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A Method for Design and Optimization of Digital Origami Based on DAG Model

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Abstract. Origami is to transform a flat square sheet of paper into a finished sculpture through folding and sculpting techniques. It is widely used in many fields such as traditional art, furniture design, solar panels and medical devices. To address the problems of complex configuration of origami, uneasy folding, and the difficult process of establishing origami model, this paper proposes a digital origami representation and design optimization method with DAG (Directed Acyclic Graph) model and directional plane. Firstly, the DAG model is constructed, whose nodes and branches represent the paper states and folding behaviors respectively. Secondly, the constraint relations are defined and established between the point-line-surface geometric elements and the folding behaviors, making it feasible to conform to the paper folding process. Lastly, combined with DAG model, the folding design process, including similar folding, reasonable folding and fewer folding operations, can be optimized to improve the computational efficiency. The method provides a digital theory for origami and is validated and tested by the software Unity3d.

Keywords: Digital origami; User interaction; DAG model; Design optimization
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1 INTRODUCTION

Origami is an ancient art of folding paper into various shapes via hand-teaching technique [1]. In recent years origami has become a branch of modern geometry with involvement of many researchers [2]. Origami-related techniques can be used not only in visual art, but also in various fields, such as solar panels, robotic arm operations [3]. In order to pursue the development of origami art and meet various industrial needs, origami requires a combination of new technologies, especially digital technologies, to design artistic and practical products.

Cundy and Rollet et al. [4] pioneered the application of institutional design with origami techniques. They first considered folding marks as hinges and cardboards between folding marks as rods. In recent years, the width and depth of research in origami technique have gained importance and flourished worldwide [5].

Tachi et al. [6] proposed an origami motion simulation software that enables human-computer interaction, utilizing the projection of the origami folding process onto the constraint space to calculate the motion trajectory of the patch. The software uses the technique for triangulating polygonal patches and adjusting crease lines to make the entire paper folding process smoother. Edwin et al.[7] designed width bands between folding marks to build and simulate most origami models for paper with significant thickness, but the implementation of the algorithm is based on the Matlab platform, which limits its calculation speed, and requires the user to manually input a series of parameters including vertex coordinates, creases and vertex information that make up the patch, leading to huge workload and error-prone process. American origami artist Robert[8] designed a mathematics software Tree Maker based on calculation of the folding marks. The software includes two parts: first, the folding marks of any goal product are converted into computer process information based on the program and mathematical knowledge; and second, the correct folding order can be calculated based on the seven axioms of origami that he discovered.

Over the past few decades, with the development of digital technology, origami techniques have attracted extensive research in the field of mathematics as well as in many areas of engineering, such as expandable and retractable wings[12], energy absorbing devices[13] and DNA folding[14]. Origami techniques adopts advanced methods[15] such as 3D prototype modeling, Unity graphics, animation playback, and geometry. However, few researches have been reported on the digital representation and optimal design with the combination of both geometric elements and topological structures in the origami process.

This paper uses the plane as the main geometric element for the folding design, addressing the complex structural process of paper folding, and proposes a digital origami representation and optimal design method combined with DAG (Directed Acyclic Graph) model and directional plane. Firstly, a DAG model is constructed with its nodes representing the paper states and its branches representing the folding behaviors. Secondly, the constraint relationship is established between geometric elements (point, line and surface) and folding behaviors, making the origami process reasonable and feasible. Lastly, DAG model is optimized to enhance the paper folding process and improve calculation efficiency.

2 OVERVIEW OF METHOD

This paper takes origami as the object to fold a two-dimensional plane into a three-dimensional object, without considering the bonding and cutting. In each folding step, the piece of paper is folded along a straight line, which is called the folding edge or folding line. Folding lines and folding surfaces are the key elements of the operation [16,17]. The folding process is sequential and each step of the folding operation can be decomposed; the pre-folding state is turned into a new state after each folding step. Considering that the structure of DAG is no loop and can present hierarchical relationship, this paper integrates the DAG model with the origami technique, which can effectively solve the complexity of the paper folding process.

