Tolerance Semantics Modeling Based on Mathematical Definition

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ABSTRACT

Tolerance is an almost ubiquitous concern during the whole product life cycle, and its management is imperative for seamless integration of CAD and CAM. In this paper, how to interpret tolerance semantics based on a mathematical definition of tolerance is presented. First, the tolerance zone is divided into three types according to four basic attributes: size, form, position, and orientation. The key to representing tolerance semantics exactly is to determine the position and orientation of the tolerance zone. Then, based on variations in degrees of freedom (DOF), the algebraic constraint equations for the tolerance zone boundary and variational features are deduced systematically, leading to an exact and complete interpretation of the tolerance semantics. Finally, an example is given to illustrate the rationality of the proposed method.

Keywords: tolerance modeling, tolerance semantics, degrees of freedom (DOF).
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1 INTRODUCTION

Tolerance modeling and representation form an important link between design and manufacturing processes [1,2]. There has been much discussion of tolerance modeling and representation in recent years. The proposed tolerance models can be divided into five kinds: the offset zone model, the parametric model, the variational surface model, the kinematic model, and the DOF model [3].

The offset zone model was proposed by Requicha [4,5] for representing geometric tolerance according to the variational class concept. The problem of tolerance zone formation was discussed, and a method that uses an offset operation to generate tolerance zones was implemented. The deviation of an object is considered acceptable if the boundary lies within the special range of the tolerance zone. This model was refined in subsequent work [6–8]. However, this model cannot deal with form tolerance and orientational tolerance because of their unfixed location. The parametric model [9,10] was then proposed, which was dimension-driven, constraint-based, and able to accommodate plus/minus variations of part dimensions. This model was proposed mainly for dimensional tolerance. In the variational surface model [11,12], tolerances were used to define the valid variational region directly, and the surfaces were allowed to vary independently with changes in model variables. The positions of the vertices and edges were calculated from the variational surfaces. Later, this approach was generalized by other researchers [13,14]. The kinematic model [15] has also been proposed, in which the tolerance zones were modeled taking into account the effects of datum features, datum precedence, and material modifiers. However, the kinematic model has proved to be too complex to use. Using a combination of kinematic joints, Chase et al. [16] put forward an improved kinematic method in which the geometric variations caused by tolerances are estimated using combinations of kinematic joints. The DOF model was developed by several research groups. Clement et al. [17] proposed the concept of topologically and technologically related surfaces (TTRS). Elementary surfaces were introduced and they were divided into seven types. From the TTRS, 28 different possible geometric relationships and their DOFs were constructed. And the displacement torsor was defined as 6-dimensional vector, consisting of 3 translational and 3 rotational variables. A novel method was proposed based on the "T-Map", which was similar to the set of model variables used in the DOF-based model developed by the ASU research group [18–20]. All possible variational surfaces can be represented in the T-Map. By using Minkowski sum, the method can deal with the accumulation of different kinds of tolerances.

However, these tolerance modeling methods do not represent well the semantics of tolerance specifications, although some work has been carried out on interpreting the engineering semantics of certain tolerance types specified on certain features such as a planar face, a hole, or a pattern of holes [3,21–24]. Hu and Xiong [25] proposed a computer-aided approach to dimensional and geometric tolerance design based on constraints. Hu et al. [26] also used features of ISO/TC 213 as the basis for the construction of a tolerance network and a tolerance model for assembly.

To interpret tolerance semantics in 3D CAD systems, the tolerance zone boundary and variational features can be presented using DOFs on the basis of the mathematical definition developed in this study. A novel tolerance classification for supporting computer interpretation of tolerance semantics is proposed. The tolerance zone can be divided into three categories: the immovable tolerance zone, the translational tolerance zone, and the float tolerance zone. Based on this classification, an interpretation method is given which can interpret almost types of tolerances uniformly.

2 MATHEMATICAL REPRESENTATION OF GEOMETRICAL FEATURES

The geometric elements widely used in tolerance specification are points, lines, and planes etc., which have six DOFs at most: three translational DOFs denoted by x, y, and z, and three rotational DOFs denoted by α, β, and γ. Therefore, dx, dy, and dz are the variations along translational DOFs x, y, and z, and dα, dβ, and dγ are the variations along rotational DOFs α, β, and γ. The actual number of DOFs is 6-n, where n is the number of invariant features. If a new feature is not created when the geometrical features move along axes X, Y, and Z or rotate along α, β, and γ, then those DOFs are called variant. The DOFs and the variations of points, lines, and planes are shown in Table 1.

