An S-curve Acceleration/Deceleration Design for CNC Machine Tools Using Quintic Feedrate Function

Jin-Hung Chen[1], Syh-Shiuh Yeh[2*] and Jin-Tsu Sun[3]

[1]National Taipei University of Technology, jhchen@ntut.edu.tw
[2]National Taipei University of Technology, ssyeh@ntut.edu.tw
[3]Industrial Technology Research Institute, ricky@itri.org.tw

ABSTRACT

The development of S-curve acceleration and deceleration (ACC/DEC) algorithms has become important for computer numerical control (CNC) machining in recent decades because it provides smoother feed motions for cutting tools and can thus shorten the machining time, maintain the machining quality, and enhance the tool life. In this study, a quintic feedrate function is developed and applied to realize an S-curve ACC/DEC algorithm with limited values of feedrate, feed acceleration, and feed jerk. This study analyzes the kinematic characteristics of the quintic feedrate function in detail and then develops a systematic scheme to determine the ACC/DEC time by referring to the given feedrate difference, feed acceleration, and feed jerk constrains. Furthermore, several simulations and experiments carried out on a three-axis CNC milling machine demonstrate the feasibility of the approaches developed in this study.

Keywords: acceleration/deceleration, feedrate function, CNC machine tools.
DOI: 10.3722/cadaps.2011.583-592

1 INTRODUCTION

With the rapid increases in the requirements of quality and productivity for manufacturing mechanical parts, computer numerical control (CNC) machine tools have been developed to operate at high speed with high acceleration to achieve accurate and rapid feed motions [4]. However, when machining segments with discontinuities exist during rapid feed motions, the motion speed of the feed drive axes can change abruptly, and the abruptly changed motion speed can cause undesirable vibration on the mechanical structure of the CNC machine tools [7]. Also, the abruptly changed motion speed can degrade the tracking and contouring accuracy of the rapid feed motions because it usually causes the applied actuators to operate in saturation conditions [3,5]. Therefore, to overcome these problems, it is important to design a proper acceleration/deceleration (ACC/DEC) algorithm to smoothen the rapid feed motions of CNC machine tools [1,5,8,9,19,20,21,24]. Because the S-curve ACC/DEC algorithm is widely used in CNC machining systems to obtain good machining accuracy and efficiency and to enhance the longevity of machine tools [2,5,6,15], this study uses the polynomial ACC/DEC before interpolation to generate feedrate profiles. Moreover, the S-curve ACC/DEC is employed to obtain proper feed acceleration and jerk characteristics during the rapid feed motions of CNC machine tools.

The development of S-curve ACC/DEC algorithms is usually based on the required machining accuracy and the physical constraints of machine tools. Leng et al. [10] presented a flexible ACC/DEC method based on the sixth-ordered displacement curve to construct the ACC/DEC functions by the quintic polynomial to reduce the residual vibration caused by the discontinuity of jerk. Leng et al. [11] proposed an ACC/DEC control model by referring to the cubic polynomial constructed speed curve to avoid the intense vibration in high-speed NC machining. Shi et al. [17] presented an S-shaped ACC/DEC algorithm equipped with a rounding error compensation tactic to compensate for the error at the transition of each segment. Rew et al. [16] proposed an efficient method to achieve the desirable motion behaviors by introducing asymmetricity to the conventional S-curve velocity profile with designated jerk magnitude. Yau et al. [23] used the S-shaped federate profile for ACC/DEC planning to make the single block achieve C1 continuity and jerk-limited capability. Weck et al. [22] used a quintic spline function to construct tool paths and achieved a cubic acceleration profile with limited feedrate and acceleration for each spline segment on a tool path. Based on the quintic spline interpolation, Erkorkmaz and Altintas [5] developed an algorithm that generates feedrate profiles with satisfied jerk limitations for high-speed CNC machining systems. Later, Altintas and Erkorkmaz [2] further developed an approach that combines feedrate optimization and quintic spline interpolation to achieve rapid and smooth motions with feedrate, acceleration, and jerk-limited cubic acceleration profiles. Meckl and Arestides [14] optimized the parameters of the asymmetric S-curve feedrate profile to minimize the residual vibration. Nevertheless, the S-curve ACC/DEC algorithms with different S-curve polynomial functions generally produce feed motions with different acceleration and jerk properties [12]. The S-curve ACC/DEC algorithms that use blended S-curve polynomial functions usually have discontinuities and can thus induce motion jerks at the junctions of blended curves during rapid feed motions [18]. Therefore, this study considered an S-curve ACC/DEC algorithm that uses only one feedrate function to achieve smooth and rapid feed motions of CNC machine tools.

