Adaptive Assembly Adjustment Strategy for Optical Systems Based on Digital Twin

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Abstract. With the increasing demand for higher quality of complex and precision mechanical products, assembly has become a critical process in the manufacturing industry. Coaxial optical systems, as a typical example of complex products, still heavily rely on manual adjustment during assembly, with the quality highly dependent on operator experience. This process is characterized by significant uncertainty. To address this issue, a feasible solution is proposed using digital twin technology. In this paper, a digital twin-driven adaptive assembly control strategy for coaxial optical systems is developed, utilizing a fuzzy control algorithm. Firstly, the impact of lens's pose on imaging quality in the assembly process of coaxial optical systems is analyzed. Secondly, a fuzzy mapping relationship between lens's pose and imaging quality is established, upon which a control strategy for lens's pose adjustment is developed. Finally, a prototype adjustable and measurable assembly system is implemented to validate the effectiveness of the proposed method. The results demonstrate that the control strategy can achieve adaptive pose control for coaxial optical system lenses and can be successfully integrated into the digital twin framework, facilitating the wider application of digital twin technology.

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1 INTRODUCTION

With the escalating complexity of mechanical products and the ever-increasing performance demands, traditional assembly approaches are ill-suited to modern requirements[17]. This holds especially true in the assembly of optical systems, where even the most negligible variations in the positioning of lenses can critically affect the system's overall performance. Such minute discrepancies pose substantial challenges to the quality control of conventional manual assembly techniques. Furthermore, the inherent uncertainty of these traditional methods conflicts with the objectives of high efficiency and superior production quality, compelling the industry to seek innovative technologies to enhance assembly precision[14].

The advent of Digital Twin (DT) technology presents a revolutionary strategy for mitigating the randomness and uncertainty inherent in assembly processes while also enhancing efficiency[11]. It accomplishes this by employing advanced techniques like real-time testing, dynamic modeling, and optimized control for an exhaustive modeling of the assembly process. Furthermore, Digital Twin constructs accurate virtual counterparts of physical products and tools, enabling the simulation and prediction of product behavior in future assembly stages, which informs further optimization efforts[12]. In recent years, several scholars have investigated the application of DT technology in the field of assembly. Qi et al.[8] explore the integration of big data and digital twin technologies by contrasting their similarities and differences at the data level. They propose a complementary synthesis of the two technologies, presenting a solution to the critical challenge of cyber-physical integration in the manufacturing process. Sun et al.[10] address the issues of low efficiency and poor consistency in the assembly and commissioning of high-precision products (HPPs). They propose an assembly-commissioning method based on DT technology by constructing a comprehensive information model. In the study by Kamil et al.[19], the assembly process of the Bowden holder in 3D printers was investigated. Modeling was performed using CAD software, followed by simulations on the Tecnomatix platform. Subsequently, data interaction between the virtual and physical environments was achieved through the integration of PLC and OPC server, which has broken down barriers between virtual and physical domains, facilitating the potential integration of digital twin technology with industrial systems. Negri et al.[7] proposed integrating simulation models with the Manufacturing Execution System (MES), thereby realizing a genuine Digital Twin for the manufacturing process. This approach addresses the shortfall of traditional DT in exercising control over physical equipment systems.

DT assembly combines the benefits of virtual and semi-physical assembly methods[16]. Despite this, most research to date has concentrated on managing and optimizing the assembly process equipment and data, with a lack of systematic study on product performance and insufficient integration of issues related to assembly itself. This paper outlines the development of a fuzzy control strategy within the digital twin framework to calibrate lens positioning in coaxial optical systems. This strategy enhances the assembly process's accuracy and facilitates the practical application of digital twin technology in complex manufacturing operations.

During the assembly process of a coaxial optical system, the analysis of assembly errors becomes extremely challenging due to the presence of errors in the lenses' surface shape, inherent system errors in the fixtures, and coupling effects between various components. These factors contribute to the random and nonlinear characteristics of deviation analysis. It is challenging to design a control system for a nonlinear system with uncertainties[5]. Fuzzy controllers can integrate sensor-detected data with human experience, and by selecting the parameters of the fuzzy controller, it is always possible to design a fuzzy controller suitable for nonlinear systems[4]. Therefore, fuzzy controllers can be used to realize the smart control of the coaxial optical system assembly. In the term of Assembly Line Balancing Problem (ALBP), Zacharia et al.[15] have developed an enhanced SALBP-2 model that accounts for the uncertain nature of operation process times through the integration of fuzzy logic. The model delineates a holistic fuzzy cost function predicated on dual fuzzy objectives, with the dual goals of enhancing the production line's efficiency and maintaining its equilibrium. Tsourveloudis et al.[13] developed three fuzzy control modules that balance the workflow by adjusting the processing rate at each stage of production. These modules are designed to manage production line networks for both single or multiple types of parts, ensuring that product inventory is maintained at low levels while achieving high machine utilization rates. Huo et al.[1] have developed a fuzzy control system in the context of the Industry 4.0 era, which utilizes two fuzzy controllers for real-time analysis of assembly line information, effectively reducing the congestion and starvation rates of the system while enhancing buffering efficiency. Li et al.[3] have developed a strategy for adaptive force and end-effector posture control. This strategy involves the analysis and modeling of contact force dynamics, which is followed by employing a fuzzy-PID control technique. The technique finely tunes both the translational and angular movements throughout the terminal assembly procedure, thus facilitating the automation of flexible cable assembly.