This paper focuses on simulating the entire folding process from its initial state to the expected state with computer-aided techniques. The main research includes the digital representation of directional plane, folding state, and folding behavior during paper folding. The process is shown in Figure 1 and steps of the process are as follows.

Step 1: Treat origami as a folding process of directional plane, which is made up of a directed loop and multiple directed edges.

Step 2: Based on the folding design process, decompose the whole process into multiple stages, each of which involves different folding behavior changing one folding state into another. Each state and behavior is represented by a node and a branch of the DAG structure, respectively.

Step 3: Define each folding behavior as a digital representation with constraints.

Step 4: Combined with DAG model, the entire folding design process can be optimized through the nodes and branches. Some optimizable operations include similar folding, reasonable folding and fewer folding operations.

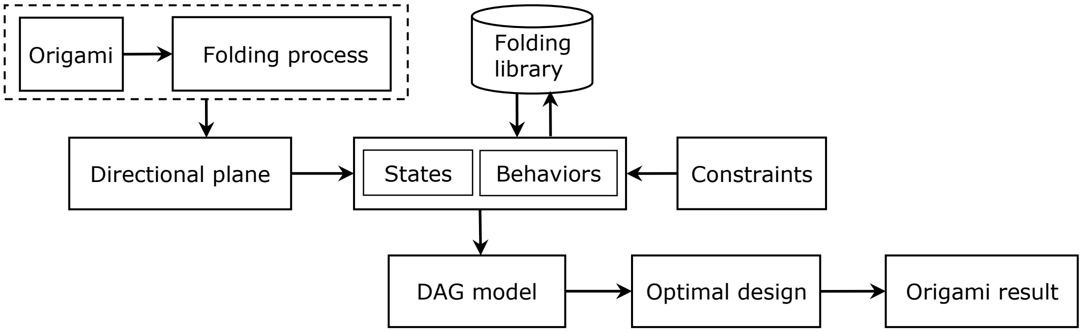


Figure 1: The basic process of the method.

3 FOLDING CLASSIFICATION AND REPRESENTATION

At present, there are many researches on origami in academic, but few researches have been reported on the combination of geometric elements and topological structures. Paper folding can be classified according to different folding rules for origami. A flat paper can be regarded as a directional plane or a directed loop consisting of multiple directed edges. In the folding process, on the one hand the geometric elements (point, line and surface) describe the folding paper states, on the other hand the change of topological structures is defined by constraint relations.

3.1 Folding Classification

Considering accuracy and intuitiveness of alignment in the actual process of folding, the basic folding operations are divided into point to position alignment, line to position alignment, angle to position alignment and composite operation: (1)Point to position alignment: Point-based determination of location, including point-to-point and point-to-line. (2)Line to position alignment: Line-based determination of location, including line-to-line, line-to-point and line-oriented. (3)Angle to position alignment: angle-based determination of location. (4)Composite operation: the operation involves two or more than two of the above-mentioned operations.

(1) Point to position alignment:

- Point-to-Point: Fold the paper along between one point and another point, shown in Figure 2(a).
- Point-to-Line: Fold the paper along a point to a specified line, shown in Figure 2(b).

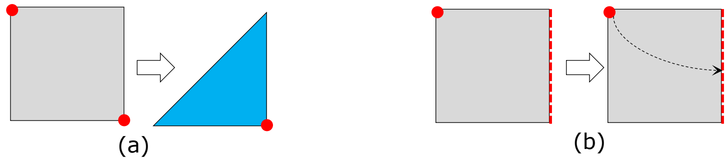


Figure 2: Point to position.

(2) Line to position alignment:

- Line-to-Line: Fold the paper along between one line and another point, shown in Figure 3(a).
- Line-to-Point: Fold the paper along a line to a specified point, shown in Figure 3(b).
- Line-oriented: Fold with folding lines, shown in Figure 3(c).