In this study, an S-curve ACC/DEC algorithm using a quintic feedrate function is developed to generate feedrate profiles with limited values of feedrate, feed acceleration, and feed jerk. The kinematic characteristics of the quintic feedrate function have been analyzed in detail to obtain the relations among the ACC/DEC time, feedrate difference, feed acceleration, and feed jerk during ACC/DEC periods. Therefore, for the given feedrate difference, feed acceleration, and feed jerk constraints, the corresponding ACC/DEC time can be obtained by applying the formula derived in this study. The obtained ACC/DEC time and the given feedrate difference are then used to plan the feedrate profiles with the desired kinematic characteristics. Several simulations and experiments were carried out on a three-axis CNC milling machine to demonstrate the feasibility of the ACC/DEC approaches proposed in this study.

This paper is organized as follows. Section 2 details the development of an S-curve ACC/DEC algorithm using the quintic feedrate function. The determination of ACC/DEC time with kinematic constraints is also described in detail in this section. Section 3 presents several simulation and experimental results to demonstrate and validate the proposed ACC/DEC approaches in this study. Finally, section 4 concludes this paper.

2 THE QUINTIC FEEDRATE FUNCTION WITH VELOCITY/ACCELERATION/JERK CONSTRAINTS

Because of its continuity, the quintic polynomial function is employed to illustrate the feedrate function in the ACC/DEC periods used in this study. Consider the quintic feedrate function as shown in Eqn. (2.1).

$$F(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \tag{2.1}$$

Then, the corresponding feed acceleration and feed jerk functions can be obtained by taking the first and second derivatives with respect to the time of Eqn. (2.1), as shown in Eqn. (2.2) and Eqn. (2.3).

$$A(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \tag{2.2}$$

$$J(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 \tag{2.3}$$

Here, $F(t)$, $A(t)$, and $J(t)$ are the feedrate function, the feed acceleration function, and the feed jerk function, respectively, applied to plan the feedrate profiles used in this study. If the start time of an acceleration period is $t = t_s$ and the corresponding preset values of feedrate, feed acceleration, and feed jerk are F_s, A_s, and J_s, respectively, then, Eqn. (2.4), Eqn. (2.5), and Eqn. (2.6) are obtained to illustrate the relationships among the start time t_s, the feedrate F_s, the feed acceleration A_s, and the feed jerk J_s.

$$F_s = F(t_s) = a_0 + a_1 t_s + a_2 t_s^2 + a_3 t_s^3 + a_4 t_s^4 + a_5 t_s^5 \tag{2.4}$$

$$A_s = A(t_s) = a_1 + 2a_2 t_s + 3a_3 t_s^2 + 4a_4 t_s^3 + 5a_5 t_s^4 \tag{2.5}$$

$$J_s = J(t_s) = 2a_2 + 6a_3 t_s + 12a_4 t_s^2 + 20a_5 t_s^3 \tag{2.6}$$

Similarly, if the end time of the acceleration period is $t = t_e$ and the corresponding preset values of feedrate, feed acceleration, and feed jerk are F_e, A_e, and J_e, respectively, then, Eqn. (2.7), Eqn. (2.8), and Eqn. (2.9), respectively represent the relationships among the end time t_e, the feedrate F_e, the feed acceleration A_e, and the feed jerk J_e.

$$F_e = F(t_e) = a_0 + a_1 t_e + a_2 t_e^2 + a_3 t_e^3 + a_4 t_e^4 + a_5 t_e^5 \tag{2.7}$$

$$A_e = A(t_e) = a_1 + 2a_2 t_e + 3a_3 t_e^2 + 4a_4 t_e^3 + 5a_5 t_e^4 \tag{2.8}$$

$$J_e = J(t_e) = 2a_2 + 6a_3 t_e + 12a_4 t_e^2 + 20a_5 t_e^3 \tag{2.9}$$

Therefore, the parameters of the quintic feedrate function, $[a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5]^T$, can be obtained by solving the matrix equation Eqn. (2.10).

$$\begin{bmatrix} 1 & t_s & t_s^2 & t_s^3 & t_s^4 & t_s^5 \\ 0 & 1 & 2t_s & 3t_s^2 & 4t_s^3 & 5t_s^4 \\ 0 & 0 & 2 & 6t_s & 12t_s^2 & 20t_s^3 \\ 1 & t_e & t_e^2 & t_e^3 & t_e^4 & t_e^5 \\ 0 & 1 & 2t_e & 3t_e^2 & 4t_e^3 & 5t_e^4 \\ 0 & 0 & 2 & 6t_e & 12t_e^2 & 20t_e^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} F_s \\ A_s \\ J_s \\ F_e \\ A_e \\ J_e \end{bmatrix} \tag{2.10}$$

For each acceleration period, the start time for acceleration is set to zero; the end time for acceleration is set to the desired acceleration time t_d; and the preset values of the feed acceleration and feed jerk at start time and end time are also zeros. Then, the matrix equation, Eqn. (2.10), can be rewritten as Eqn. (2.11).