To summarize, there is a necessity for additional investigation into the incorporation of digital twin technology within the domain of coaxial optical system assembly. In this paper, we have developed a digital twin model for the assembly of the coaxial optical system. This model facilitates the analysis of the relationship between system performance parameters and the pose of lenses. Furthermore, an innovative fuzzy control strategy has been proposed for the precise adjustment of the lenses' pose, ensuring rapid alignment in response to feedback to achieve the target position.

The rest of this paper is organized as follows: Sec. 2 introduces the assembly digital twin framework; In Sec. 3, the development of a fuzzy control strategy for lens positioning within the digital twin framework is introduced; Sec. 4 reports on the assembly experiment and corresponding results for a coaxial optical system; Conclusions is shown in.Sec. 4.

2 ASSEMBLY DIGITAL TWIN FRAMEWORK FOR OPTICAL SYSTEM

Assembly digital twin is a technology that integrates the physical, data, and virtual spaces of the whole assembly process to enhance its quality and efficiency[18]. In order to apply digital twin technology to the assembly process of coaxial optical systems, it is necessary to construct a digital twin framework for assembly, as shown in Fig. 1.

Aligning an optical system is a multifaceted and intricate procedure, influenced by a myriad of factors, including the positioning, orientation, contour, fastening force of bolts, as well as ambient temperature and humidity. These factors are classified into macroscopic and microscopic elements based on their level of impact on system performance. The macroscopic factors pertain to the pose of each lens throughout assembly, whereas microscopic factors involve the impact of the bolts' fastening force on the contour of the lens surface. This study focuses on the macroscopic factors affecting the assembly of optical systems and incorporates the assembly digital twin framework into the assembly process.

In assembly digital twin framework, the physical space of the assembly process includes the objects and their environment, encompassing the five-axis adjustment fixture, the optical system with its constituent lenses, and the laser interferometer. The performance of optical system depends on the poses of its lenses. In the assembly process, the five-axis adjustment fixture is used to accurately control the lens's pose along the X, Y, Z axes and around the X, Y axes. Meanwhile, the laser interferometer serves as an instrument to measure the wavefront of the optical system across various lens pose, employing metrics such as Peak-to-Valley (PV) and Root Mean Square (RMS).

A key feature of digital twin assembly is the ability to analyze and make decisions based on actual data during the assembly process, which is completed in the virtual space. The effect of the lenses pose on its optical performance is analyzed, and the performance relationship of the optical system is obtained in this model.

The data interchange space is used for filtering and transmitting data during the assembly process. The Open Platform Communications Unified Architecture (OPC UA) is an open, cross-platform, secure, reliable and efficient communication protocol that can enable the information exchange and communication between the physical and virtual spaces. We use the OPC UA gateway to upload the pose and the wavefront aberration data of the optical system from the physical space to the virtual space, and send the control instructions from