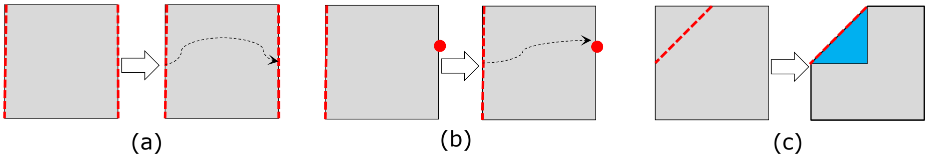


Figure 3: Line to position alignment.

(3) Angle to position alignment: Define an angle and use the angle as a reference to rotate, such as an equal angle, shown in Figure 4.

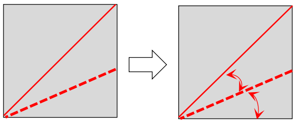


Figure 4: Angle to position alignment.

(4) Composite operation: the operation involves two or more than two of the above-mentioned operations. As shown in Figure 5, three edges and four surfaces (including positive and negative surfaces) are folded through three basic operations.

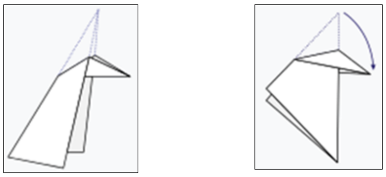


Figure 5: Composite operations.

The above folding classification can be decomposed into basic operation steps, while supporting new folding types and gradually expanding the folding library. Validate each folding type to ensure the reasonableness and feasibility of the folding process. The common folding types[18] can all be transformed into the above four types, as shown in Figure 6, (a)illustrates valley fold/mountain fold => line alignment (line to position); (b) illustrates crease => line alignment (line to position), and so forth.

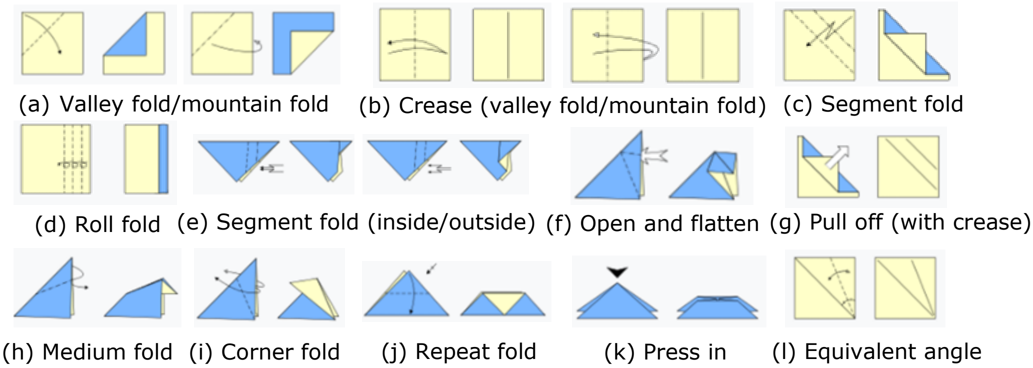


Figure 6: Basic types of paper folding.

3.2 Digital Representation of Folding

For the complex changing process of folding[19,20], combined with computer-aided design, the folding process is digitally represented in terms of both geometric elements change and topological structures change, and the plane is the basic unit of folding representation in this paper. On the one hand, the plane is defined as a directional plane, which forms a directed loop by multiple directed edges, and the folding process is essentially a decomposition and merging process of directional plane and directed edges. On the other hand, the folding process is divided into static folding states and dynamic folding behaviors. Combined with the DAG structure, the nodes of the DAG represent the folding states while the branches represent folding behaviors, and at the same time the folding dynamic process is represented layer by layer.

(1) On geometric elements, paper folding includes the point-line-surface geometric elements, operation types and constraint relations, which are defined formally as follows.

$$F = (G, P, R) \tag{3.1}$$

G is the set of geometric elements, P is the set of basic operations, and R is the constraint relationships between geometric elements during the operations.