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & t_d & t_d^2 & t_d^3 & t_d^4 & t_d^5 \\ 0 & 1 & 2t_d & 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 0 & 0 & 2 & 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} F_s \\ 0 \\ 0 \\ F_e \\ 0 \\ 0 \end{bmatrix} \tag{2.11}$$

Defining the sub-matrices, A_{11}, A_{21}, and A_{22} as

$$A_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \; A_{21} = \begin{bmatrix} 1 & t_d & t_d^2 \\ 0 & 1 & 2t_d \\ 0 & 0 & 2 \end{bmatrix} \text{ and } A_{22} = \begin{bmatrix} t_d^3 & t_d^4 & t_d^5 \\ 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix},$$

the matrix equation, Eqn. (2.11), can be further rewritten as

$$A_{11} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \text{ and } A_{22} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} F_e \\ 0 \\ 0 \end{bmatrix} - A_{21} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}.$$

The parameters of the quintic feedrate function, $[a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5]^T$, can thus be obtained as Eqn. (2.12).

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} A_{11}^{-1}\begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \\ A_{22}^{-1}\left\{ \begin{bmatrix} F_e \\ 0 \\ 0 \end{bmatrix} - A_{21}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \right\} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1}\begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} t_d^3 & t_d^4 & t_d^5 \\ 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix}^{-1}\left\{ \begin{bmatrix} F_e \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & t_d & t_d^2 \\ 0 & 1 & 2t_d \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \right\} \end{bmatrix}$$
$$= \begin{bmatrix} \begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} t_d^3 & t_d^4 & t_d^5 \\ 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix}^{-1}\begin{bmatrix} F_e - F_s \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \tag{2.12}$$

By setting the feedrate difference F_d as $F_d = F_e - F_s$, which also denotes the feedrate variation during the acceleration period, Eqn. (2.12) can be rewritten as Eqn. (2.13).

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} A_{11}^{-1}\begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \\ A_{22}^{-1}\left\{ \begin{bmatrix} F_e \\ 0 \\ 0 \end{bmatrix} - A_{21}\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \right\} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} F_s \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} t_d^3 & t_d^4 & t_d^5 \\ 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix}^{-1}\begin{bmatrix} F_d \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \tag{2.13}$$

The feedrate function $F(t)$ is thus obtained as Eqn. (2.14).

$$F(t) = F_s + a_3t^3 + a_4t^4 + a_5t^5 \tag{2.14}$$

The corresponding feed acceleration function $A(t)$ and feed jerk function $J(t)$ are then obtained as shown in Eqn. (2.15) and Eqn. (2.16).

$$A(t) = 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \tag{2.15}$$

$$J(t) = 6a_3t + 12a_4t^2 + 20a_5t^3 \tag{2.16}$$

It is obvious that the feed acceleration function $A(t)$ and the feed jerk function $J(t)$ are independent of the parameter a_0, and they are closely related to the desired acceleration time t_d and the feedrate difference F_d. Here, solving the matrix equation, Eqn. (2.13), obtains the parameters a_3, a_4, and a_5 as Eqn. (2.17).

$$\begin{bmatrix} a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} t_d^3 & t_d^4 & t_d^5 \\ 3t_d^2 & 4t_d^3 & 5t_d^4 \\ 6t_d & 12t_d^2 & 20t_d^3 \end{bmatrix}^{-1}\begin{bmatrix} F_d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{10}{t_d^3} & \frac{-4}{t_d^2} & \frac{1}{2t_d} \\ \frac{-15}{t_d^4} & \frac{7}{t_d^3} & \frac{-1}{t_d^2} \\ \frac{6}{t_d^5} & \frac{-3}{t_d^4} & \frac{1}{2t_d^3} \end{bmatrix}\begin{bmatrix} F_d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{10F_d}{t_d^3} \\ \frac{-15F_d}{t_d^4} \\ \frac{6F_d}{t_d^5} \end{bmatrix} \tag{2.17}$$