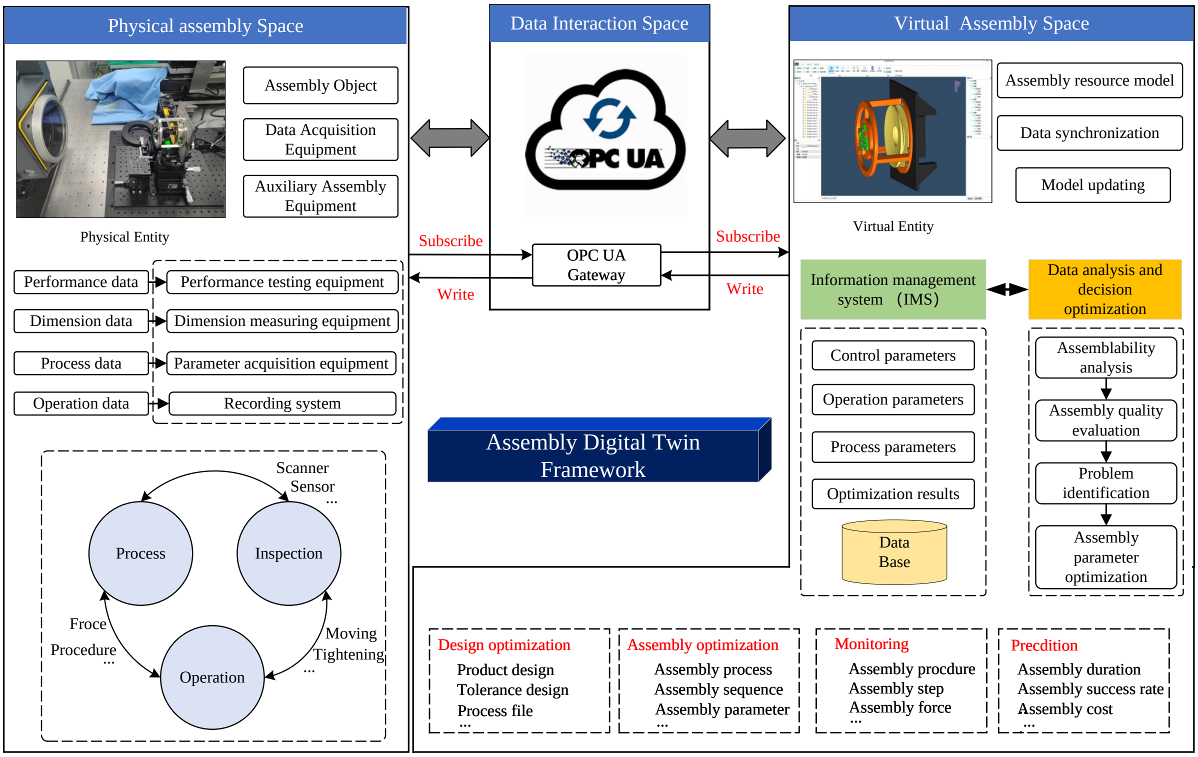


Figure 1: Assembly digital twin framework

the virtual space to the physical space, achieving the function of the data interaction space.

At this point, the construction of the digital twin framework for the coaxial optical system assembly is completed. Leveraging this framework, a fuzzy control algorithm based alignment strategy is designed, which handles uncertainty with fuzzy logic, achieves the closed-loop control of error with a controller, and realizes the adaptive attitude control of the optical system.

3 ADAPTIVE ASSEMBLY OF COAXIAL OPTICAL SYSTEM

Fuzzy system is a control method for complex and nonlinear models that can handle the uncertainty and fuzziness of the models[2]. It is widely applied in multi-axis control applications such as robotics. In the assembly process of coaxial optical system, it is essential to ensure that each lens reaches a relatively ideal assembly pose, mainly to maintain the optical axis consistency of each lens. However, machining errors cause the alignment parameters of each lens to be inconsistent. Therefore, the relationship between the performance parameters of the optical system and the pose of the lens needs to be analyzed to quickly achieve the ideal assembly pose of the lens.

In this paper, the assembly performance of the coaxial optical system is measured by a laser interferometer, and expressed by the PV and RMS of the wavefront aberration. The wavefront aberration of the lens surface is represented by the weighted superposition of Zernike polynomial, which are a set of orthogonal polynomials that are complete in the unit circle[9]. The wavefront aberration of any field point can be expanded as:

$$W(\rho,\phi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}C_n^m Z_n^m(\rho,\phi). \quad (1)$$

where $W(\rho,\phi)$ represents the aberration of the field point, ρ and ϕ are the radial and angular coordinates in pupil coordinate system, C_n^m are the Zernike polynomial coefficients, and $Z_n^m(\rho,\phi)$ are the Zernike polynomials, defined as:

$$Z_n^m(\rho,\phi)=\begin{cases} N_n^m R_n^{|m|}(\rho)\cos(m\phi), m\geq 0 \\ -N_n^m R_n^{|m|}(\rho)\cos(m\phi), m<0 \end{cases}. \quad (2)$$

where N_n^m is the normalization factor described in more detail below and $R_n^{|m|}(\rho)$ is given by:

$$R_n^{|m|}(\rho)=\sum_{s=0}^{(n-|m|)/2}\frac{(-1)^s(n-s)!}{s!\,[0.5(n+|m|)-s]!\,[0.5(n-|m|)-s]!}\times\rho^{n-2s}. \quad (3)$$

Different terms of the Zernike polynomials correspond to different types of wavefront aberrations. During the precise calibration of lens pose, focus should be confined to the critical 7-9 parameters that are relevant to the pose. The 7th ($(3\rho^2-2)\rho\cos\phi$) and 8th ($(3\rho^2-2)\rho\sin\phi$) terms represent the X and Y directions of the third-order coma, respectively, while the 9th ($6\rho^4-6\rho^2+1$) term pertains to third-order spherical. The $X-coma$ is caused by misalignment due to displacement along the X axis and rotation around the Y axis, while the $Y-coma$ results from displacement along the Y axis and rotation around the X axis. Spherical aberration is triggered by misalignment in displacement along the Z axis. Therefore, calibrating the pose of lens using the five-axis adjustment fixture, corrections for the X axis and Y axis displacements and rotations are based on the respective comas, whereas Z axis displacement adjustments is informed by the spherical aberration. A fuzzy system is used in this paper to analyze the Zernike polynomial coefficients in real time, and generate control commands according to the coma and spherical values, adjust the lens pose, minimize the aberrations, and improve the optical system assembly quality and efficiency.