Example: The change and representation of geometric elements when rotary folding, as shown in Figure 5.

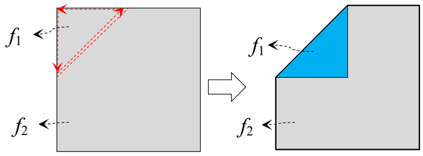


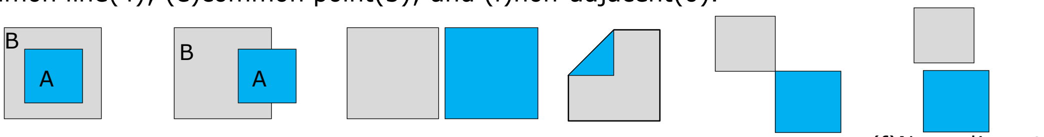
Figure 7: Geometric elements.

$G=\{f_1, f_2, l_{1,2}\}$, $P=\{\rho_1 (f_1, l_{1,2}, \alpha)\}$, $R=\{-\pi \leq \alpha \leq \pi\}$, ρ_1 for rotational folding operation.

In this, $f_1= f_1 (l_{1\_1}, l_{1\_2}, l_{1,2})$, $f_2= f_2 (l_{2\_1}, l_{2\_2}, l_{2\_3}, l_{2\_4}, l_{1,2})$, f_i is a directed loop consisting of directed edges that form a directional face, and f_2 is similar; $l_{1,2}$ is the common edge between f_1 and f_2.

(2) On topological structure, the types are divided into: surface-surface, surface-line, surface-point, line-line, line-point, and point-point topological relationships according to the topological changes and correlations of point-line-surface folding operations; each type is further divided into:

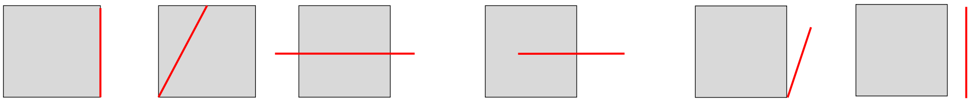
Surface-surface topological relations: (a)surface A belongs to surface B(represented by numerical value 1, and the like); (b)surface A partly belongs to surface B(2); (c)common surface(3); (d)common line(4); (e)common point(5); and (f)non-adjacent(0).



(a)Belong to (b)Partly belong to (c)Coplaner (d)Colinear (e)Copoint (f)Non-adjacent

Figure 8: Topological relation between surface and surface.

Surface-line relations: (a)line at the boundary of the surface(1); (b)line inside the surface(2); (c) line through surface(3); (d)inside and outside(4); (e)intersection point(5); (f)line outside the surface(0).



(a)Boundary (b)Inside (c)Through (d)Inside & outside (e)Intersection (f)Outside

Figure 9: Topological relation between line and surface.

Surface-point relations: (a)point at the boundary of the surface (1); (b)point inside the surface(2); (c)point outside the surface(0).

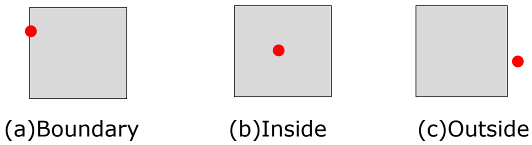


Figure 10: Topological relation between point and surface.

Line-line relations: (a)common line(1); (b)line A(green) belongs to line B(red)(2); (c)line A(green) partly belongs to line B(red)(3); (d)common point(4); (e)parallel(5); (f)non-adjacent(6).

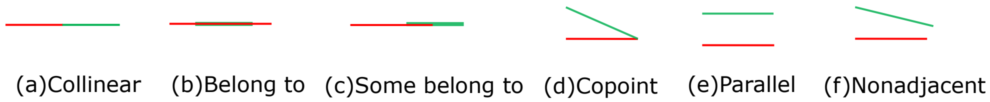


Figure 11: Topological relation between line and line.

Line-point relations: (a)point on the line(1); (b)point at the end of the line(2); (c)point outside the line(0).

