



## Validation of Medial Axis Transform Objects

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### ABSTRACT

Medial Axis Transform (MAT) satisfies most of the demands of present day solid modeling and product development methods as it captures the geometric proximity of the boundary elements in a simple form. However, Euler type topology-verification equations are not available for MAT. Different features of 3D MAT like seam-end points, junction points, seams, etc. are described. Available algorithms are used to generate the 3D MATs. MATs available in the literature are also used. From the generated and available MATs, basic relations between number of seam-end points (NSEP), number of junction points (NJP) and number of seams (NS) are developed experimentally. These equations are generalized in such a way that they are valid for both 2D and 3D MATs. The applications of these equations include MAT validation and validation of MAT generation methods. Limitations of the work is presented. The relationships developed are experimental only and mathematical proof is not yet developed.

**Keywords:** MAT, validating MAT, MAT characteristics.

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### 1 INTRODUCTION

While working on animal visual systems, Harry Blum proposed a symmetric representation, which he called the **Medial Axis Transform (MAT)** and later he suggested it for image analysis and object representation. [2-5]. In later years, Harry Blum wrote that he preferred to call it the **Symmetric axis transform** but others continue to call it the medial axis as part of the larger class of medial representations. The medial axis with its associated radius function is known as the **Medial Axis Transform (MAT)**. It can be viewed as the locus of the center of a maximal ball as it rolls inside an object.

Since its introduction, the MAT has found use in a wide variety of applications that primarily involve reasoning about geometry or shape. The MAT has been used in pattern analysis and image analysis, finite element mesh generation, mold design and path planning, to name a few. There exists a

bi-jjective mapping between the MAT and object boundary. Moreover, it is possible to reconstruct the object given its MAT. Thus MAT can potentially be used as a representation scheme in geometric modelers, along with more popular schemes, such as constructive solid geometry (CSG) and boundary representation (B-rep) [23]. More importantly, dimensional reduction and topological equivalence make MAT a simplified, abstract representation of the geometry. Since the MAT also provides details with respect to the symmetry of the object, it can be used in applications where this property is considered important.

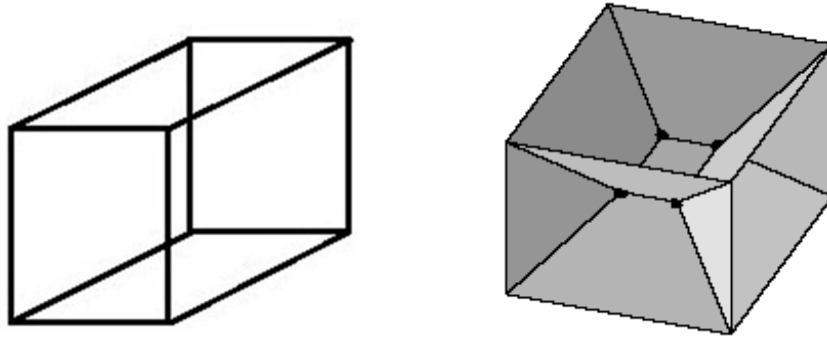


Fig. 1: A rectangular box and its MA.

Though large amount of work has been done to develop algorithm for generating MAT [6] [7], [8], [11],[12], [14], [16], [15], [18], [20], [21] and for MAT to object mapping, Some work [9], [13], [17], [22] to find out the properties of MAT but little work has been done so far in developing Euler type equations for MAT topology. The present work is an effort to fill this requirement.

## 1.1 Medial Axis (MA)

Blum's medial axis can be defined in a few different intuitive ways. All the analogies given below are equivalent.

### 1.1.1 Grassfire Analogy

Consider a fire started at the boundary that spreads with uniform speed and burns everything in its path. As the fire spreads into the middle, each part eventually meets other parts of the fire started at other parts of the boundary. When these two wave fronts meet, they quench each other because there is nothing left for either to burn. Since the fire spreads at uniform speed, these quench points must be equidistant from two different parts of the boundary. The locus of these quench points is the medial axis. Fig. 2 shows the medial axis of the objects with the quench points as the fire progresses from the boundary towards its centre [1].