Geometrical feature	*DOF*	*Variation*
Point	x, y, z	α, β, γ
Line	x, y, α, β	z, γ
Plane	z, α, β	x, y, γ

Tab. 1: DOFs and variations of geometrical features.

Fig. 1(a) shows an arbitrary plane in its local coordinate system (LCS) which has three DOFs, α, β, and z. The variational plane obtained by slightly varying the normal surface is shown in Fig. 1(d). with its DOFs. For small variations of the model variables, the topology of the geometry remains unchanged. The LCSs of a normal line and a normal point are shown as Fig. 1(b)., Fig 1.(c), respectively. Hence, the following equations can be obtained:

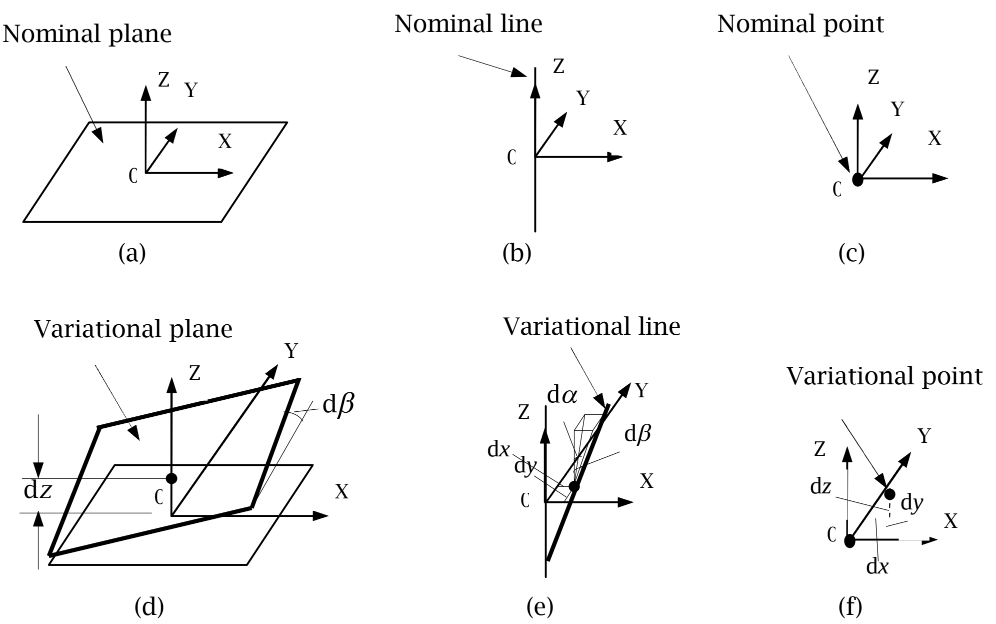

Fig. 1: Geometrical features and variations.

Plane:

$$z = 0 \tag{1}$$

Line:

$$\begin{cases} x = 0 \\ y = 0 \end{cases} \tag{2}$$

Point:

$$P(x, y, z) = 0 \tag{3}$$

For a plane, dα, dβ, and dz are the variational values of variables α, β, and z respectively, as shown in Fig. 1(d). The variational plane equation can be written as follows:

$$z = \mathrm{d}z + x \cdot \mathrm{d}\beta + y \cdot \mathrm{d}\alpha \tag{4a}$$

that is,

$$Ax + By + Cz + D = 0 \tag{4b}$$

where $A = \mathrm{d}\beta$, $B = \mathrm{d}\alpha$, $C = -1$, $D = \mathrm{d}z$.

Similarly, for a line, dx, dy, dα, and dβ are the variational values of variables x, y, α, and β respectively, as shown in Fig. 1(e). The variational line equation can be written as follows:

$$\begin{cases} x = \mathrm{d}x + z \cdot \mathrm{d}\beta \\ y = \mathrm{d}y + z \cdot \mathrm{d}\alpha \end{cases} \tag{5a}$$

that is,

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases} \tag{5b}$$

where $A_1 = 1, B_1 = 0, C_1 = \mathrm{d}\beta, D_1 = \mathrm{d}x, A_2 = 0, B_2 = 1, C_2 = \mathrm{d}\alpha, D_2 = \mathrm{d}y$.

For a point, d*x*, d*y*, and d*z* are the variational values of variables *x*, *y*, and *z* respectively, as shown in Fig. 1(f). The variational point equation can be written as follows:

$$P(x, y, z) = P(\mathrm{d}x, \mathrm{d}y, \mathrm{d}z) \tag{6}$$

3 CLASSIFICATION BASED ON TOLERANCE SEMANTICS

In the definition of ASME Y14.5.1 [27], a mathematical definition was developed for tolerances based on the point set that determines the tolerance zone. The semantics of different types of tolerance with the same tolerance zone can be considered according to this definition.