Therefore, for the given values of the desired acceleration time t_d, the start feedrate F_s, and the feedrate difference F_d, the feedrate function $F(t)$ during an acceleration period becomes well defined and can be obtained by Eqn. (2.14) and Eqn. (2.17). Suppose $t_{\max}^A$ is the time for achieving the maximum feed acceleration and $t_{\max}^J$ is the time for achieving the maximum feed jerk, $t_{\max}^A$ and $t_{\max}^J$ can be obtained by solving Eqn. (2.18) and Eqn. (2.19), respectively.

$$\left.\frac{dA(t)}{dt}\right|_{t=t_{\max}^A} = 0 \Rightarrow 6a_3 t_{\max}^A + 12a_4 (t_{\max}^A)^2 + 20a_5 (t_{\max}^A)^3 = 0 \tag{2.18}$$

$$\left.\frac{dJ(t)}{dt}\right|_{t=t_{\max}^J} = 0 \Rightarrow 6a_3 + 24a_4 (t_{\max}^J) + 60a_5 (t_{\max}^J)^2 = 0 \tag{2.19}$$

Eqn. (2.20) and Eqn. (2.21) respectively show the obtained $t_{\max}^A$ and $t_{\max}^J$.

$$t_{\max}^A = \left\{0, \quad \frac{t_d}{2}, \quad t_d\right\} \tag{2.20}$$

$$t_{\max}^J = \{0.211 t_d, \quad 0.789 t_d\} \tag{2.21}$$

By referring to the profile of a quintic polynomial function, Eqn. (2.22) and Eqn. (2.23) respectively show the exact solutions.

$$t_{\max}^A = \frac{t_d}{2} \tag{2.22}$$

$$t_{\max}^J = 0.211 t_d \tag{2.23}$$

Moreover, by respectively substituting Eqn. (2.22) and Eqn. (2.23) into Eqn. (2.15) and Eqn. (2.16), the maximum feed acceleration $A_{\max}$ and the maximum feed jerk $J_{\max}$ can be obtained as shown in Eqn. (2.24) and Eqn. (2.25), respectively.

$$A_{\max} = A(t_{\max}^A) = 3a_3 (t_{\max}^A)^2 + 4a_4 (t_{\max}^A)^3 + 5a_5 (t_{\max}^A)^4 = 1.875 \frac{F_d}{t_d} \tag{2.24}$$

$$J_{\max} = J(t_{\max}^J) = 6a_3 (t_{\max}^J) + 12a_4 (t_{\max}^J)^2 + 20a_5 (t_{\max}^J)^3 = 5.773 \frac{F_d}{t_d^2} \tag{2.25}$$

Suppose $A_{\lim}$ and $J_{\lim}$ are the limit values of feed acceleration and feed jerk respectively during an acceleration period. It is then required to design the acceleration time t_d according to the given feedrate difference F_d so that the maximum feed acceleration $A_{\max}$ is less than the limit value $A_{\lim}$ and the maximum feed jerk $J_{\max}$ is less than the limit value $J_{\lim}$. In other words, the inequalities, Eqn. (2.26) and Eqn. (2.27), must be held during the acceleration period.

$$1.875 \frac{F_d}{t_d} < A_{\lim} \ \text{ or } \ 1.875 \frac{F_d}{A_{\lim}} < t_d \tag{2.26}$$

$$5.773 \frac{F_d}{t_d^2} < J_{\lim} \ \text{ or } \ \sqrt{5.773 \frac{F_d}{J_{\lim}}} < t_d \tag{2.27}$$

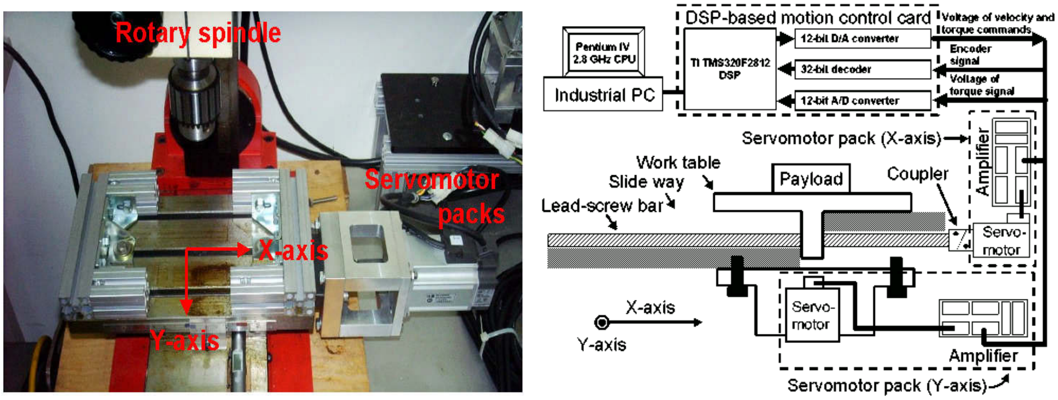
The desired acceleration time t_d can thus be chosen as Eqn. (2.28).