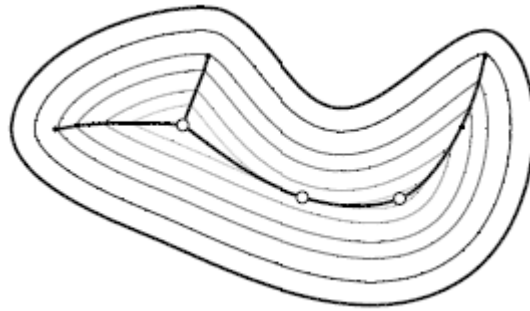


Fig. 2: Grassfire analogy.

### 1.1.2 Maximal Discs

Suppose that at each point of the boundary we placed a circle inside of the boundary such that it was tangent to it. If we start with a very small circle and expand it outwards, it will eventually bump into other parts of the boundary. When we have grown it as large as possible, we will have a circle that has the following properties:

- It is entirely within the object, and
- It is tangent to the object at more than one point

The locus of the centers of all such circles is the medial axis. For solid objects we consider maximal balls. Fig. 3 shows the medial axes of planar and solid objects respectively.

### 1.1.3 Ridges in a Distance Transform

As we progress from away from a particular point on the boundary, we find increasing values in the distance map until we reach a point at which we are now closer to some other part of the boundary than the place that we left. So, the distance increases as we progress inwards until it forms a ridge, a locus of points with values higher than those just off the ridge on either side. The locus of these ridges in the distance transform is the medial axis.

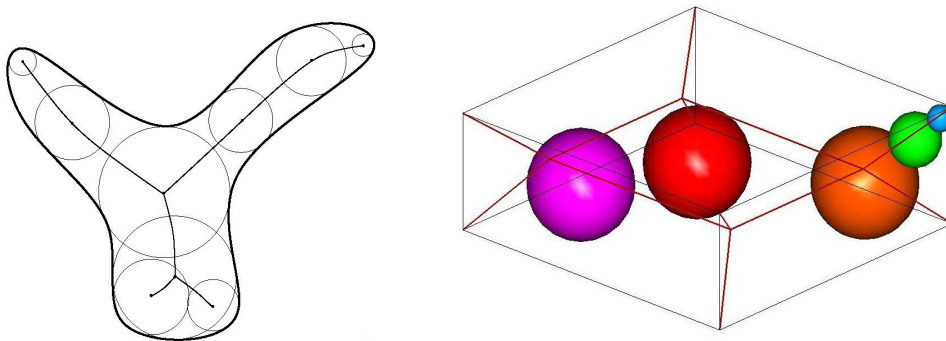


Fig. 3: Medial axis and maximal ball/discs.

Mathematically, MA is the closure of the locus of centers of maximal discs (balls) that are at least tangent to the surface at two places. Let  $D$  be a subset of  $R^n$ . A closed ball is said to be maximal in  $D$  if it is contained in  $D$  but is not a proper subset of any other ball contained in  $D$ .

The MA of a subset  $D$  of  $R^n$ , denoted  $MA(D)$ , is the locus of points which lie at the centers of all closed balls which are maximal in  $D$ , together with the limit points of this locus.

$$MA(D) = \{p \in R^n \mid \exists r \geq 0, s.t.(p, r) \in MAT(D)\}$$

## 1.2 Medial Axis Transform (MAT)

Medial axis with its associated radius function is known as Medial Axis Transform (MAT). The radius function of the medial axis of  $D$  is a continuous, real-valued function defined on  $MA(D)$  whose value at each point on the medial axis is equal to the radius of the associated maximal ball.

$$MAT(D) = \{(p, r) \in R^n \times [0, \infty) \mid B_r(p) \text{ is a maximal ball contained in } D\}$$

Medial Axis Transform is invertible, that is given the MAT of an object; the original object can be obtained from it. But we cannot trace the original object from its MA.

## 1.3 Classification of MAT Points

### 1.3.1 Classification of Points on 2D MAT

Points on 2D MAT can be classified based on the properties of their maximal disks. The classification is as follows: [7]

- **Normal point:** a point whose maximal disc touches exactly two separate boundary segments is called normal point. Point N (or any point on the line segment (A, E1) excluding the end points A and E1) in Fig. 4(a) is a normal point. Its underlying maximal disk is shown in Fig. 4(b)
- **Branch point:** a point whose maximal disc touches the domain boundary in three or more separate segments is called branch point. Points E1 and F1 in Fig. 4(a) are branch points. Fig. 4(c) shows the maximal disk corresponding to the branch point E1.
- **End point:** a point whose maximal disc touches the boundary in exactly one contiguous set is called an end point. Fig. 4(a) shows the end points A, B, C and D. These points touch the boundary at a point and the corresponding maximal disc is of radius zero.
- **Foot point:** a point of contact with the domain boundary, of the underlying disk of a point on the MAT is called the foot point (Fp1, Fp2, Fp3) of the point on the MAT. From the definition of the point types in a MAT, a normal point will have two foot-points (Fp1 and Fp2 in Fig. 4(b)), a branch point will have three or more (Fp1, Fp2, and Fp3 in Fig. 4(c)) and an end point will have one or more foot-points.

