Visualization and Surface Rendering Based on Medical Image

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ABSTRACT

Visualization for medical volumetric data by using of computer-aided design and application techniques has become an important method to aid diagnosis and treatment, which has attracted a wide field of research and application. For implementing 3-D reconstruction of medical images, the problem of constructing a parametric triangular patch to smoothly connect three surface patches is studied in this paper, which has premise of image segmentation and triangular surfaces. We present a new method to define boundary conditions, which are defined by the new method have the same parameter space if the three given surface patches can be converted into the same form through parameter transformation. Consequently, any of the classic methods for constructing functional triangular patches can be used directly to construct a parametric triangular patch to connect given surface patches with G^1 continuity. Reconstruction effects prove this method easy to get satisfied results with good quality in a short time, and the resulting parametric triangular patch preserves precision of the applied classic method.

Keywords: 3-D Visualization, parametric triangular patch, medical image.
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1 INTRODUCTION

Construction of surfaces plays an important role in computer aide design (CAD), free-form surface modeling and computer graphics (CG). To make the process of constructing complex surfaces simple, piecewise techniques are frequently used, with four-sided and triangular patches being the most popular choices. This paper studies the problem of boundary condition determination in the process of constructing parametric triangular patches to smoothly connect three given surface patches, for the purpose of 3-D reconstruction by medical images.

A curved triangular patch that interpolates the boundary interpolation conditions was first proposed by Barnhill, Birkhoff and Gordon [1]. The triangular patch is constructed using the *Boolean sum scheme*. Gregory [2] used the *convex combination method* to construct a triangular patch. The triangular patch is formed by the convex combination of three interpolation operators, each of which satisfies the interpolation conditions on two sides of a triangle. The idea [2] was further extended in papers [3, 4]. Nielson [5] presented a *side-vertex method* to construct a curved triangular patch using combination of three interpolation operators, each satisfying the given boundary conditions at a vertex and its opposite side. Hagen [6] extended Nielson's approach to construct *geometric patches*. These results have been generalized to triangular patches with first and second order geometric continuity [7, 8]. The problem of constructing non-four-sided patches including curved triangular patches was also studied in [9, 10]. In [11] a method to construct a curved triangular patch by combining four interpolation operators: an *interior interpolation operator* and three *side-vertex operators* [5] is presented. The constructed triangular patch reproduces polynomial surfaces of degree four. Another method proposed recently [12] constructs a triangular patch by a *basic approximation operator* and an *interpolation operator*. The constructed triangular patch satisfies C^1 boundary condition and reproduces polynomial surfaces of degree five.

In general, the above methods all work on the assumption that the interpolation conditions on the boundary of the triangle are defined on the same parameter space. In practice, however, this is usually not the case. It is therefore necessary to have a method to determine suitable interpolation conditions so that the methods [1]-[12] can be used directly to construct triangular patches. In [13], a method is presented to construct the cross-boundary conditions. The constructed cross-boundary conditions have suitable magnitudes, but not suitable directions on the boundary of the triangle. This paper overcomes this problem by presenting a simple but efficient method to construct cross-boundary conditions which have both suitable magnitudes and directions. The combination of the new method and the classic functional triangular patch construction methods [1]-[12] can be used to construct a G^1 parametric triangular patch to connect three surface patches. The constructed parametric triangular patch has the same interpolation precision as the classic methods [1]-[12].

2 THE PROBLEM OF CONNECTING THREE PATCHES

Suppose $P_i(s_i,t_i)=(x_i(s_i,t_i),y_i(s_i,t_i),z_i(s_i,t_i)),0\le s_i,t_i\le 1,i=1,2,3$ are three given surface patches, which defined on different s_it_i-parametric planes. The three patches meet in the way shown in Fig.1. The goal is to construct a triangular patch $P_T(s,t)$ to connect the three patches $P_i(s_i,t_i),i=1,2,3$ with G^1 continuity. $P_T(s,t)$ and $P_i(s_i,t_i),i=1,2,3$ being G^1 continuous means that they have a common boundary and the normal vectors of them on the common boundary have the same direction.