The control strategy aims to decrease the error and keep the alignment amount of the optical system within the desired range. The adjustment range of each axis was determined by previous experiments and experience, with the upper limit F_{dU} and the lower limit F_{dL}, to ensure that the lenses can form images during the alignment process. As shown in Fig. 2, a fuzzy controller is designed based on the above analysis, which takes the 7th ($X-coma$), 8th ($Y-coma$) and 9th ($spherical$) terms of the Zernike polynomials coefficients as the input, and the adjustment amount of each axis of the five-axis adjustment fixture as the output. Both input and output variables are quantified with the universes, with the domain ranges defined as $[F_{dL},F_{dU}]$ for each variable. The five-axis adjustment fixture and its coordinate system of each axis are shown in Fig. 3.

In the fuzzification process, the input and output data sets are converted into fuzzy data sets through the fuzzy membership functions. Fig. 4 shows the membership function of each input and output parameters. Then, a rule matrix base with several control rules is established based on the experience and logic of the human operators to reduce the error value to a reasonable range quickly. A large adjustment amplitude is used when the error value is large; a small adjustment amplitude is used when the error value is small, to avoid overshooting. The stability of the control system is also considered, to ensure the effect and safety of the attitude adjustment. A rule matrix of the five-axis adjustment fixture parameters is used to represent these rules, as shown in Table 1. For the first fuzzy control rule, when both input variables $X-coma$ and $Y-coma$ are classified as negative big (NB) and the variable $spherical$ is classified as negative (N), the resultant output variables are as follows: X-T and Y-R should be set to positive big (PB), Y-T to PB, X-R to NB, and Z-T to N.