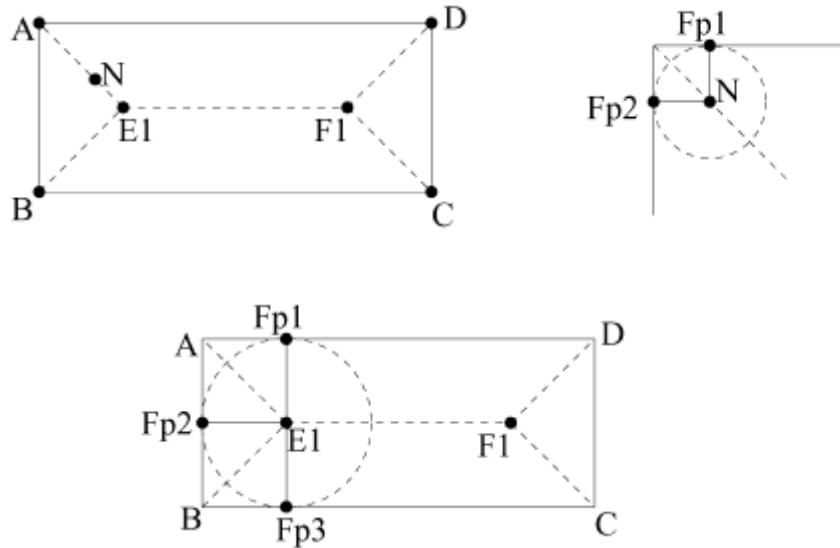


Fig. 4: Classification of points on 2D MAT: (a), (b) and (c).

### 1.3.2 Classification of Points on 3D MAT

The various elements that generally comprise the 3D MAT have been defined by Sherbrooke et al. [22]. These are reproduced here and are illustrated in Fig. 5.

- **Touch point:** a point at which a maximal sphere is tangent to the boundary surface.
- **Seam:** a connected space curve consisting of points that have three touch points and are non-manifold. Points on the seam are called seam points.
- **Junction point:** a point where the seams intersect.
- **Seam-End point:** these points do not arise from the maximal ball condition of the MAT, but are actually on the limit points of the MAT. A seam-end point generally results when a seam runs into the boundary of the solid. Therefore, vertices with convex edges incident are seam end points.
- **Skeletal edge:** a connected space curve consisting of points whose maximal spheres have a single touch point with the surface of the object. A skeletal edge is possible only if a profile edge contains a point that has a locally maximal positive curvature (LMPC).
- **Sheet:** sheet is a manifold subset of the MAT which is maximal in the sense that it is connected and bounded only by seams and skeletal edges. A seam corresponds to the intersection of sheets. Alternatively, sheets can be obtained as the connected pieces of the MAT created by dissolving all MAT connections across seams.

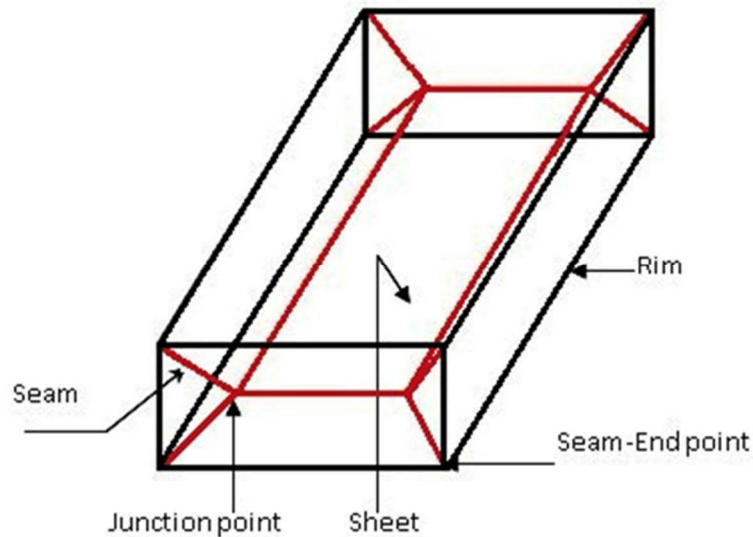


Fig. 5: Classification of points on 3D MAT.