Point-point relations: (a)co-planar(1); (2)co-linear(2); (3)non-adjacent(0).

Example: Changes of topological structure during folding are shown in Figure 12. The main steps of the process from the state of Figure 12(a) to Figure 12(e) are as follows.

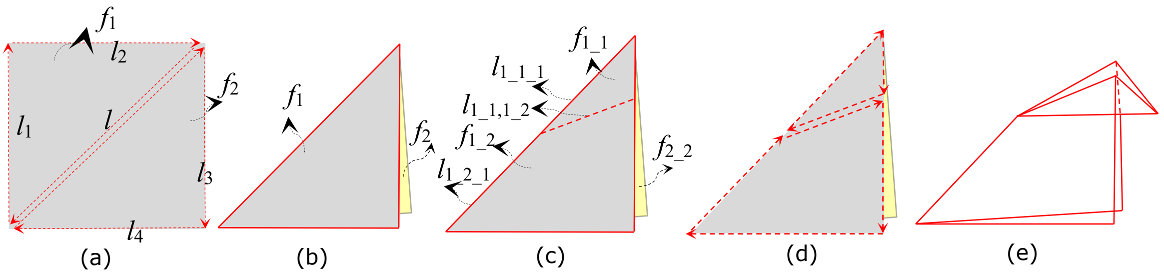


Figure 12: Process of topological structure change.

Step 1: Increase the crease l, as shown in Figure 12(a), dividing the plane f into f_1 and f_2, co-planar: (f_1, f_2, co-planar) = ($f_1(l_1, l_2, l), f_2(l_3, l_4, l)$, co-planar).

Step 2: Fold along the crease, as shown in Figure (b), f_1 and f_2 are co-linear: (f_1, f_2, co-linear) = ($f_1(l_1, l_2, l), f_2(l_3, l_4, l)$, co-linear).

Step 3: Increase the crease $l_{1\_1,1\_2}$, as shown in Figure (c), f_{1_1} is co-planar with f_{1_2}: (f_{1_1}, f_{1_2}, co-planar)=(f_{1_1} ($l_{1\_1\_1}, l_{1\_1\_2}, l_{1\_1,1\_2}$)$, f_{1\_2}$($l_{1\_2\_1}, l_{1\_2\_2}, l_{1\_2\_3}, l_{1\_1,1\_2}$), co-planar) , and the other side is similar.

Step 4: Fold along the crease, as shown in Figure (d,e), f_{1_1} is co-linear with f_{1_2}: (f_{1_1}, f_{1_2}, co-linear)=(f_{1_1} ($l_{1\_1\_1}, l_{1\_1\_2}, l_{1\_1,1\_2}$)$, f_{1\_2}$($l_{1\_2\_1}, l_{1\_2\_2}, l_{1\_2\_3}, l_{1\_2,1\_1}$), co-linear) , and the other side is similar.

4 OPTIMIZATION OF PAPER FOLDING BASED ON DAG MODEL

Considering the similarity and reusability of the folding process, this paper uses an adjacency matrix to represent the relationships between point-line-surface geometric elements, topological structures and constraints. Combined with the DAG structure to represent the folding state and behaviors, the design and optimization of folding process is implemented.

