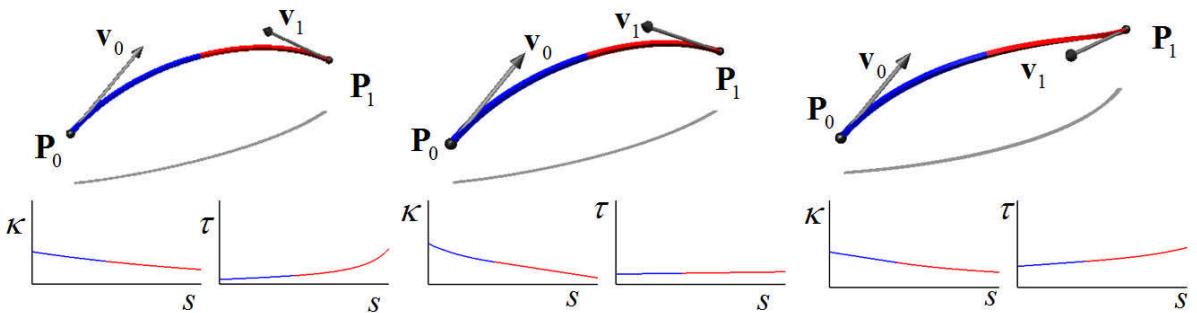
(a) $\alpha_0 = 1, \alpha_1 = 0, \beta_0 = 0, \beta_1 = 1, \Omega_0 = 0.3$ (b) $\alpha_0 = 1, \alpha_1 = -1, \beta_0 = 1, \beta_1 = -2, \Omega_0 = 0.3$ (c) $\alpha_0 = -1, \alpha_1 = 1, \beta_0 = -1, \beta_1 = 2, \Omega_0 = 0.15$
Fig. 6: Compound-rhythm log-aesthetic space curves (*type* 2).

(a) $r_\theta = 0.2$ (b) $r_\theta = 0.4$ (c) $r_\theta = 0.6$ (d) $r_\theta = 0.8$
Fig. 7: Changing r_θ modifies the connection point of the two segments.

    The computation time for generating a compound-rhythm log-aesthetic space curve segment is around 10ms on a Core 2 Duo 3.0GHz processor. Thus, compound-rhythm curves can be fully interactively generated.

6   CONCLUSIONS

This paper proposed a method for drawing a compound-rhythm log-aesthetic space curve segment. A compound-rhythm log-aesthetic space curve segment is composed of two monotonic-rhythm log-aesthetic space curve segments whose LCGs and LTGs are continuous. We can interactively draw a compound-rhythm log-aesthetic space curve segments by specifying two endpoints, their tangents, the slopes α_0, α_1 of the LCG, the slopes β_0, β_1 of the LTG, the torsion parameter Ω_0, and r_θ which determines the connection point, and the type of the curve. We showed that all the parameters for the second curve segment can be computed from the continuity condition of the two segments. Thus, the computation time of a compound-rhythm log-aesthetic space curve segment is as efficient as log-aesthetic curve segment. One of the advantages of using compound-rhythm log-aesthetic space curve

segments is that the drawable space of user-specified parameters is larger than monotonic-rhythm curves. It is trivial to connect more than two segments using the method proposed in this paper.

Future work includes an approximation by free-form curves such that the monotonicity of curvature and torsion as well as the linearity of LCGs and LTGs is guaranteed.

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