- **Sheet point:** a point in the interior of a sheet. Sheet points have precisely two touch points.
- **Sheet-End point:** these points also are among the limit points of the MAT, but whereas a seam-end point is a limit point of seam, a sheet-end point is a limit point of a sheet. All seam points are sheet-end points but all sheet-end points need not be seam points.
- **Rim:** connected components of sheet-end points are called rims. Convex edges, resulting from the intersection of sheets with the boundary are rims.

From the classification of the points on 3D MAT, any seam point should be equidistant to three boundary segments, any sheet point should be equidistant to two boundary segments and the junction point should be equidistant to four or more boundary segments.

#### 1.4 Conditions for a MAT Point

From the definition of the MAT the following two conditions that have to be satisfied by points on a MAT segment can be derived [7], [21].

##### 1.4.1 Distance Criterion

Any point on a seam (apart from the other points of the MAT) should be equidistant to three different boundary segments. This is equivalent to saying that any point on the simplified segment of MAT in 2D (apart from the terminal points) should be equidistant to two boundary segments.

##### 1.4.2 Curvature Criterion

In 2D, for free-form boundaries, it is the radius of curvature of the disk at any MAT point should be less than or equal to the minimum of the local radius of curvature of the boundary segments. In 3D, the radius of curvature of the ball (or sphere) at any point on MAT should be less than or equal to the minimum of the radius of curvatures (reciprocal of the maximum normal curvatures) at the touch

points of the boundary (This condition is similar to the condition to avoid gouging (cutter interference) when a ball-end mill is used. Otherwise, the ball will not satisfy the maximal ball criterion (the ball will pierce the boundary in the same way as the ball-end mill will cause gouging). So the point is no longer a MAT point even though the equidistant criterion is satisfied (one can call such points as bisector points since those points have to be only equidistant. Curvature criterion becomes very important in the case of determining MAT points for free form entities. Without this condition, the generated points belong to a bisector segment and not to a MAT segment. This is also one reason why any approach based on generating bisectors of free-form edge pairs would require additional processing.

### 1.5 Properties of MAT

Medial axis transform exhibits several properties that enable representation of an object in an unambiguous and simpler manner. In the literature on the MAT and its applications, many properties have been suggested. Some of them are [21, 22]

- **Uniqueness:** there is unique MAT for a given object, and it is invariant with respect to the coordinate system.
- **Continuity:** MA of continuous boundary is also continuous. Choi et al. [23] proved that if a plane domain has finite number of boundary curves each of which consists of finite number of real analytic pieces, then the medial axis is a connected geometric graph in  $\mathbb{R}^2$  with finitely many vertices and edges, and each edge is a real analytic curve which can be extended in the  $C^1$  manner at the end vertices. Chazal et al. [18] proved that the medial axis transform is also continuous when  $C^2$ -perturbations are applied on shapes in  $\mathbb{R}^3$ .
- **Decomposition:** the dimensionality of a MA is lower than that of its object. 2D MAT transforms planar figures into lines and 3D MAT decomposes an object into surface patches.
- **Topological Equivalence:** the MA is topologically (homotopically) equivalent to the object. That is, all the features of the given object such as holes, slots, enclosed voids, etc., are retained in its MAT.
- **Recomposition (Invertibility):** given a MAT, its corresponding object can be reconstructed by taking the union of all the maximal discs/balls defined by that MAT.
- The MAT has no interior.
- Small change in the boundary of an object causes a large change in its MAT. Some times a large number of variations on the object boundary results in a discontinuous MAT.

## 2 GENERATION OF MAT

This paper uses the work done by other authors [11], [12], [16], [20], [21], as well as few mats generated in-house. Medial axis was created for large number of 2D and 3D objects that included revolved, extruded and non-extruded objects also. Few MA were created manually to verify the equations for varied shapes.

Few representative objects and their Medial Axis is shown in Fig. 6 (for 2D objects) and Fig. 7 (for 3D objects). The caption on the objects give the information about the topological characteristics of the objects.