To interpret tolerance semantics effectively, the tolerance zone is first classified. Generally, the tolerance zone has four basic attributes: size, form, position, and orientation. Among these attributes, the size is specified by the designer, and all the possible forms are determinate (the forms for freeform elements are not considered here). Therefore, the essence of tolerance semantics is represented mainly by the position and orientation of the tolerance zone. By analyzing the position and orientation, tolerance can be divided into three categories based on the characteristics of the position and orientation of the tolerance zone, as follows [28]:

(1) Immovable tolerance zone (ITZ)

In the ITZ, the position and orientation of the tolerance zone are both immovable.

(2) Translational tolerance zone (TTZ)

In the TTZ, the position of the tolerance zone is movable, but the orientation of the tolerance zone is fixed.

(3) Float tolerance zone (FTZ)

In the FTZ, the position and orientation of the tolerance zone are both movable.

The classification of common tolerance types is given in Tab. 2. It can be seen from the table that geometrical and dimensional tolerance types can be dealt with in the same way.

New tolerance types	*Traditional tolerance types*
ITZ tolerance	Dimensional tolerance*, coaxiality, symmetry, positional tolerance, surface(linear) profile
TTZ tolerance	Dimensional tolerance*, parallelism, perpendicularity, angularity
FTZ tolerance	Straightness, flatness, circularity, cylindricity

Note: * means that dimensional tolerance can be either ITZ tolerance or TTZ tolerance.

Tab. 2: Classification of tolerances.

4 INTERPRETATION OF TOLERANCE SEMANTICS

4.1 ITZ Tolerance Semantics

The main characteristic of ITZ tolerance is that both position and orientation are determinate. The boundaries of this tolerance zone can be of four forms:

(1) Parallel planes or lines;

(2) Cuboids;

(3) Cylinders;

(4) Equidistant envelopes.

Among these conditions, the second condition already includes the first, while the fourth condition is closely related to the form of features. Hence, this study concentrates on the second and third conditions. The forms of the tolerance zone are shown in Fig. 2.

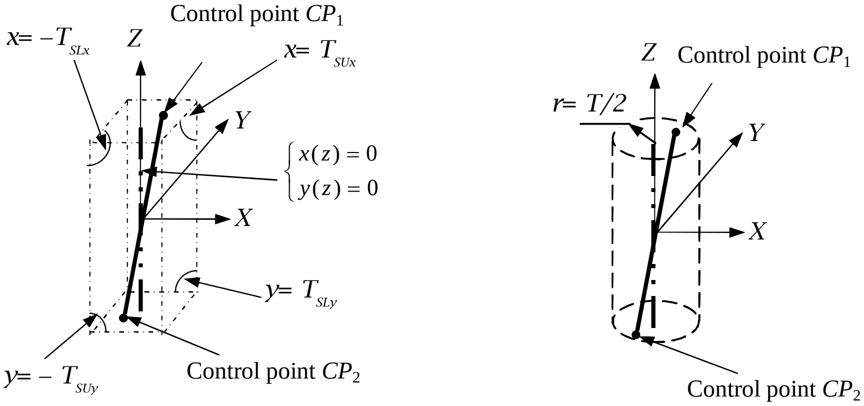

Fig. 2: Forms of ITZ: (a) Cuboid tolerance zone, (b) Cylinder tolerance zone.

As shown in Fig. 2., it is not necessary to consider tolerance interaction for features with only ITZ tolerance. Therefore, the determination of the resultant tolerance zone of this feature becomes very simple. The boundaries of the tolerance zone in Fig. 2(a). are represented by four planes parallel to the x and y axes, while the boundary of the tolerance zone in Fig. 2(b). is represented directly by a cylinder.

For the cuboid tolerance zone, the following equations can be derived:

$$\begin{aligned} x_U &= T_{SUx} \\ x_L &= -T_{SLx} \\ y_U &= T_{SUy} \\ y_L &= -T_{SLy} \end{aligned} \tag{7}$$

Here the variables T_{SLx} and T_{SUx} are the upper and lower bounds in the x-direction, while the variables T_{SLy} and T_{SUy} are the upper and lower bounds in the y-direction.

For the cylindrical tolerance zone, the equation is the following:

$$r = T/2 \tag{8}$$

The key to determining the DOF values of variable features is that the variable feature should satisfy the following condition according to the tolerance semantics:

Condition 1: There exists at least one variable feature whose projection on the active tolerance region has the same length or the same area as that of the active tolerance region.