If these three patches are defined on the same parametric *st*-plane, then the methods for constructing functional triangular patches can be used directly to construct a parametric triangular patch to connect these patches with C^1 continuity. In most applications of CAD, CG and related areas, however, these three patches usually are not defined on the same parameter space. In this case, one needs to define C^1 boundary conditions by the three patches so that the constructed parametric triangular patch can smoothly connect these patches with a "visually pleasing shape" suggested by these three patches. After the C^1 boundary conditions are defined, the functional methods of constructing triangular patches can be used to construct parameter triangular patch directly. As $P_T(s,t)$ and $P_i(s_i,t_i),i=1,2,3$ are defined on different parameter spaces, $P_T(s,t)$ satisfying C^1 boundary conditions, will connect these three patches with G^1 continuity.

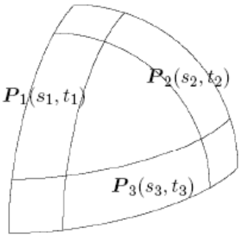

Fig.1: Three surfaces meet

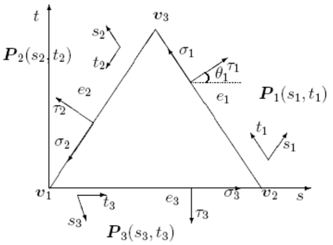

Fig.2: Three patches meet on *T*

Let *T* be an equilateral triangle with vertices $v_1=(0,0),v_2(1,0)$ and $v_3=(\frac{1}{2},\frac{\sqrt{3}}{2})$ in the *st*-parametric space, e_i denote the opposite side of v_i and τ_i is the unit outward normal vector of e_i, as shown in Fig.2. Let σ_i denote the unit vector from v_2 to v_3. σ_2 and σ_3 are defined similarly. The sides $e_i, i=1,2,3$ can be parameterized as follows:

$$\begin{aligned} e_1(u) &= (1-u)v_2 + uv_3, \\ e_2(u) &= (1-u)v_1 + uv_3, \qquad 0 \le u \le 1 \\ e_3(u) &= (1-u)v_2 + uv_2, \end{aligned} \tag{2.1}$$

The parametric triangular patch $P_T(s,t)$ to be constructed will be defined on the equilateral triangle *T*, as shown in Fig.2. On the three sides of *T*, the boundary curve and cross-boundary slope conditions given by the three surfaces, $P_i(s_i,t_i), i=1,2,3$ are as follows:

$$P_i(e_i(u)), \qquad \frac{\partial P_i}{\partial s_i}(e_i(u)), \qquad i=1,2,3 \tag{2.2}$$

where $e_i(u)$ are defined in Equ. (2.1), $P_i(e_i(u))$ and $\frac{\partial P_i}{\partial s_i}(e_i(u))$ denote the boundary value and the cross-boundary slope of $P_i(s_i,t_i)$ on the side e_i respectively.

As the boundary conditions (2.2) cannot be used directly to construct the triangular patch on *T*, we will use them to define the new boundary conditions. Let the new boundary conditions be:

$$P_T(e_i(u)), \qquad \frac{\partial P_T}{\partial \tau_i}(e_i(u)), \qquad i=1,2,3 \tag{2.3}$$

The new boundary conditions (2.3) should be defined in a way so that if the three patches $P_i(s_i,t_i), i=1,2,3$ are defined by the same surface $P(s,t)$, but with different parameter spaces, then $P_T(e_i(u))$ on the three sides of *T* in Fig.2 can be defined by $P(s,t)$, i.e., by:

$$\begin{aligned} P_T(e_i(u)) &= P(e_i(u)), \\ \frac{\partial P_T}{\partial \tau_i}(e_i(u)) &= \frac{\partial P}{\partial \tau_i}(e_i(u)), \qquad i=1,2,3 \end{aligned} \tag{2.4}$$