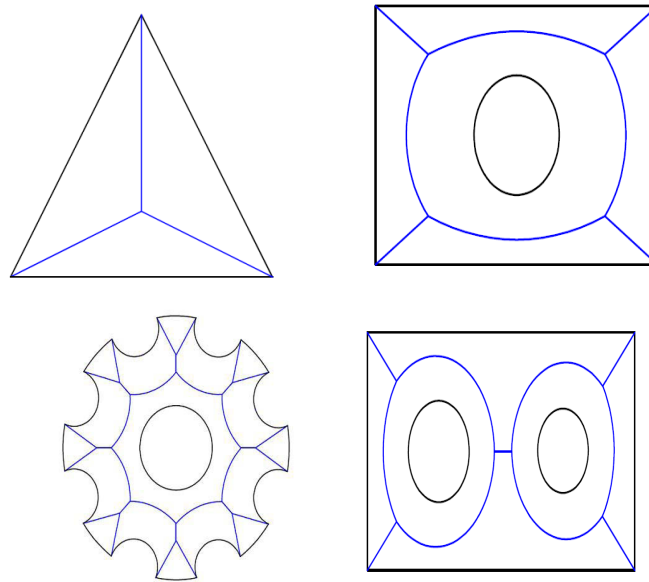


Fig. 6: Referred from left to right and top to bottom: Representative 2D objects and their MA (a) Triangle  $NEP=3$ ,  $NBP=1$ ,  $NS=3$ ,  $n=0$  (b) Object with hole  $NEP=4$ ,  $NBP=1$ ,  $NS=8$ ,  $n=1$  (c) Object with hole  $NEP=16$ ,  $NBP=16$ ,  $NS=32$ ,  $n=1$  and (d) Object with two holes  $NEP=4$ ,  $NBP=6$ ,  $NS=11$ ,  $n=2$ .

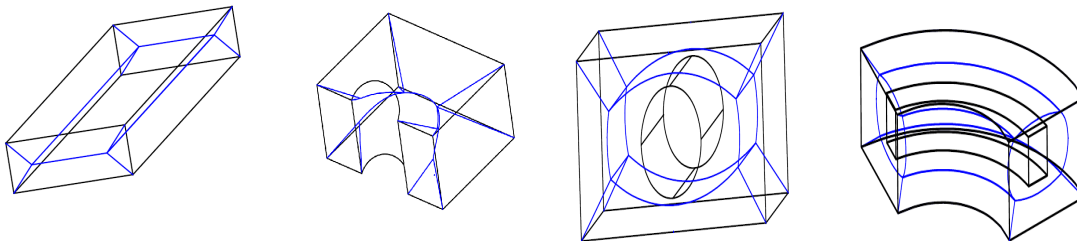


Fig. 7: Referred from left to right and top to bottom: Representative 3D objects and their MA: (a) Seam end Points  $NSEP=8$ , Junction Points  $NJP=4$ , Seams  $NS=12$ , Holes  $n=0$  (b) Seam end Points  $NSEP=12$ , Junction Points  $NJP=8$ , Seams  $NS=22$ , Holes  $n=0$  (c) Object with hole, Seam end Points  $NSEP=8$ , Junction Points  $NJP=8$ , Seams  $NS=20$ , Holes  $n=1$  and (d) Seam end Points  $NSEP=8$ , Junction Points  $NJP=8$ , Seams  $NS=20$ , Holes  $n=1$ .

### 3 RELATIONSHIP BETWEEN THE FEATURES OF MAT

In an effort to find out a relationship between different topological elements of MAT, initially two dimensional objects are considered.



### 3.1 Relationship between the Features of 2D MAT

Large number of MATs of two dimensional objects were generated and different topological elements were considered and analyzed. Finally, two relationships between various elements of two dimensional MAT were proposed and verified. These relationships give relationship among the number of end points (NEP), number of branch points (NBP), number of sections (NS) and number of holes (n). These equations are

$$NBP = NEP + 2(n-1) \quad (4.1)$$

$$NS = NEP + NBP + (n-1) \quad (4.2)$$

### 3.2 Relationship between the Features of 3D MAT

Similar to the effort for two dimensional objects, large number of MATs of three dimensional elements were generated and collected from available literature. The independent topological elements were scrutinized and finally a relationship between the number of seam-end points (NSEP), number of junction points (NJP), number of seams (NS) of MAT and the number of through holes (n) in the object, are developed, as follows.

$$NJP = NSEP + 4(n - 1) \quad (4.3)$$

$$NS = NSEP + a NJP + b(n-1) \quad (4.4)$$

Where a and b are constants which depend on whether NJP is even or odd.

$$a = 1, b = 1 \quad \text{if NJP is odd.}$$

$$a = 1.5, b = 2 \quad \text{if NJP is even.}$$

The above equations give the relationship between the number of various features of 3D MAT. The equations are named **3D MAT-point equations**.