According to Fig. 2, the variational feature has four DOFs, that is, dx, dy, dα, and dβ. From Fig. 2(a)., based on condition 1, the values of the model variables can be determined by the following inequalities and constraints:

Variations:

$$\begin{aligned} -\frac{T_{Sy}}{2a} &\le \mathrm{d}\alpha \le \frac{T_{Sy}}{2a} \\ -\frac{T_{Sx}}{2a} &\le \mathrm{d}\beta \le \frac{T_{Sx}}{2a} \\ -T_{SLx} &\le \mathrm{d}x \le T_{SUx} \\ -T_{SLy} &\le \mathrm{d}y \le T_{SUy} \end{aligned} \tag{9}$$

Constraints:

$$\begin{aligned} -T_{SLx} &\le \mathrm{d}x + \mathrm{d}\beta \cdot z \le T_{SUx} \\ -T_{SLy} &\le \mathrm{d}y + \mathrm{d}\alpha \cdot z \le T_{SUy} \end{aligned} \tag{10}$$

where $T_{Sx} = T_{SLx} + T_{SUx}, T_{Sy} = T_{SLy} + T_{SUy}$.

In Fig. 2(b), based on equations (9) and (10), let $T_{SLx} = T_{SUx} = T/2$, $T_{SLy} = T_{SUy} = T/2$, and add another constraint as follows:

$$\sqrt{(\mathrm{d}y + \mathrm{d}\alpha \cdot z)^2 + (\mathrm{d}x + \mathrm{d}\beta \cdot z)^2} \le T_{SUy} \tag{11}$$

Therefore the tolerance semantics of ITZ tolerance can be interpreted clearly and rigorously.

4.2 TTZ Tolerance Semantics

The main characteristic of TTZ tolerance is that the orientation of the tolerance zone is fixed, whereas the position is movable. TTZ tolerance is specified only when the ITZ tolerance specification cannot satisfy the functional requirements of the tolerance design rule. This means that for every TTZ, a corresponding ITZ must be defined, and therefore the interaction between them should be considered. Therefore the resultant tolerance zone of TTZ and ITZ will be the intersection of ITZ and TTZ. Here the ITZ tolerance and TTZ tolerance are denoted by T_P and T_O respectively, and their tolerance zones are denoted by Z_P and Z_O respectively.

A 3D tolerancing system based on a mathematical definition was proposed in the authors' previous work [29]. However, this 3D tolerancing system involves many Boolean intersection operations which must be performed to obtain the resultant tolerance zone and which are very time-consuming. These difficulties may even make tolerance analysis in 3D CAD systems take an unacceptably long time. The method proposed in this work can improve the efficiency dramatically in two steps: (1) Determine the maximum variational zone for the TTZ. (2) Then determine the position of the TTZ within the maximum zone and obtain the resultant tolerance zone of both the ITZ and the TTZ by performing a Boolean intersection operation between the ITZ and the TTZ.

In most cases, the forms of T_P and T_O are a cuboid and a cylinder. Considering that T_O can be translated, the resultant tolerance zone, denoted by Z_{PO}, is multiform, as shown in Fig. 3., where RTZ means resultant tolerance zone. Because a method for determining the resultant tolerance zone has been presented in previous work by the authors, this paper presents a way of defining efficiently the variations and boundary constraints of the resultant tolerance zone.

As shown in Fig. 3, the interpretation of the resultant tolerance zone is very complex because of the multiform nature of this zone. It can be seen that the determined features involved in the resultant tolerance zone are a circle and a straight line. There exist three types of Boolean intersection operations: circle-to-circle intersection, circle-to-line intersection, and line-to-line intersection. It is obvious that for all the cases enumerated, the intersection points can all be analytically calculated and parameterized given the position of the tolerance zone and the tolerance value. With the tolerance value and the position of the resultant tolerance zone, the intersection points can be calculated easily and efficiently.

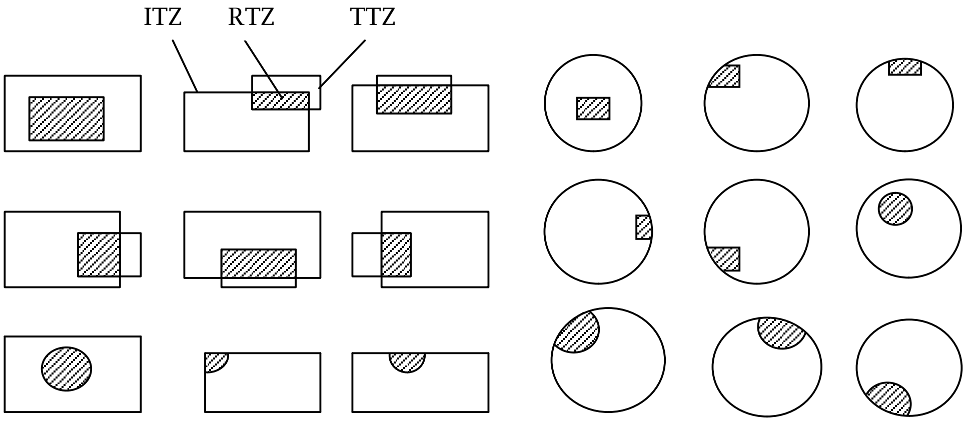

Fig. 3: Some degenerate resultant tolerance zones.