3 ALGORITHM OF CONSTRUCTING THE BOUNDARY CONDITIONS

In this section, we show how to determine $P_T(e_i(u))$, $\frac{\partial P_T}{\partial \tau_i}(e_i(u)), i=1,2,3$. As $P_T(s,t)$ and $P_i(s_i,t_i)$ are G^1 continuous on the common boundary, $P_T(e_i(u))$, $\frac{\partial P_T}{\partial \tau_i}(e_i(u)), i=1,2,3$ can be defined by $P_i(s_i,t_i), i=1,2,3$ as follows:

$$\begin{aligned} P_T(e_i(u)) &= P_i(e_i(u)), \\ \frac{\partial P_T}{\partial \tau_i}(e_i(u)) &= \alpha_i(e_i(u))\frac{\partial P_i}{\partial s_i}(e_i(u)) + \beta_i(e_i(u))\frac{\partial P_i}{\partial t_i}(e_i(u)), \qquad i=1,2,3 \end{aligned} \tag{3.1}$$

where $\alpha_i(e_i(u))$ and $\beta_i(e_i(u))$ are functions of *u* to be constructed, respectively.

For simplicity, we shall show the construction process of $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$ only. The $\alpha_i(e_i(u))$ and $\beta_i(e_i(u)), i=2,3$ can be constructed similarly.

As $\frac{\partial P_T}{\partial \tau_1}(e_1(u))$ and $\frac{\partial P_T}{\partial t_1}(e_1(u))$ satisfy:

$$\langle \frac{\partial P_T}{\partial \tau_1}(e_1(u)) \cdot \frac{\partial P_T}{\partial t_1}(e_1(u)) \rangle = 0$$

where $\langle a \cdot b \rangle$ denotes the dot product of vectors *a* and *b*.
It follows from Eqn.(3.1) that

$$A_1\alpha_1(e_1(u)) + B_1\beta_1(e_1(u)) = 0 \tag{3.2}$$

where

$$A_1 = \langle \frac{\partial P_1}{\partial s_1}(e_1(u)) \cdot \frac{\partial P_1}{\partial t_1}(e_1(u)) \rangle$$
$$B_1 = \langle \frac{\partial P_1}{\partial t_1}(e_1(u)) \cdot \frac{\partial P_1}{\partial t_1}(e_1(u)) \rangle$$

The Equ.(3.2) gives the function relation between $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$. If one of $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$ is defined, the rest one is defined. In the following we show how to construct $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$. At point v_2, we have:

$$\frac{\partial P_T}{\partial \tau_1}(v_2) = \alpha_1(v_2)\frac{\partial P_1}{\partial s_1}(v_2) + \beta_1(v_2)\frac{\partial P_1}{\partial t_1}(v_2) \tag{3.3}$$

The angle θ_1 between vectors τ_1 and t_3 is $30°$, thus

$$\frac{\partial P_3}{\partial t_3}(v_2) = \frac{\sqrt{3}}{2}\frac{\partial P_T}{\partial \tau_1}(v_2) - \frac{1}{2}\frac{\partial P_T}{\partial \sigma_1}(v_2)$$

From

$$\frac{\partial P_T}{\partial \sigma_1}(v_2) = \frac{\partial P_1}{\partial t_1}(v_2)$$

we have

$$\frac{\partial P_T}{\partial \tau_1}(v_2) = \frac{2\sqrt{3}}{3}\frac{\partial P_3}{\partial t_3}(v_2) + \frac{\sqrt{3}}{3}\frac{\partial P_1}{\partial t_1}(v_2) \tag{3.4}$$

It follows from Equ.(3.3) and Equ.(3.4) that $\alpha_1(v_2)$ and $\beta_1(v_2)$ in Equ.(3.1), denoted α_1^0 and β_1^0, can be determined by the following equations:

$$\begin{aligned} &\langle \frac{\partial P_1}{\partial s_1}(v_2) \cdot \frac{\partial P_1}{\partial s_1}(v_2) \rangle \alpha_1^0 + \langle \frac{\partial P_1}{\partial t_1}(v_2) \cdot \frac{\partial P_1}{\partial s_1}(v_2) \rangle \beta_1^0 = \langle \frac{\partial P_T}{\partial \tau_1}(v_2) \cdot \frac{\partial P_1}{\partial s_1}(v_2) \rangle \\ &\langle \frac{\partial P_1}{\partial s_1}(v_2) \cdot \frac{\partial P_1}{\partial t_1}(v_2) \rangle \alpha_1^0 + \langle \frac{\partial P_1}{\partial t_1}(v_2) \cdot \frac{\partial P_1}{\partial t_1}(v_2) \rangle \beta_1^0 = 0 \end{aligned} \tag{3.5}$$