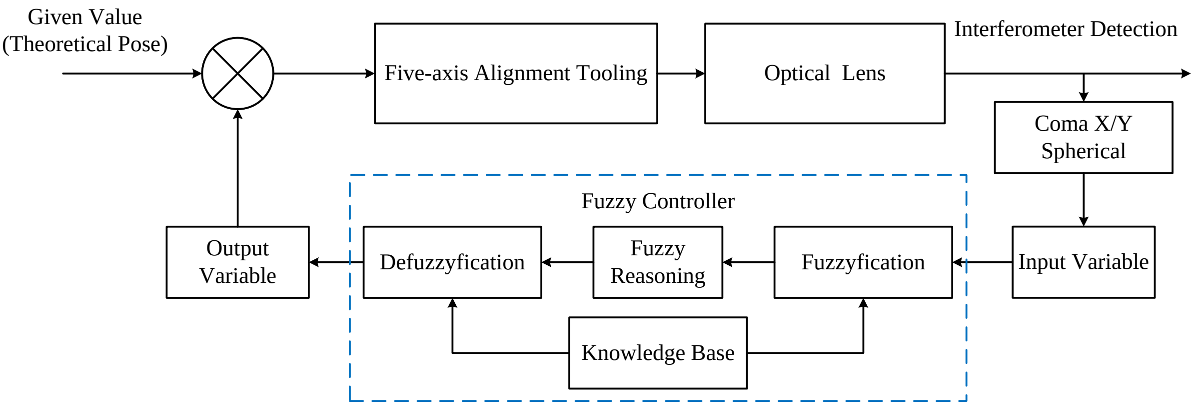


Figure 2: Five-axis fuzzy controller

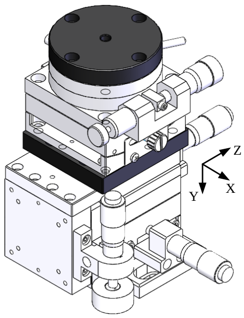
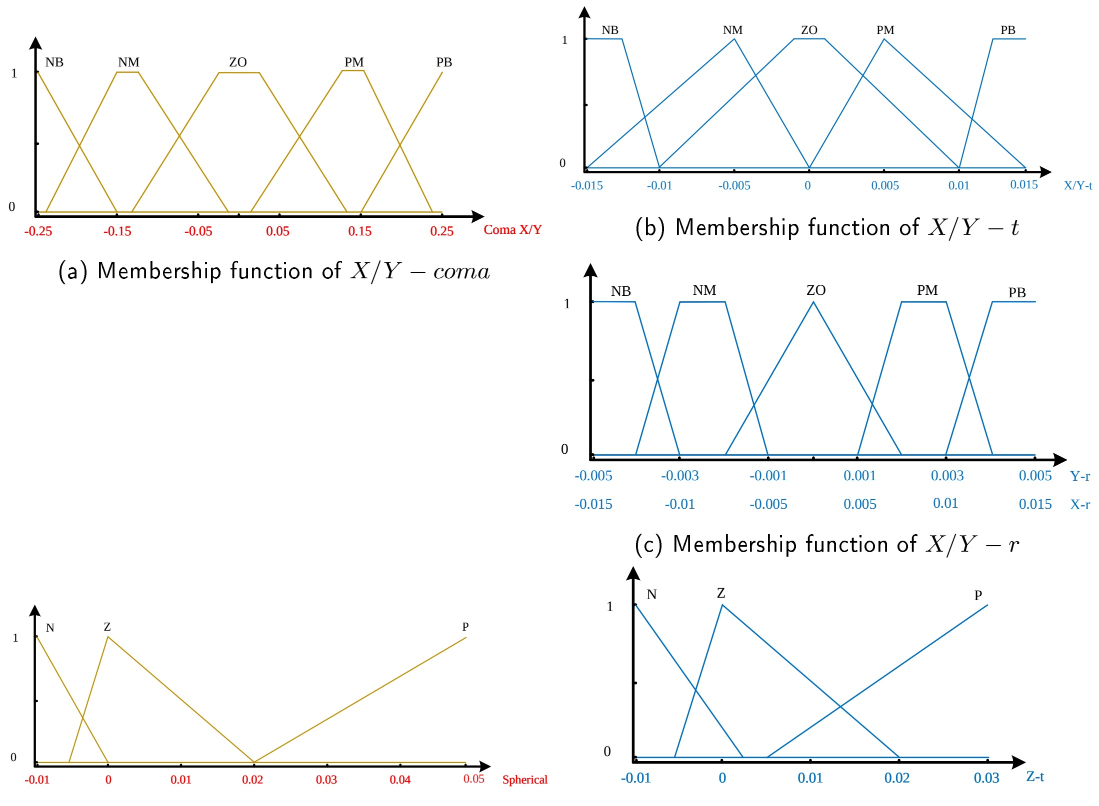


Figure 3: Five-axis adjustment fixture

There are 75 rules for the five-axis adjustment fixture fuzzy controller, and the control rules are constructed by the Mamdani Reasoning Method. After the completion of fuzzy reasoning, the resulting output consists of a set of fuzzy membership values. Fuzziness facilitates the expression of experience; however, the final output is typically expected to be a definite value. Therefore, all outputs in this paper undergo defuzzification using the center of gravity method[6], which is expressed as Equation (4).

$$y = \frac{\sum_{r=1}^{R_l} A^{\alpha r} C_{A^{\alpha r}}}{\sum_{r=1}^{R_l} A^{\alpha r}}. \tag{4}$$

where $A^{\alpha r}$ represents the area of the consequent's fuzzy subset acquired through the α-level membership as dictated by the r^{th} rule. The term $C_{A^{\alpha r}}$ denotes the centroid of the area $A^{\alpha r}$, while R_l signifies the total count of fuzzy rules employed.



(a) Membership function of $X/Y - coma$

(b) Membership function of $X/Y - t$

(c) Membership function of $X/Y - r$

(d) Membership function of $spherical$

(e) Membership function of $Z - t$

Figure 4: Membership functions of input and output parameters

Input			Output				
$X - coma$	$Y - coma$	$spherical$	X-T	Y-R	Y-T	X-R	Z-T
NB/PB	NB/PB		PB/NB	PB/NB	PB/NB	NB/PB	
NB/PB	NM/PM		PB/NB	PB/NB	PM/NM	NM/PM	
NB/PB	ZO		PB/NB	PB/NB	ZO	ZO	
NM/PM	NB/PB		PM/NM	PM/NM	PB/NB	NB/PB	
NM/PM	NM/PM	N/Z/P	PM/NM	PM/NM	PM/NM	NM/PM	N/Z/P
NM/PM	ZO		PM/NM	PM/NM	ZO	ZO	
ZO	NB/PB		ZO	ZO	PB/NB	NB/PB	
ZO	NM/PM		ZO	ZO	PM/NM	NM/PM	
ZO	ZO		ZO	ZO	ZO	ZO	