A distinct characteristic can be observed when only two types of tolerance, T_P and T_O, are affected according to the form of the resultant tolerance zone, as shown in Fig. 3. The section which is perpendicular to the central axes is generated along the central line. Each form of this section is the same. Therefore, it is convenient to present the tolerance zone boundary and the 3D tolerance zone in the form of a 2D tolerance zone. At the same time, when presented in the form of a 2D tolerance zone boundary, the number of boundary lines is four at most and one at least. The boundary is Z_P or Z_O respectively, or Z_O alone. The following rules can be deduced:

(1) When there is only one boundary line, it must be formed by circular boundary Z_O;

(2) When there are four boundary lines, they must be formed by four boundaries Z_O;

(3) When there are two or three boundary lines, they must be formed by combining Z_O and Z_P.

If the boundary is a straight line, then the DOF variation in the first variable is the distance from the boundary lines to the coordinate origin, the second variation is the angle at which the boundary line is rotated along the *Z*-axis, and the third variation is zero. If the boundary is an arc, then the first variation is the radius of this arc, the second is the variable distance from the center of this arc to the normal center, and the third is the angle at which the center of this arc is rotated relative to the normal center.

The tolerance zone can be expressed conveniently using algebraic constraints. Taking any condition in Fig. 3 as an example, a representation for algebraic constraints of variational features can be deduced. As shown in Fig. 4., the variable limit regions of a variational feature along its DOF direction should be determined, such as T_{xL}, T_{xU}, T_{yL} and T_{yU}. Similarly to the previous section, the algebraic constraints can be stated as follows:

Variations:

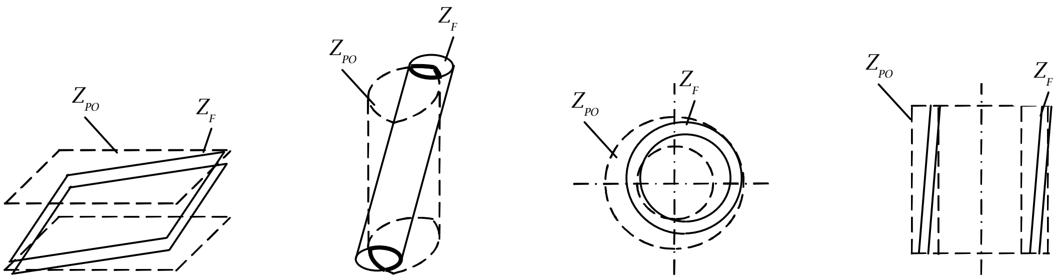

Fig. 4: Relationships between Z_F and Z_{PO}. (a) ZF are two parallel planes, (b) Z_F is a cylindrical surface, (c) Z_F are two concentric circles, (d) Z_F are concentric cylinders.

$$
\begin{aligned}
&-\frac{T_{xL}}{2a} \le \mathrm{d}\alpha \le \frac{T_{xU}}{2a} \\
&-\frac{T_{yL}}{2a} \le \mathrm{d}\beta \le \frac{T_{yU}}{2a} \\
&-T_{xL} \le \mathrm{d}x \le T_{xU} \\
&-T_{yL} \le \mathrm{d}y \le T_{yU}
\end{aligned}
\tag{12}
$$

Constraints:

$$
\begin{aligned}
&(\mathrm{d}x + \mathrm{d}\beta \cdot z) \in Z_{PO} \\
&(\mathrm{d}x + \mathrm{d}\beta \cdot z + \mathrm{d}y + \mathrm{d}\alpha \cdot z) \in Z_{PO}
\end{aligned}
\tag{13}
$$

4.3 FTZ Tolerance Semantics

The main characteristic of this tolerance zone is that both its orientation and position can vary. These can be translated or rotated arbitrarily. According to the tolerance design rule, FTZ tolerance is specified only when neither ITZ tolerance nor TTZ tolerance can satisfy the functional requirements. That means, for every FTZ, there must be a resultant tolerance zone formed by ITZ and TTZ, and thus the interaction between FTZ and the resultant tolerance zone should be considered.