In these equations only through holes of 3D objects are considered. By observing a large number of MATs it is concluded that the other features of the object need not be considered for developing these relations. The other features whose effect is observed in this work are slots, flanges, and blind holes. It is observed that the slots, flanges, and blind holes in the object affect the number of seam-end points (NSEP), and therefore affect the number of junction points (NJP) and number of seams (NS). Hence they are not included in these equations.

First 3D MAT-point equation gives the maximum number of junction points possible in MAT for the given number of seam-end points (NSEP). For the pentagonal prism shown in Fig. 8:

$$NSEP = 10, n = 0,$$

$$NJP = 10 + 4(0 - 1)$$

$$NJP = 6$$

Therefore the maximum possible number of junction points in a pentagonal prism is six. In this particular prism shown in Fig. 8, the number of junction points (NJP) is four.

Second 3D MAT-point equation gives the number of seams (NS) possible for a known number of seam-end points (NSEP) and junction points (NJP). For the same pentagonal prism shown in Fig. 9, the number of seams (NS) is fourteen. If the number of seam-end points (NSEP), number of junction points (NJP), number of seams (NS) of an object MAT are known, the number of through holes (n) can be calculated by using these MAT-point equations.

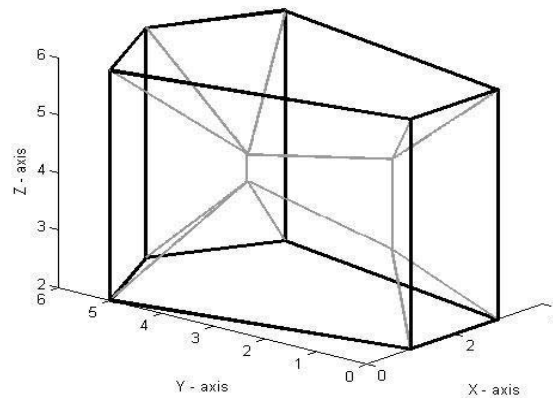


Fig. 8: 3D MA of a pentagonal prism NSEP = 10, NJP = 4, NS = 14, n = 0.

#### 4 GENERALIZED MAT-POINT EQUATIONS

An effort was done to combine Bhatt-Anil equations with the 3D MAT-point equations. It was possible to generalize these equations by introducing three new constants to take into account the dimensionality of the objects. These generalized MAT-point equations are

$$NJP = NSEP + 2 d_1 (n-1) \quad (4.5)$$

$$NS = NSEP + d_2 NJP + d_3 (n-1) \quad (4.6)$$

Here  $d_1$ ,  $d_2$ , and  $d_3$  are constants which are named as dimensionality constants. Their values depend on whether the MAT is 2D or 3D.

$$[d_1 \ d_2 \ d_3] = [1 \ 1 \ 1] \quad \text{if the MAT is 2D.}$$

$$[d_1 \ d_2 \ d_3] = [2 \ 1 \ 1] \quad \text{if the MAT is 3D and NJP is odd.}$$

$$[d_1 \ d_2 \ d_3] = [2 \ 1.5 \ 2] \quad \text{if the MAT is 3D and NJP is even.}$$

While generalizing, the nomenclature used for 3D MAT is applied for 2D MAT also. That is NSEP, NJP, and NS also represent the number of end points, branch points and sections of 2D MAT respectively.

#### 5 VALIDATION OF MAT

As discussed in the Literature Review, there are different methods available for the generation of MAT. Sometimes inconsistencies occur in the generated MATs. These inconsistencies are in the form of excess or insufficient number of junction points and seams. If the junction points and seams are in excess (insufficient), then the object representation is incorrect and the original object cannot be retrieved by using such MAT.