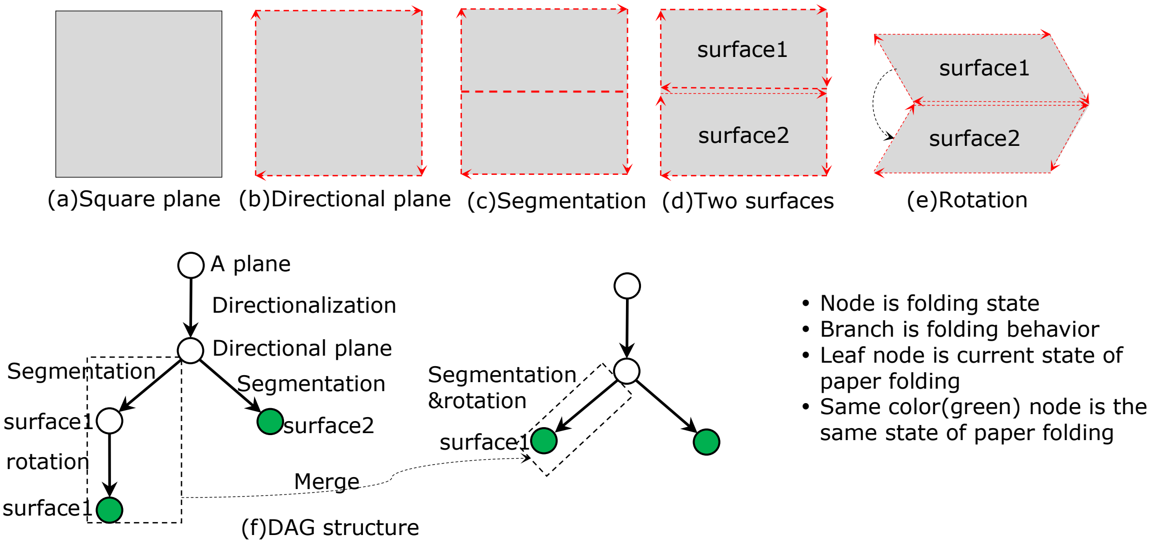


Figure 13: Paper folding with DAG structure and operations merging.

In addition, a folding library has been defined. On the one hand, it can support the process decomposition and constraint decomposition for a new type of folding, and it can be expanded; on the other hand, it can validate the reasonableness of the new folding process through the existing types in the library. This paper combines the DAG structure and the adjacency matrix to realize the verification and optimization design of the folding process. It mainly involves the following three aspects.

(1) Folding representation of the DAG structure: taking the plane as the basic unit, the folding process mainly includes folding states and folding behaviors, with the node representing the folding state and the branch representing the folding behavior, as shown in Figure 13.

(2) The storage of the adjacency matrix: with topological structure, it is to store the surface-line geometry elements, the intra-surface element association relations, and the inter-surface association relations. For example, the first step of folding, as shown in Figure 12(a), has an adjacency matrix stored as follows:

$$\begin{matrix} f \\ l_1 \\ l_2 \\ l_3 \\ l_4 \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ & 0 & 4 & 0 & 1 \\ & & 0 & 4 & 1 \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix} => \begin{matrix} f_1 \\ l_{1\_1} \\ l_{1\_2} \\ l_{1,2} \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ & 0 & 4 & 1 \\ & & 0 & 1 \\ & & & 0 \end{bmatrix} + \begin{matrix} f_2 \\ l_{2\_1} \\ l_{2\_2} \\ l_{2,1} \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ & 0 & 4 & 1 \\ & & 0 & 1 \\ & & & 0 \end{bmatrix} + \begin{matrix} f_1 \\ f_2 \\ l_{1,2} \end{matrix}\begin{bmatrix} 0 & 4 & 1 \\ & 0 & 4 \\ & & 0 \end{bmatrix} \qquad (4.1)$$

(3) Folding optimization design: the types of folding operations in the folding library are divided into basic operations and combined operations. Combined operations are generated from multiple basic operations. These operations that have already been validated are stored into the library. Each new operation will not only be searched from the library, but also be validated whether it is reasonable. The optimization operations mainly involve similar folding steps, reasonable folding steps and fewer folding steps. Similar folding steps mainly include repetitive operations, symmetric operations, and their common characteristics are that the operation is made up of the basic operations which are the same or similar; reasonable folding steps mean that the rationality of every step can be validated via existing folding lib; fewer folding steps imply that folding steps can be optimized to fewer steps based on the folding traces.