For all kinds of FTZ tolerance denoted by T_F in Tab. 2., the two previous tolerances of the resultant tolerance zone, Z_{PO} and the tolerance zone Z_F of T_F, must be deduced first, as shown in Fig. 5. The position and orientation of T_F must be determined to define the algebraic constraint of the tolerance zone boundary. As the tolerance zone is floated, it behaves similarly to the variational features described in the previous section. The algebraic constraint definition of the variational features can be used. Here the condition is:

Condition 2: There must exist at least one variational feature that lies in Z_{PO} and Z_F at the same time, and whose projection on the active tolerance region has the same length or the same area as that of the active tolerance region.

After the position and orientation of Z_F are determined, the resultant tolerance zone denoted by Z_{POF} can be derived by intersecting Z_{PO} and Z_F. The form of Z_{POF} is more complex than that of Z_{PO}, and the other conditions are all the complex 3D solid models except for that shown in Fig. 4(c). For Z_{POF} in Fig. 4(c)., its boundary can be presented expediently by combining the boundary of Z_{PO} and Z_F and each boundary's start and end positions.

To present the variational feature conveniently, the approximate representation method based on descending dimension is proposed here. The Z_{POF} is sliced along certain orientations to create a series of 2D tolerance zones. For example, as shown in Fig. 4(a)., a series of sections is generated along perpendicular parallel planes. In Fig. 4(c)., Fig. 4(d)., the series of sections is generated along perpendicular axes. Similarly, for those 2D sections, boundaries can be developed by combining the boundaries of Z_{PO} and Z_F and each boundary's start and end position, as shown in Fig. 5.

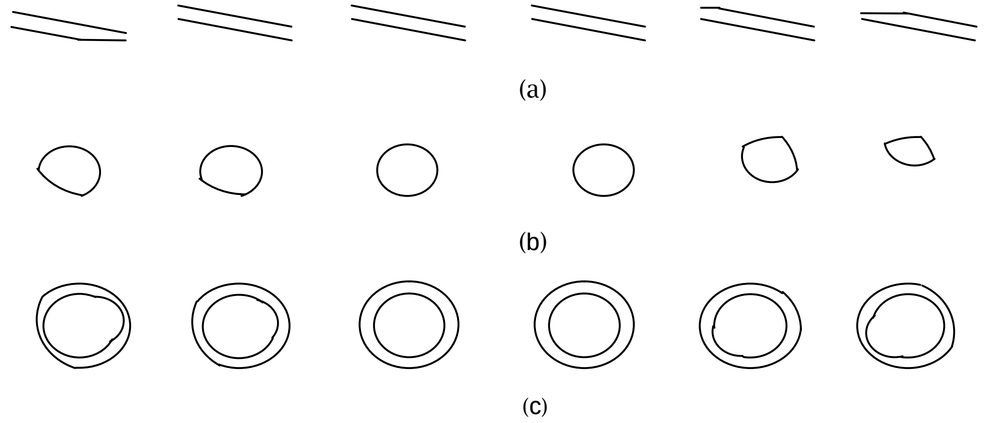

Fig. 5: Series of sections for all kinds of Z_{POF}. (a) Series of sections with Z_F as two parallel planes, (b) Series of sections with Z_F as a cylindrical surface, (c) Series of sections with Z_F as concentric cylinders.

It should be noted that the representation of the FTZ tolerance semantics is different from the interpretation of the ITZ and TTZ tolerance semantics. Therefore, the variational feature type should be different from that of the nominal element, because the variations of the FTZ tolerance cannot be expressed using the same variational feature type. Non-uniform rational B-spline (NURBS) curves and surfaces are used to represent the variational feature uniformly with the deviations of the FTZ tolerance. A series of random points located in Z_{POF} are first generated according to a given sequence and position. Then the representation of variational features is generated using NURBS curves and surfaces.

A NURBS curve is defined by the following equation [30]:

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \qquad 0 \le u \le 1 \tag{14}$$

where P_i are the control points, w_i are the weights corresponding to the control points P_i, and $N_{i,p}(u)$ are the basis functions.

Similarly, a NURBS surface can be defined as following

$$S(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j} P_{i,j}}{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j}} \qquad 0 \le u, v \le 1 \tag{15}$$

where $P_{i,j}$ are the control points, $w_{i,j}$ are their corresponding weights, and $N_{i,p}(u)$ and $N_{i,q}(u)$ are the basis functions in the *u* and *v* directions respectively.