Table 1: Rule matrix for fuzzy controller

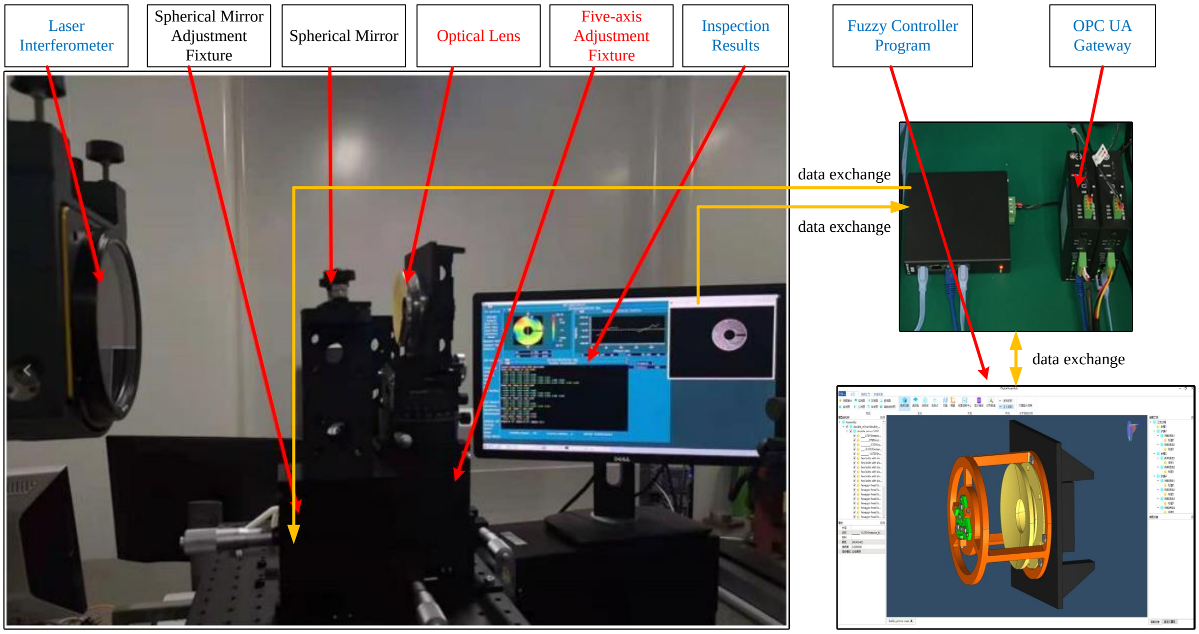


Figure 5: Optical lens assembly prototype system

4 A CASE STUDY

In the optical system assembly process, a prototype system utilizing fuzzy control was developed for the precise positioning of lenses, as shown in Fig. 5. This system includes a laser interferometer as a detection unit, adjustable lenses, a five-axis adjustment fixture, and a spherical mirror with corresponding fixture to form the optical system inspection path. To ensure optical system performance, the optical axis of the lens and spherical mirror must be co-axialized, ensuring the alignment of their focal points. Conventional methods, which depend on interferometric data and empirical adjustments, are significantly influenced by uncontrollable variables, compromising assembly efficiency. Leveraging the digital twin framework for assembly, the detection data from the optical system is channeled through the OPC UA gateway and processed by the fuzzy controller to compute the lens misalignments. Then, the necessary adjustment parameters for the respective axes of the five-axis adjustment fixture are derived.

This study conducted experiments on a prototype alignment system to verify the effectiveness of the proposed method. The initial step of the experiment was to position the lenses based on the design parameters of the optical system, ensuring the integrity of the optical path so that the laser interferometer could detect the image of the optical system. Subsequent adjustments were made according to the detection results. It should be noted that due to the presence of micro errors (such as the influence of bolt pre-tightening forces on the surface shape of the lens) and processing errors of lens, the adjustment of the lens pose will only reduce rather than completely eliminate coma and spherical aberrations. Therefore, it's sufficient to adjust these errors within an acceptable range. In this study, the target range for controlling coma was set between $[-0.05, 0.05]$, and for spherical aberration, between $[-0.01, 0.01]$.

Subsequently, it is essential to define the universe for the input and output variables, along with their respective linguistic value sets. The input variables encompass the wavefront aberration parameters $X-coma$, $Y-coma$ and $spherical$, where the universe of discourse for $X-coma$ and $Y-coma$ are expressed as $I=$