On the other hand, at v_3 we have:

$$\begin{aligned} &\frac{\partial P_T}{\partial \tau_1}(v_3) = \alpha_1(v_3)\frac{\partial P_1}{\partial s_1}(v_3) + \beta_1(v_3)\frac{\partial P_1}{\partial t_1}(v_3) \\ &\frac{\partial P_T}{\partial \tau_1}(v_3) = -\frac{2\sqrt{3}}{3}\frac{\partial P_2}{\partial t_2}(v_3) - \frac{\sqrt{3}}{3}\frac{\partial P_1}{\partial t_1}(v_3) \end{aligned} \tag{3.6}$$

Thus $\alpha_1(v_3)$ and $\beta_1(v_3)$ in Equ.(3.1), denoted α_1^1 and β_1^1 can also be determined by the following equations:

$$\langle \frac{\partial P_1}{\partial s_1}(v_3)\cdot\frac{\partial P_1}{\partial s_1}(v_3)\rangle\alpha_1^1+\langle \frac{\partial P_1}{\partial t_1}(v_3)\cdot\frac{\partial P_1}{\partial s_1}(v_3)\rangle\beta_1^1=\langle \frac{\partial P_T}{\partial \tau_1}(v_3)\cdot\frac{\partial P_1}{\partial s_1}(v_3)\rangle$$
$$\langle \frac{\partial P_1}{\partial s_1}(v_3)\cdot\frac{\partial P_1}{\partial t_1}(v_3)\rangle\alpha_1^1+\langle \frac{\partial P_1}{\partial t_1}(v_3)\cdot\frac{\partial P_1}{\partial t_1}(v_3)\rangle\beta_1^1=0 \quad (3.7)$$

Now $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$ can be defined by a linear interpolation as follows:

$$\alpha_1(e_1(u))=(1-u)\alpha_1^0+u\alpha_1^1$$
$$\beta_1(e_1(u))=(1-u)\beta_1^0+u\beta_1^1 \qquad 0\le u\le 1 \quad (3.8)$$

where α_1^i and $\beta_1^i, i=0,1$ are defined by Equ. (3.5) and (3.6).

Based on Equ. (3.2) and (3.8), there are two ways to define $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$. They are shown below:

$$\alpha_1(e_1(u))=(1-u)\alpha_1^0+u\alpha_1^1$$
$$\beta_1(e_1(u))=-A_1\alpha_1(e_1(u))/B_1 \qquad 0\le u\le 1 \quad (3.9)$$

$$\beta_1(e_1(u))=(1-u)\beta_1^0+u\beta_1^1$$
$$\alpha_1(e_1(u))=-B_1\beta_1(e_1(u))/A_1 \qquad 0\le u\le 1 \quad (3.10)$$

Then, the final definition of $\alpha_1(e_1(u))$ and $\beta_1(e_1(u))$ is formed by the combination of (3.9) and (3.10), i.e., by:

$$\alpha_1(e_1(u))=\frac{(A_1\alpha_1^0-B_1\beta_1^0)(1-u)+(A_1\alpha_1^1-B_1\beta_1^1)u}{2A_1}$$
$$\beta_1(e_1(u))=\frac{(B_1\beta_1^0-A_1\alpha_1^0)(1-u)+(B_1\beta_1^1-A_1\alpha_1^1)u}{2B_1} \quad (3.11)$$

Similarly, one can define $\alpha_i(e_i(u))$ and $\beta_i(e_i(u))$ for $i=2,3$ as follows:

$$\alpha_2(e_2(u))=\frac{(A_2\alpha_2^0-B_2\beta_2^0)(1-u)+(A_2\alpha_2^1-B_2\beta_2^1)u}{2A_2}$$
$$\beta_2(e_2(u))=\frac{(B_2\beta_2^0-A_2\alpha_2^0)(1-u)+(B_2\beta_2^1-A_2\alpha_2^1)u}{2B_2}$$
$$\alpha_3(e_3(u))=\frac{(A_3\alpha_3^0-B_3\beta_3^0)(1-u)+(A_3\alpha_3^1-B_3\beta_3^1)u}{2A_3}$$
$$\beta_3(e_3(u))=\frac{(B_3\beta_3^0-A_3\alpha_3^0)(1-u)+(B_3\beta_3^1-A_3\alpha_3^1)u}{2B_3} \quad (3.12)$$

4 EXPERIMENTAL RESULTS

Experimental results presented in this section are carried out by constructing a parametric triangular patch to connect three patches or to interpolate the G^1 interpolation conditions on the sides of the triangle.