5 CASE STUDY

The proposed method of interpreting the tolerance semantics is implemented using Microsoft Visual C++ 6.0 and the geometric modeling engine ACIS 6.0. ACIS has an important class-attribute class which enables the extension of the ACIS data structure. Through defining tolerance as an attribute of geometric features, the tolerance information can be attached to geometric elements of the solid model.

The part with tolerance specifications on the hole with the dimension of $\Phi 15$, as shown in Fig. 6., is used as an example to test the proposed method. The generation algorithm is deduced as follows:

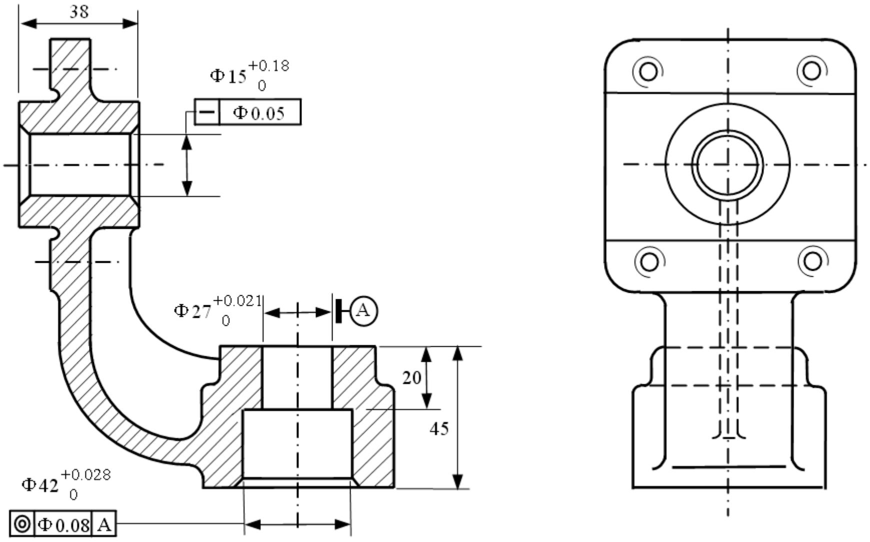

Fig. 6: Nominal geometry of a bracket.

Step 1: Establish the LCS. The center of the LCS is located on the centerline of the hole. The direction of axis *Z* is the same as the centerline of the hole. The directions of axes *x* and *y* can be selected to form a plane defining the bottom surface of the hole.

Step 2: Determine the resultant tolerance zone of the centerline of the hole. Here, the resultant tolerance zone is the intersection of the position tolerance zone and the straightness tolerance zone.

(1) Determine the position tolerance zone of the centerline. The position tolerance zone has a fixed position and orientation and is therefore easy to determine. Here, the default value is 0.1 mm.

(2) Determine the straightness tolerance zone of the centerline. The straightness tolerance zone does not have a fixed position and orientation, but its form and size are determined. Hence the position and orientation can be determined by determining the position and orientation of the centerline of the straightness tolerance zone. Suppose the center positions of the straightness tolerance zone are d*x* and d*y*; then the following equations can be deduced according to Fig. 7,

$$-\frac{T_P+T_F}{2}\le \mathrm{d}x\le \frac{T_P+T_F}{2} \tag{16a}$$

$$-\frac{T_P+T_F}{2}\le \mathrm{d}y\le \frac{T_P+T_F}{2} \tag{16b}$$

and will satisfy:

$$\sqrt{(\mathrm{d}x)^2+(\mathrm{d}y)^2}\le \frac{T_P+T_F}{2} \tag{16c}$$

where T_p is the position tolerance, that is, T_p=0.1.

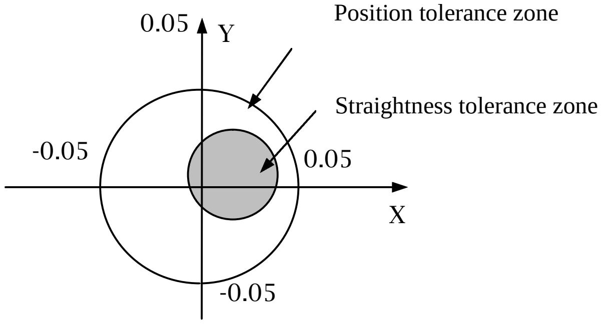

Fig. 7: Relationship between position tolerance zone and straightness tolerance zone of the hole.