$$t_d = \max\left\{1.875 \frac{F_d}{A_{\lim}}, \quad \sqrt{5.773 \frac{F_d}{J_{\lim}}}\right\} \tag{2.28}$$

Therefore, for the given feedrate difference F_d, the limit feed acceleration $A_{\lim}$, and the limit feed jerk $J_{\lim}$, Eqn. (2.28) determines the desired acceleration time t_d so that the maximum feed acceleration $A_{\max}$ and the maximum feed jerk $J_{\max}$ is respectively bounded by the limit feed acceleration $A_{\lim}$ and the limit feed jerk $J_{\lim}$. Moreover, the feedrate function $F(t)$ during the acceleration period can be obtained by Eqn. (2.14), Eqn. (2.17), and Eqn. (2.28). The determination of the desired deceleration time during a deceleration period can also be obtained by applying Eqn. (2.28) because of the symmetry of the quintic polynomial function.

3 SIMULATIONS AND EXPERIMENTAL RESULTS

Simulations and experiments were carried out on a three-axis CNC milling machine comprising an industrial PC, a digital signal processor (DSP)–based motion control card, and a mechanical system equipped with commercial servomotor packs. Fig. 1. presents a photograph and a schematic drawing of the experimental setup. The industrial PC with a Pentium IV 2.8 GHz CPU is used to provide functions and includes an interface for simulations, human and machine operations, implementation of the proposed approaches, and recording data values during experiments. The DSP-based motion control card with a high-performance TI TMS320F2812 DSP is used as the interface for sending motion commands to and for receiving feedback signals from the servomotor packs with a sampling period of 1.0 ms. The mechanical system of the milling machine is mainly composed of a vertical axis and a biaxial moving table. The biaxial moving table is driven by two Panasonic A4-type AC servomotor packs [13] equipped with built-in velocity and torque control loops. Here, torque and velocity command signals are sent to each amplifier of the servomotor packs through a 12-bit D/A converter installed on the DSP-based motion control card. A rotary incremental encoder is directly coupled to the motor of a servomotor pack, and the generated encoder signals, used to indicate the angular positions of the motor, are received through a 32-bit decoder implemented on the DSP-based motion control card. A 12-bit A/D converter implemented on the DSP-based motion control card is used to receive analog signals from the servomotor packs to monitor the torque values that actuate the biaxial moving table. Tab. 1. lists some specifications of the main components used in this study.



(a) Photograph (b) Schematic drawing

Fig. 1: The experimental setup.

Milling machine	Work-table area (mm × mm)	240 × 145
	Size (mm × mm × mm)	500 × 350 × 690
	X/Y/Z stroke (mm × mm × mm)	130 × 180 × 230
	Maximum speed (mm/min)	1,200
	Screw lead (mm)	1.2
Servomotor pack	Rated power (kW)	0.2
	Rated torque (Nm)	0.64
	Maximum torque (Nm)	1.9
	Rated angular velocity (rpm)	3,000
	Maximum angular velocity (rpm)	5,000
	Inertia of rotor (kg-m^2)	0.42×10^{-4}
	Resolution of encoder (encoder increments/revolution)	10,000

Tab. 1: Specifications of main components in this study.

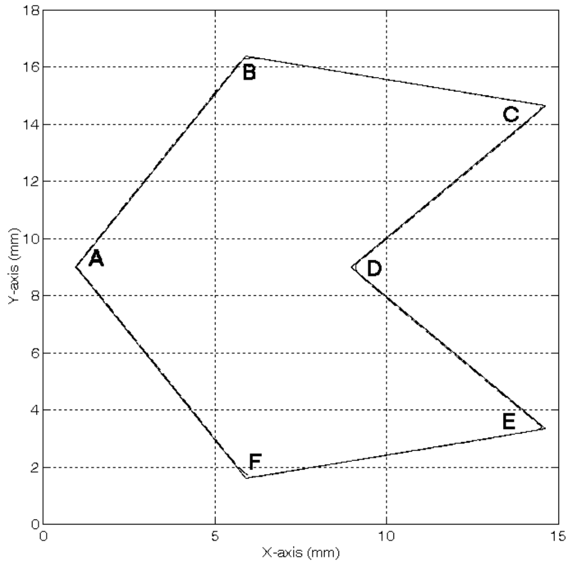
An effective motion test is used to validate the proposed approaches in this study. The performance index is obtained by referring to the maximum of absolute values. Moreover, some motion conditions are further applied to the motion test to constrain the motions of the cutting tool, including:

- The normal feedrate: 800 mm/min
- The limit acceleration/deceleration: 12.5 m/sec^2
- The limit jerk: 25000 m/sec^3

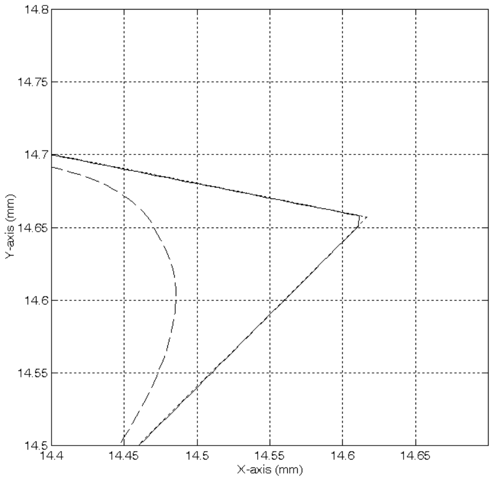
Fig. 2. shows the test path and the corresponding motion results. Here, the model path is the path obtained by linearly connecting the cutter locations; the interpolated path is obtained by sequentially interpolating the model path; and the actual path is the actual motion result of the cutting tool. The cutter starts its motion from cutter location A, sequentially passes through cutter locations B to F, and finally returns to cutter location A. Tab. 2. further lists the designate corner feedrates at different corners. The motion feedrate must be lower than the corner feedrate when the cutting tool moves passing through the corresponding corner to retain the limit values of acceleration and jerk. Fig. 2(b). and Fig. 2(c). show that the interpolated path can undercut the model path and that the amount of undercut depends on the value of corner feedrate. Usually, larger corner feedrate generates larger undercut at corners. However, the actual path significantly undercuts the model path at corners mainly because of the servo-lags that exist in the applied axial servomechanisms. Fig. 3. shows the corresponding kinematic profiles of the motion test. Here, the feed values denote the values obtained along the travel distance, such as the feedrate, feed acceleration, and the feed jerk; the axial feed values are the values of each individual axis, such as the axial feedrate, axial acceleration, and axial jerk of X and Y axes. Tab. 3. lists the summarized motion results. Fig. 3(a). shows the S-curve ACC/DEC periods in each motion segment. The corresponding feed acceleration and feed jerk are clearly smooth and have maximum values that are significantly lower than the limited values, as shown in Fig. 3(b). and Fig. 3(c). However, because the discontinuities of the segments at corners abruptly change axial feed motions, the axial acceleration and axial jerk fluctuate significantly and the corresponding maximum values are significantly larger than feed values, as shown in Tab. 3. Nevertheless, the axial feed values retain the limit values of acceleration and jerk. The motion test thus validates the proposed approaches in this study.

Corner	B	C	D	E	F
Corner feedrate (mm/min)	674	425	530	425	674

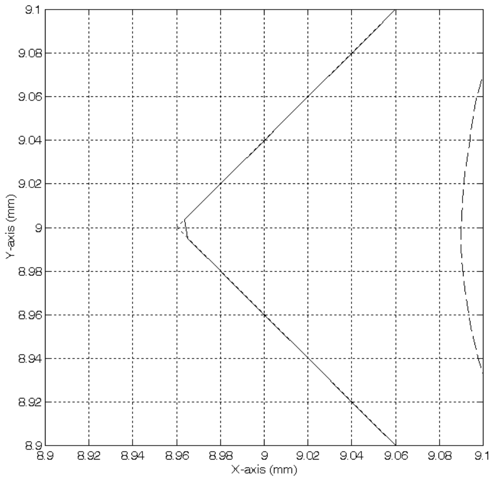
Tab. 2: The designate corner feedrates.