The first experiment is to compare the new method using the six functions presented by Frank[15]. The six functions are expressed by the following parametric form:

$$
\begin{aligned}
F_1(u,v) &= 3.9\exp[-0.25(9u-2)^2-0.25(9v-2)^2]+3.9\exp[-(9u+1)^2/49-(9v+1)/10] \\
&\quad +2.6\exp[-0.25(9u-7)^2-0.25(9v-3)^2]-1.04[-(9u-4)^2-(9v-7)^2] \\
F_2(u,v) &= 5.2\exp[18v-18u]/(9\exp(18v-18u]+9) \\
F_3(u,v) &= 5.2\exp[1.25+\cos(5.4v)]/[6+6(3u-1)^2] \\
F_4(u,v) &= 5.2\exp[-81((u-0.5)^2+(v-0.5)^2)/16]/3 \\
F_5(u,v) &= 5.2\exp[-81((u-0.5)^2+(v-0.5)^2)/4]/3 \\
F_6(u,v) &= 5.2\ \mathrm{sqrt}[64-81((u-0.5)^2+(v-0.5)^2)]/9-2.6 \\
x(u,v) &= u \\
y(u,v) &= v
\end{aligned}
\tag{4.1}
$$

The set of data points (including 33 points) presented in Ref. [15] is used to produce triangles for comparison. The triangulation of the data set is performed by two steps. First, the data set is projected to *xy* plane, then the data set on *xy* plane is triangulated using the max-min criterion proposed by Lawson [16]. The boundary curves of 3-D triangles are defined by $(x, y, F_i(x,y)), 1 \le i \le 6$, where (x,y) is the point on the side of the triangles in Fig.3, $F_i(x,y)$ is obtained by replacing (u,v) in $F_i(u,v)$ (Equ.(4.1)) with (x,y).

The interpolation conditions for the test cases are as mentioned boundary curves and cross-boundary slopes on the 3-D triangles, taken from $F_1(u,v)$ to $F_6(u,v)$ above. Let *S* be a side of the 3-D triangles, the interpolation conditions on *S* are normalized by defining them on the unit region, i.e. on the region [0, 1]. The cross-boundary slopes on *S* are defined by $\frac{\partial F_i(u,v)}{\partial n} \times L$, where *L* denotes the length of *S*, *n* denotes the out normal vector of *S*, $1 \le i \le 6$. Based on the interpolation conditions on the 3-D triangles, the comparison is carried out by applying the new method to the method [12] to construct surfaces. The comparison results are shown in Figs.3-4, respectively. In Figs.3-4, for $1 \le i \le 6$, the surface (*a*) $F_i(u,v)$ is produced by using Equ. (4.1), the surface (*b*) $F_i(u,v)$ is produced with the method[12] by directly using the interpolation conditions, while the surface (*c*) $F_i(u,v)$ is produced with the method[12] by the boundary curves, and the cross-boundary conditions which are redefined by the new method. We can see that the surfaces (*a*) $F_i(u,v)$ and (*b*) $F_i(u,v)$ visually have no difference.

5 CONCLUSION

For purpose of satisfied medical image 3-D reconstruction, a new method has been proposed, which uses functional triangular patch construction method to construct parametric triangular patches and. Our study has shown that the new method improves previous methods in both surface shape and surface quality, which is verified by examining six functions models of the resulting surface patches and 3-D reconstruction result based on medical images. The key in achieving the improvement is a technique to define the cross-boundary conditions. The resulting cross-boundary conditions have not only suitable magnitudes but also suitable directions.

With this new method, one can directly apply any of the classic functional triangular patch construction methods to construct a C^1 parametric triangular patch to smoothly connect three surface patches. It is clear that the new method preserves precision of the classic methods. If the applied classic method has a precision of polynomials of degree *n*, then the constructed parametric triangle patches have a precision of parametric polynomials of degree *n*.