For example, the symmetric operation is shown in Figure 14. First, Figure 14(a) shows a square flat segmented into two same triangles f_1 and f_2; f_1 is rotated by angle π. Second, f_1 is segmented into f_{1_1}, f_{1_2} and f_{1_3}; f_{1_1} and f_{1_2} are respectively rotated by angle π, as shown in Figure 14(b,c). Last, f_{1_1}, f_{1_2}, f_{1_3} on f_1 are similar to f_{2_1}, f_{2_2} and f_{2_3} on f_2, their operation is symmetric, as shown in Figure 14(d).

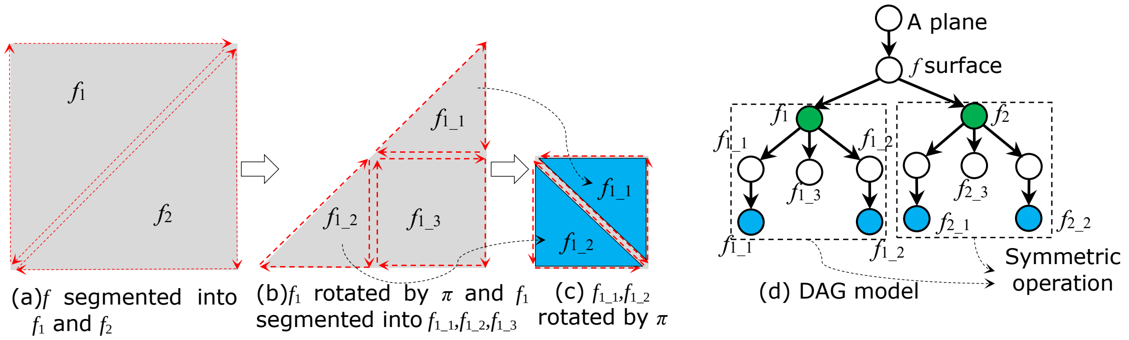


Figure 14: Symmetry operation of paper folding and its DAG structure.

5 EXPERIMENT AND ANALYSIS

The system was designed and realized, combined with 3-D engine Unity3D platform [21], and several typical examples of folding were simulated and tested. The computer configuration was 2.80 GHz Intel(R) Xeon(R) CPU, memory 32 GB, and Win 10 operating system.

The first example is to fold a paper boat and to optimize design with DAG model. The folding process is used to verify the reasonableness of the nodes and branches with DAG structure which represent the folding states and behaviors, respectively. The root node is the initial state of the paper, and the leaf nodes represent the current state of active surfaces, as shown in Figure 15(j). Firstly, Figure 15(b) is generated through line-oriented average folding from Figure 15(a); secondly, f_1 is segmented into 6 regions, 2 is rotated into 4 to generate 2’, and 3 is rotated towards the opposite direction, as shown in Figure 15(b,c); then 6 and 7 are folded and rotated, as shown in Figure 15(c,d), and other steps are similar. Figure 15(j) illustrates the process of paper folding from Figure 15(a) to Figure 15(e); the green nodes represent frontal states every step.

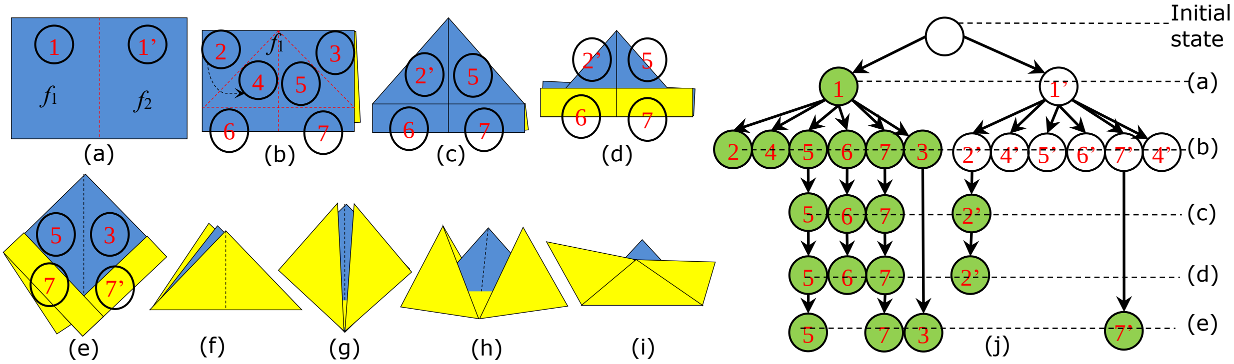
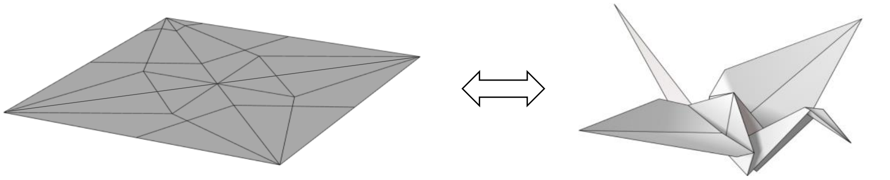