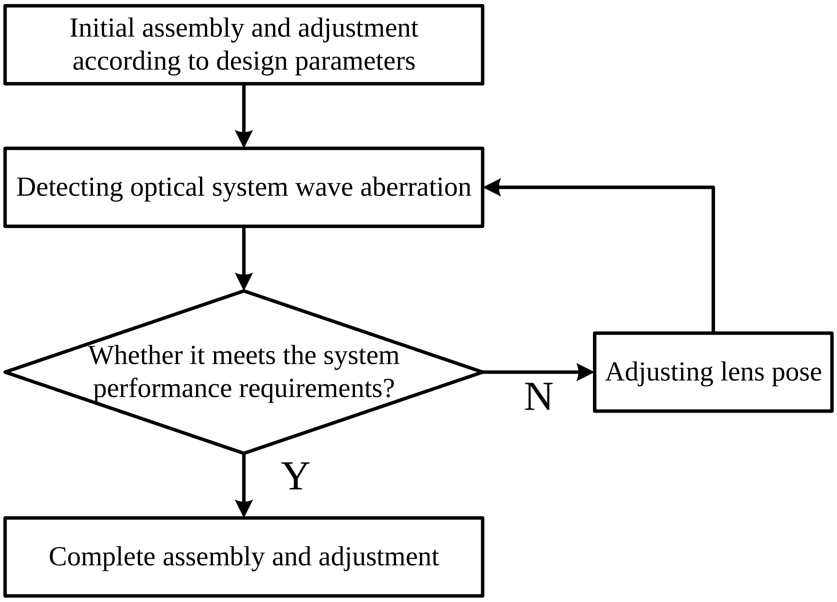


Figure 6: Fuzzy control assembly experimental process

$[-0.25, 0.25]$, corresponding to the values in the linguistic value set, $L = \{NB, NM, ZO, PM, PB\}$, denoting the values of $X-coma$ and $Y-coma$ as NB (negative big), NM (negative middle), ZO (zero), PM (positive middle), and PB (positive big). The universe of discourse for $spherical$ is denoted as $I = [-0.01, 0.05]$, and corresponding linguistic value set is $L = \{N, Z, P\}$. The output variables are the adjustment amount of each axis in the five-axis adjustment fixture, with their universe defining the maximum adjustment that the optical system can undergo for interferometric detection. Specifically, the universe of discourse for the translation along the X and Y axes are both $I = [-0.015, 0.015]$, the universe of discourse the rotation around the X and Y axes are $I = [-0.015, 0.015]$ and $I = [-0.005, 0.015]$ respectively, corresponding linguistic variables are also L. The universe of discourse for the translation along the Z axis is $I = [-0.01, 0.03]$, its corresponding linguistic value set is same as $spherical$. It is important to note that the input consists of Zernike Fringe coefficients, which are unitless, while the output represents the adjustment amounts for each axis, with units in millimeters(mm).

The alignment experiment was conducted and the experimental process is shown in the Fig. 6.Initially, a preliminary installation is performed according to the optical system design parameters. Following this, wavefront aberrations detection is conducted to determine whether the system performance meet the requirements. In the event of nonconformity, the lens alignments are iteratively adjusted in accordance with the aberration analysis findings, ensuring that all aberration metrics satisfy the predefined standards. Throughout this study, we have executed three distinct experiments and the results of the detection and adjustment are recorded in Table 2.

The results indicate that, in the state of initial misalignment, a mere two adjustments suffice to bring the wavefront aberration within the acceptable ranges specified for all three experiments. For example, consider the first experiment, where the initial coefficients of the $w7$, $w8$ and $w9$ wavefront aberration coefficients were measured at 0.121, 0.168, and 0.01, respectively. Following the adjustments, these coefficients were reduced to -0.012, 0.001, and -0.003, respectively. Its demonstrate that the fuzzy control strategy proposed

Num.	Original wavefront aberration			Adjustment parameters					Adjusted wavefront aberration		
	$w7$	$w8$	$w9$	X-T	Y-R	Y-T	X-R	Z-T	$w7$	$w8$	$w9$
1	0.121	0.168	0.01	-0.006	-0.002	-0.008	0.008	0.005	-0.073	-0.089	-0.003
	-0.073	-0.089	-0.003	0.001	0.001	0.002	-0.001	0.000	-0.012	0.001	-0.003
2	-0.115	-0.212	-0.005	0.004	0.002	0.011	-0.010	-0.005	-0.033	0.057	0.001
	-0.033	0.057	0.001	0.000	0.000	-0.003	0.002	0.000	-0.043	0.001	0.001
3	-0.186	0.084	0.012	0.007	0.003	-0.006	0.005	0.006	-0.045	-0.054	0.002
	-0.045	-0.054	0.002	0.000	0.000	0.003	0.002	0.000	-0.027	0.012	0.002

Table 2: The experimental result

in this paper can quickly adjust the lens's pose towards an optimal state. Notably, in case 2 and 3, under the premise that the $X - coma$ meets the set threshold, the amount of movement along the X-axis and the amount of rotation around the Y-axis were zero during the new adjustment process. However, the value of the $X - coma$ changed to a certain degree, these variations arise primarily from the computational process of wavefront aberrations, which relies on a specific set of sampling points. Such variations do not significantly alter the overall results.

5 CONCLUSIONS

This paper proposes a method based on fuzzy control algorithms to enhance the efficiency and automation of the alignment process for co-axial optical systems. The method analyzes interference images to establish a mapping relationship between wavefront aberration coefficients and adjustment values. A fuzzy control system is then developed based on this mapping relationship. This system acts as a computational module and can be integrated into the digital twin framework for assembly, facilitating the rapid adjustment of control model parameters and the deployment of the method for various types of optical systems.

There are some limitations in this study, although a highly practical fuzzy control strategy is designed, it currently only considers coaxial optical systems. Research on a more diverse range of optical systems will be conducted in the next phase. Additionally, the impact of bolt preload on the shape of lens surfaces was not adequately addressed in this study and will be thoroughly investigated in future work.

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REFERENCES

[1] Huo, J.; Chan, F.T.; Lee, C.K.; Strandhagen, J.O.; Niu, B.: Smart control of the assembly process with a fuzzy control system in the context of industry 4.0. Advanced Engineering Informatics, 43, 101031, 2020. ISSN 1474-0346. http://doi.org/https://doi.org/10.1016/j.aei.2019.101031.

[2] Lee, H.; Park, J.: Contact states estimation algorithm using fuzzy logic in peg-in-hole assembly. In 2020 17th International Conference on Ubiquitous Robots (UR), 355–361. IEEE, 2020.