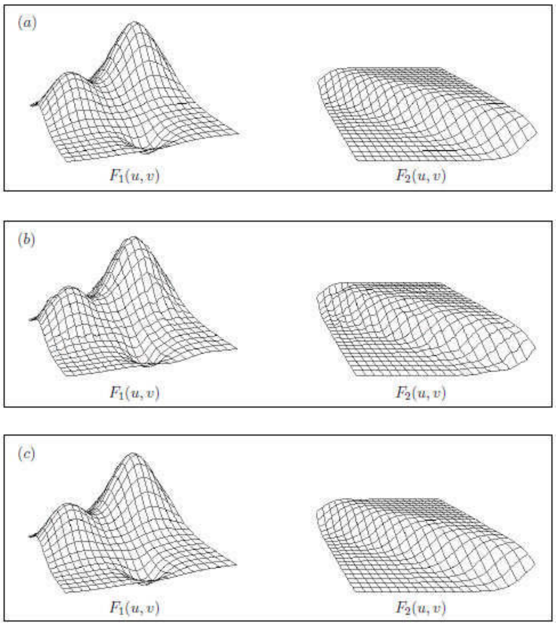

Fig. 3: Plots of $F_1(u,v)$ and $F_2(u,v)$.

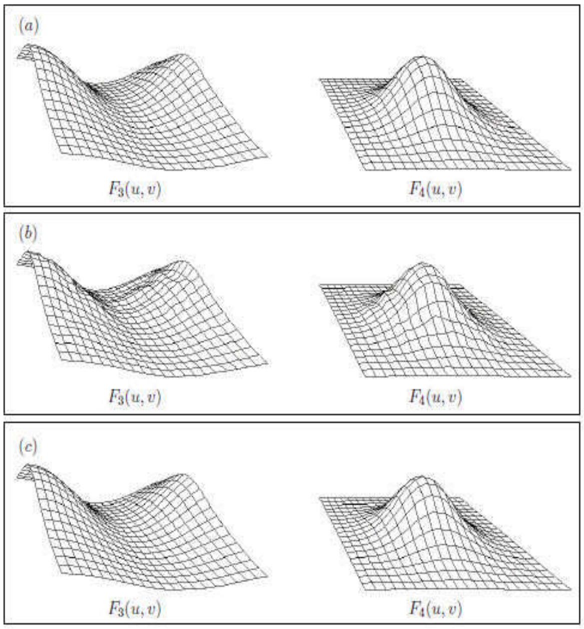

Fig. 4: Plots of $F_3(u,v)$ and $F_4(u,v)$.

The second experiment is applying this method to carry out 3-D reconstruction, which based on a large of medical images, Fig.5 illustrates that one can satisfied results with good 3-D visualization surface rendering quality, which more clearly shows the internal object, structural details, and the upper and lower spatial relationship.

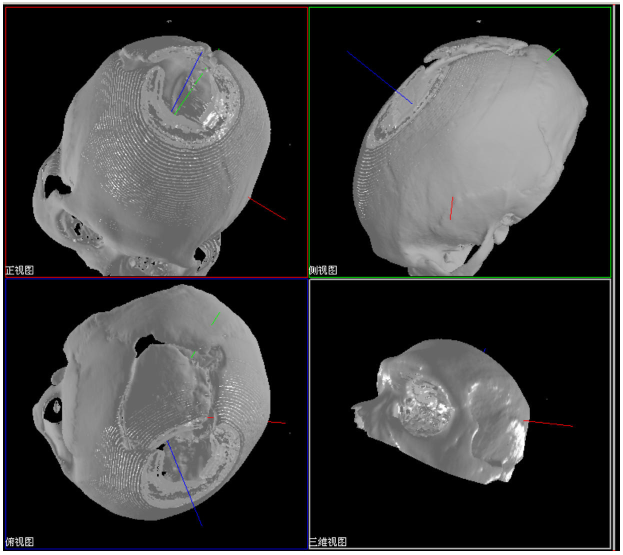

Fig. 5: 3-D reconstruction sample based on medical images.

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