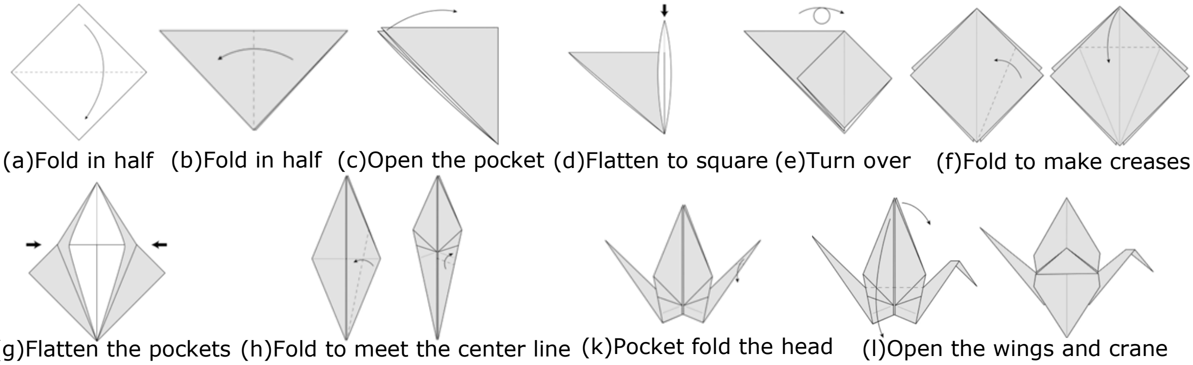
Figure 15: The process of paper boat folding and its DAG structure changing.

The second example is to fold a paper crane. To verify the folding design from 2D to 3D, the correlation is built between 2D creases and 3D physical objects, through the creases of paper crane and their folding process, as shown in Figure 16(a,b). There are a few advantages based on digital origami: (1) it shows that numerical simulation can provide an accurate prediction of the intermediate folding states and behaviors. (2) it can tell user how to reasonably fold at every stage, especially at a critical stage. The main folding stages are shown in Figure 17.



(a)A square paper containing 2D creases (b)A 3D crane of origami

Figure 16: A crane paper folding from two to three dimensions.



(a)Fold in half (b)Fold in half (c)Open the pocket (d)Flatten to square (e)Turn over (f)Fold to make creases

(g)Flatten the pockets (h)Fold to meet the center line (k)Pocket fold the head (l)Open the wings and crane finished

Figure 17: The simulation of folding process of a paper crane.

6 CONCLUSION

In this paper, with DAG model and directional plane, we propose a digital folding representation and optimal design method for the complex paper folding. The main work is as follows.

- Digital representation simulates the folding process by using the point-line-surface elements.
- The nodes and branches of DAG structure are used to analyze and represent the states and behaviors of paper folding process.
- Expandable folding library is to verify the reasonableness and feasibility of the folding process.

Some methods of folding need to be further improved. One of them is to perform the cutting function on the paper to increase the positive and negative feature operation. Another concerns how to merge multiple folding papers together.

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