(a) The overall test motions

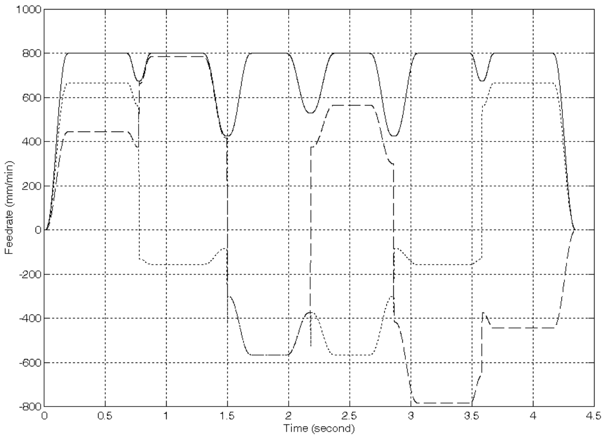


(b) The test motions at corner C

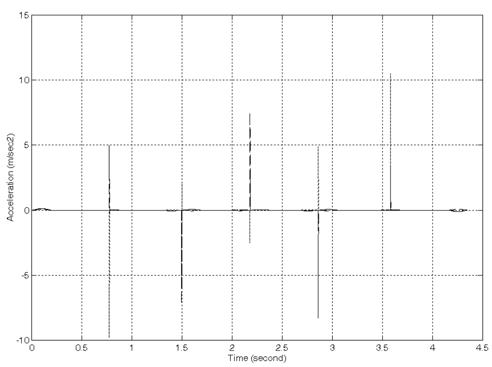


(c) The test motions at corner D

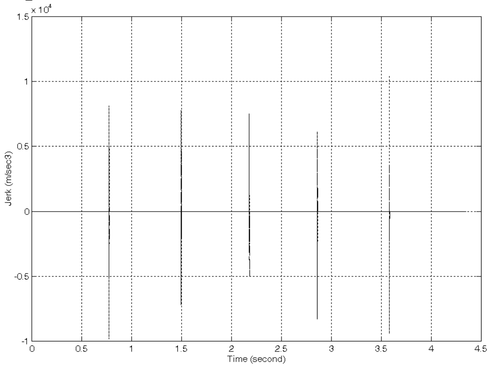
Fig. 2: Test path and the corresponding motion results (dotted line: the model path; solid line: the interpolated path; dashed line: the actual path).



(a) Feedrate profiles



(b) Acceleration profiles



(c) Jerk profiles

Fig. 3: The corresponding kinematic profiles of the motion test (solid line: feed values; dashed line: the axial feed values of X axis; dotted line: the axial feed values of Y axis).

Feedrate (mm/min)	800.000	
Feed acceleration (m/sec^2)	0.125	
Feed jerk (m/sec^3)	1.924	
	X-axis	Y-axis
Axial feedrate (mm/min)	784.628	665.176
Axial acceleration (m/sec^2)	8.281	10.480
Axial jerk (m/sec^3)	8280.809	10479.651

Tab. 3: The summarized motion results.

4 CONCLUSIONS

This study aims at developing an S-curve ACC/DEC algorithm that uses a quintic polynomial to illustrate the feedrate profiles in acceleration and deceleration periods to provide smooth feed motions for cutting tools with limited kinematic constrains. In this study, the relationships among the ACC/DEC time, feedrate difference, feed acceleration, and feed jerk of a quintic feedrate function are derived and analyzed in detail to obtain a well-defined quintic feedrate function that can be applied to construct S-curve ACC/DEC periods. Then, a systematic scheme is developed to determine a proper ACC/DEC time by referring to the given feedrate difference, feed acceleration, and feed jerk constraints so that the well-defined quintic feedrate function has bounded maximum values of feed acceleration and feed jerk during the ACC/DEC periods. An effective motion test was carried out on a three-axis CNC milling machine to validate the approaches developed in this study. Moreover, the simulation and experimental results demonstrate the feasibility of the proposed approaches for application to S-curve ACC/DEC design of CNC machine tools using a quintic feedrate function.

ACKNOLEDGEMENTS

This project is supported in part by the National Science Council of the Republic of China under Contract NSC 99-2221-E-027-009 and the Industrial Technology Research Institute, ROC, under project number 9353C72000, which is subcontracted from the Ministry of Economic Affairs, ROC. The authors would like to thank Mr. Shih-Chung Liang and Dr. Hao-Wei Nien (Mechanical and Systems Research Laboratories, Industrial Technology Research Institute, ROC) for making valuable comments.

REFERENCES

[1] Altintas, Y.: Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design, Cambridge University Press, Cambridge, UK, 2000.

[2] Alintas, Y.; Erkormaz, K.: Feedrate optimization for spline interpolation in high speed machine tools, CIRP Annals-Manufacturing Technology, 52(1), 2003, 297-302.

[3] Butler, J.; Haack, B.; Tomizuka, M.: Reference generation for high speed coordinated motion of a two axis system, American Society of Mechanical Engineers, Dynamic Systems and Control Division (Publication) DSC, 11, 1988, 457-470.

[4] Dewes, R. C.; Aspinwall, D. K.: Review of ultra high speed milling of hardened steels, Journal of Materials Processing Technology, 69(1-3), 1997, 1-17.