The MAT-point equations give the maximum possible junction/branch points for a particular number of seam-end/end points (NSEP) and number of holes ( $n$ ) of an object. And they also specify the number of seams/sections (NS) for given number of seam-end/end points (NSEP), junction/branch points (NJP) and number of holes ( $n$ ). Therefore these two MAT-point equations can be used for simple check to establish the validity of MAT generated. This will help in standardizing MAT as a representation technique in commercial CAD databases.

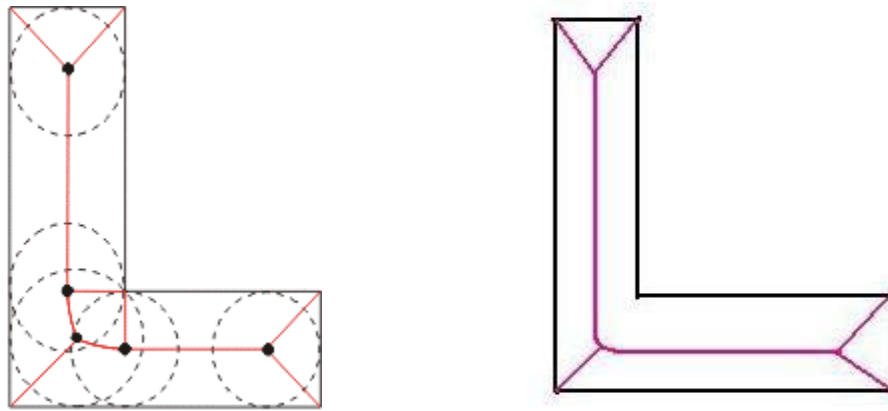


Fig. 9: Inconsistent and Correct MA (a) and (b).

For the 2D MAT shown in Fig. 9(a),  $NSEP = 6$ ,  $NJP = 5$ . From the first MAT-point equations  $NJP \leq 4$ . That is the number of branch points in the given MAT exceeds the maximum possible branch points. Therefore the given MAT is inconsistent (incorrect). The correct MAT is shown in Fig. 9(b), in which  $NSEP = 3$ ,  $NJP = 3$  ( $NJP \leq 3$ , from the first MAT-point equation).

## 6 OTHER APPLICATIONS OF GENERALIZED MAT-POINT EQUATIONS

If all the MATs generated by using a particular method are inconsistent, then the method itself is wrong. By establishing the correctness of MATs, these equations in turn check the correctness of the method by which the MATs are generated.

These equations have the capability to find out the number of holes in an object. This is an important topological information, which is otherwise very difficult to deduct from the MAT. These equations also provide a hint to utilize the  $NSEP$ ,  $NJP$ , and  $NS$  as a comparison parameters for shape matching.

## 7 LIMITATIONS OF GENERALIZED MAT-POINT EQUATIONS

These MAT equations hold good for all the objects, except two for degenerate MA. These equations does not apply for those objects whose MA is degenerate, that is the case in which MA reduce to a point or straight line. For example in case of a sphere the MA is the center point of the sphere (Fig. 10(a)), and for cylinders (cones), MA reduces to a straight line along the length of the cylinder (cone) as shown in Fig. 10(b).

It is important to mention here that all the equations thus developed are based on experimental results only and they are not yet proven mathematically.

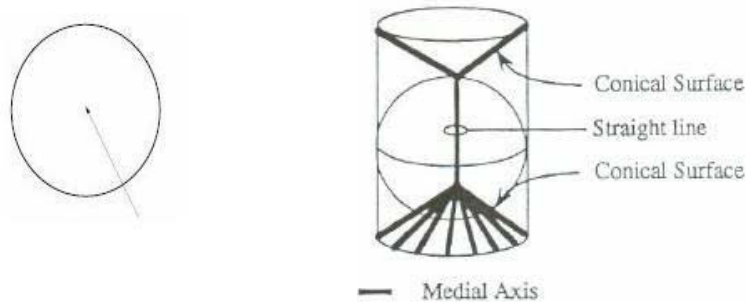


Fig. 10: Degenerate (a) and (b).

## 8 CONCLUSION

The present work is focused on finding topological validity of MAT constituents. After generating MAT for large number of objects, a correlation between different constituents is found in the form of generalized MAT point equations. Except for few degenerate cases, which are the limitations of MAT itself, these equations found valid. This work can be applied to validate some of the existing algorithms to generate MAT.

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