Consider the different variational order of dx and dy. The variations are derived as follows:

$$-\frac{T_P+T_F}{2}\le \mathrm{d}x\le \frac{T_P+T_F}{2} \tag{17a}$$

$$-\sqrt{(\frac{T_P+T_F}{2})^2-(\mathrm{d}x)^2}\le \mathrm{d}y\le \sqrt{(\frac{T_P+T_F}{2})^2-(\mathrm{d}x)^2} \tag{17b}$$

or,

$$-\frac{T_P+T_F}{2}\le \mathrm{d}y\le \frac{T_P+T_F}{2} \tag{17c}$$

$$-\sqrt{(\frac{T_P+T_F}{2})^2-(\mathrm{d}y)^2}\le \mathrm{d}x\le \sqrt{(\frac{T_P+T_F}{2})^2-(\mathrm{d}y)^2} \tag{17d}$$

When the variational feature is generated, an appropriate distribution function should be satisfied. In this example, the normal distribution is used. The results can be stated as follows:

$$\mathrm{d}x=0.03 \tag{18a}$$

$$\mathrm{d}y=-0.005 \tag{18b}$$

After determining the position of the straightness tolerance zone, the orientations of dα and dβ can be stated as follows:

$$\mathrm{d}\alpha=-0.002 \tag{19a}$$

$$\mathrm{d}\beta=0.0008 \tag{19b}$$

(3) Obtain the resultant tolerance zone of the centerline of the hole. The resultant tolerance zone is the intersection of the position tolerance zone and the straightness tolerance zone.

Step 3: Generate the variational centerline. Because the contributing elements are points on the variational centerline, the variational centerline can be divided equally into many segments along the direction of the centerline. Then the variational cylindrical surface of each slice can be generated so as to satisfy the tolerance requirement.

Step 4: Obtain the boundary of the variational hole surface. Taking point p_i as the center point and $r+T_{SU}$ and $r-T_{SL}$ as the radii, circles are generated as the boundaries of variational hole surfaces. The following equations can be derived:

$$r+T_{SU}=15+0.18=15.18 \tag{20a}$$

$$r-T_{SL}=15-0=15 \tag{20b}$$

Step 5: Generate the entire variational hole surface. Fig. 8. shows the result for the bracket with the variational hole.

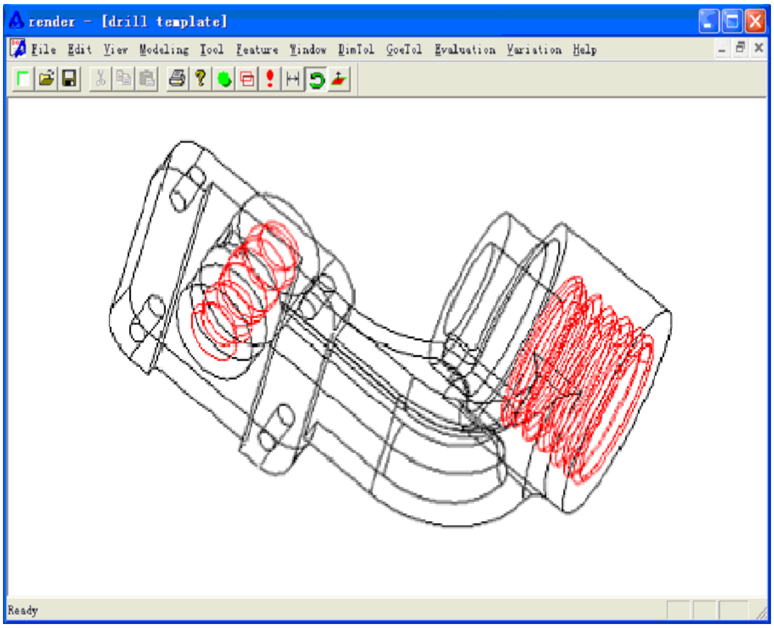

Fig. 8: Result for the bracket with the variational hole.

6 CONCLUSIONS

Tolerance greatly influences quality, process planning, measurement, cost, and assembly of products. Interpretation of tolerance semantics in the modeling and representation of tolerance is becoming more and more important. Many researchers have therefore been working on tolerance modeling.

In this work, a reasonable method of interpreting tolerance semantics systematically is proposed based on a mathematical definition. According to the types of attributes of tolerance zones, all common tolerance types, including dimensional and geometric tolerances, can be classified into three categories: ITZ tolerance, TTZ tolerance, and FTZ tolerance. The mathematical models ensure that the tolerance semantics can be represented expressively and unambiguously. The effectiveness of the proposed method is tested using a practical example. The result indicates that the method proposed here is a reasonable way to present the tolerance semantics and that the proposed method has very important applications for seamless integration of CAD and CAM. Future work will focus on more complicated situations such as mixed tolerance types under different tolerance principles using the proposed modeling method.

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