[3] Li, J.R.; Wang, Z.J.; Wang, Y.Y.; Wang, Q.H.: An adaptive force and posture control strategy for automated wiring terminal assembly. International Journal of Computer Integrated Manufacturing, 35(8), 873–889, 2022.

[4] Li, Y.; Zhuang, X.; Qiang, S.; Liu, G.; Mei, X.: Stable fuzzy adaptive control for a class of nonlinear system. In Proceedings of the 4th World Congress on Intelligent Control and Automation (Cat. No.02EX527), vol. 3, 1891–1895 vol.3, 2002. http://doi.org/10.1109/WCICA.2002.1021412.

[5] Liu, W.; Lim, C.C.; Shi, P.; Xu, S.: Backstepping fuzzy adaptive control for a class of quantized nonlinear systems. IEEE Transactions on Fuzzy Systems, 25(5), 1090–1101, 2017. http://doi.org/10.1109/TFUZZ.2016.2598360.

[6] Nakandala, D.; Samaranayake, P.; Lau, H.C.: A fuzzy-based decision support model for monitoring on-time delivery performance: A textile industry case study. European journal of operational research, 225(3), 507–517, 2013.

[7] Negri, E.; Berardi, S.; Fumagalli, L.; Macchi, M.: Mes-integrated digital twin frameworks. Journal of Manufacturing Systems, 56, 58–71, 2020. ISSN 0278-6125. http://doi.org/https://doi.org/10.1016/j.jmsy.2020.05.007.

[8] Qi, Q.; Tao, F.: Digital twin and big data towards smart manufacturing and industry 4.0: 360 degree comparison. IEEE Access, 6, 3585–3593, 2018. http://doi.org/10.1109/ACCESS.2018.2793265.

[9] Schwiegerling, J.: Scaling zernike expansion coefficients to different pupil sizes. JOSA A, 19(10), 1937–1945, 2002.

[10] Sun, X.; Bao, J.; Li, J.; Zhang, Y.; Liu, S.; Zhou, B.: A digital twin-driven approach for the assembly-commissioning of high precision products. Robotics and Computer-Integrated Manufacturing, 61, 101839, 2020. ISSN 0736-5845. http://doi.org/https://doi.org/10.1016/j.rcim.2019.101839.

[11] Tao, F.; Xiao, B.; Qi, Q.; Cheng, J.; Ji, P.: Digital twin modeling. Journal of Manufacturing Systems, 64, 372–389, 2022. ISSN 0278-6125. http://doi.org/https://doi.org/10.1016/j.jmsy.2022.06.015.

[12] Tao, F.; Zhang, H.; Liu, A.; Nee, A.Y.C.: Digital twin in industry: State-of-the-art. IEEE Transactions on Industrial Informatics, 15(4), 2405–2415, 2019. http://doi.org/10.1109/TII.2018.2873186.

[13] Tsourveloudis, N.; Dretoulakis, E.; Ioannidis, S.: Fuzzy work-in-process inventory control of unreliable manufacturing systems. Information Sciences, 127(1), 69–83, 2000. ISSN 0020-0255. http://doi.org/https://doi.org/10.1016/S0020-0255(00)00030-X.

[14] Yin, Y.; Zheng, P.; Li, C.; Wang, L.: A state-of-the-art survey on augmented reality-assisted digital twin for futuristic human-centric industry transformation. Robotics and Computer-Integrated Manufacturing, 81, 102515, 2023.

[15] Zacharia, P.T.; Nearchou, A.C.: Multi-objective fuzzy assembly line balancing using genetic algorithms. Journal of Intelligent Manufacturing, 23, 615–627, 2012.

[16] Zhang, C.; Sun, Q.; Sun, W.; Shi, Z.; Mu, X.: Performance-oriented digital twin assembly of high-end equipment: a review. The International Journal of Advanced Manufacturing Technology, 126(11), 4723–4748, 2023.

[17] Zhang, X.; Yan, Q.; Yang, J.; Zhao, J.; Shen, Y.: An assembly tightness detection method for bolt-jointed rotor with wavelet energy entropy. Measurement, 136, 212–224, 2019. http://doi.org/https://doi.org/10.1016/j.measurement.2018.12.056.

[18] Zhuang, C.; Gong, J.; Liu, J.: Digital twin-based assembly data management and process traceability for complex products. Journal of Manufacturing Systems, 58, 118–131, 2021. ISSN 0278-6125. http://doi.org/https://doi.org/10.1016/j.jmsy.2020.05.011. Digital Twin towards Smart Manufacturing and Industry 4.0.

[19] Zidek, K.; Pitel, J.; Adamek, M.; Lazorik, P.; Hosovsky, A.: Digital twin of experimental smart manufacturing assembly system for industry 4.0 concept. Sustainability, 12(9), 2020. http://doi.org/10.3390/su12093658.