[5] Erkorkmaz, K.; Altintas, Y.: High speed CNC system design. Part I: Jerk limited trajectory generation and quintic spline interpolation, International Journal of Machine Tools and Manufacture, 41(9), 2001, 1323-1345.

[6] Erkorkmaz, K.; Altintas, Y.: Quintic spline interpolation with minimal feed fluctuation, Journal of Manufacturing Science and Engineering, Transactions of the ASME, 127(2), 2005, 339-349.

[7] Guo, X.-G.; Wang, D.-C.; Li, C.-X.; Liu, Y.-D.: A rapid and accurate positioning method with linear deceleration in servo system, International Journal of Machine Tools and Manufacture, 42(7) , 2002, 851-861.

[8] Hu, J.; Xiao, L.; Wang, Y.; Wu, Z.: An optimal feedrate model and solution algorithm for a high-speed machine of small line blocks with look-ahead, International Journal of Advanced Manufacturing Technology, 28(9), 2006, 930-935.

[9] Jeon, J. W.; Ha, Y. Y.: A generalized approach for the acceleration and deceleration of industrial robots and CNC machine tools, IEEE Transactions on Industrial Electronics, 47(1), 2000, 133-139.

[10] Leng, H.; Wu, Y.; Pan, X.: Research on flexible acceleration and deceleration method of NC system, Proceedings of the 2006 International Technology and Innovation Conference, 524, 2006, 1953-1956.

[11] Leng, H.-B.; Wu, Y.-J.; Pan, X.-H.: Research on cubic polynomial acceleration and deceleration control model for high speed NC machining, Journal of Zhejiang University: Science A, 9(3), 2008, 358-365.

[12] Luo, F.-Y.; Zhou, Y.-F.; Yin, J.: A universal velocity profile generation approach for high-speed machining of small line segments with look-ahead, International Journal of Advanced Manufacturing Technology, 35(5-6), 2007, 505-518.

[13] Panasonic MINAS A4-series instruction manual (DV0P4210), Matsushita Electric Industrial Co. Ltd., Japan, 2004.

[14] Meckl, P.H.; Arestides, P.B.: Optimized S-curve motion profiles for minimum residual vibration, Proceedings of the 1998 American Control Conference, 1998, 2627-2631.

[15] Nam, S.-H.; Yang, M.-Y.: A study on a generalized parametric interpolator with real-time jerk-limited acceleration, CAD Computer Aided Design, 36(1) , 2004, 27-36.

[16] Rew, K.-H.; Ha, C.-W.; Kim, K.-S.: A practically efficient method for motion control based on asymmetric velocity profile, International Journal of Machine Tools and Manufacture, 49(7-8), 2009, 678-682.

[17] Shi, X.; Xu, B.; Xie, W.; Li, B.: Design and implementation of S-shape acceleration/deceleration algorithm based on rounding error compensation tactic, Proceedings of the 7th World Congress on Intelligent Control and Automation, 2008, 7912-7916.

[18] Spong, M.W.; Hutchinson, S.; Vidyasagar, M., Robot Modeling and Control, John Wiley & Sons Inc., New York, 2006.

[19] Timar, S. D.; Farouki, R. T.; Smith, T. S.; Boyadjieff, C. L.: Algorithms for time-optimal control of CNC machines along curved tool paths, Robotics and Computer-Integrated Manufacturing, 21(1), 2005, 37-53.

[20] Tounsi, N.; Bailey, T.; Elbestawi, M.A.: Identification of acceleration deceleration profile of feed drive systems in CNC machines, International Journal of Machine Tools and Manufacture, 43(5), 2003, 441-451.

[21] Tsai, M.-C.; Cheng, M.-Y.; Lin, K.-F.; Tsai, N.-C.: On acceleration/deceleration before interpolation for CNC motion control, Proceedings of the 2005 IEEE International Conference on Mechatronics, 2005, 2005, 382-387.

[22] Weck, M.; Meylahn, A.; Hardebusch, C.: Innovative algorithms for spline-based CNC controller: production engineering research and development in Germany, Annals of the German Academic Society for Production Engineering VI/1, 1999, 83–86.

[23] Yau, H.-T.; Wang, J.-B.; Hsu, C.-Y.; Yeh, C.-H.: PC-based controller with real-time look-ahead NURBS interpolator, Computer-Aided Design and Applications, 4(1-6), 2007, 331-340.

[24] Yong, T.; Narayanaswami, R.: A parametric interpolator with confined chord errors, acceleration and deceleration for NC machining, CAD Computer Aided Design, 35(13), 2003, 1249-1259.